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Cooperation under Fear, Greed and Prison: the Role of Redistributive Inequality in the Evolution of Cooperation

César Mantilla¹²

Abstract

This work offers an analysis of cooperation dilemmas making emphasis in the role of the unequal outcomes. Increases in the benefit from leaving mutual cooperation are associated to the greed dimension, while increases in the cost from leaving mutual defection are associated to fear dimension. The manipulation of these dimensions allows defining two cooperation dilemmas derived from the standard Prisoner's Dilemma. Using two different frameworks, classical game theory and evolutionary game theory, is shown that the magnitude and the direction of these inequalities have an effect over the decision of cooperation.

Keywords: Prisoner's Dilemma - Payoff Inequality - Evolutionary Game Theory

JEL Classification: C72, C73, C91, D03, D30, D71, D74

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Cooperación bajo Miedo y Ambición: el Papel de la Desigualdad Redistributiva en la Evolución de la Cooperación

César Mantilla³⁴

Resumen

Este trabajo ofrece un análisis de los dilemas de cooperación haciendo énfasis en el papel de los *outcomes* desiguales. Aumentos en los beneficios de abandonar la cooperación mutua están asociados a la dimensión de ambición, mientras incrementos en el costo de abandonar la no cooperación mutua están asociados a la dimensión de miedo. La manipulación de este par de dimensiones permite definir dos dilemas de cooperación derivados del Dilema del Prisionero estándar. Utilizando dos marcos teóricos diferentes, teoría de juegos clásica y teoría de juegos evolutiva, se muestra que la magnitud y la dirección de la desigualdad tienen efecto sobre la decisión de cooperación.

Palabras Clave: Dilema del prisionero - Desigualdad de pagos - Teoría de juegos evolutiva

Códigos JEL: C72, C73, C91, D03, D30, D71, D74

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1. Introduction

The analysis of incentives in strategic interactions has been focused in the ranking of the attainable payoffs as the main decision criteria in the equilibrium prediction, neglecting the role of preferences over different levels of inequality in the analysis of incentives that define the behavior inside the strategic framework.

In this work I present how the potential redistributive inequality, measured as the difference between the payoffs obtained by the two players across different cooperation dilemmas, has an effect in the agents' decision to cooperate. Here I describe how it is possible to identify the effect of these differences, in magnitude and direction, using analytical tools from classical game theory (CGT) and evolutionary game theory (EGT).

Cooperation dilemmas share a key feature: Mutual cooperation will be always a better outcome for both players than mutual defection; however, a straightforward analysis shows that defection dominates cooperation. The incentive structure guarantees that if the counterpart will cooperate, defection guarantees a higher payoff; and if the counterpart will defect, defection minimizes the loss given the lack of cooperation of both parts.

The most common cooperation dilemma, which also is used as baseline in this work, is the well-known Prisoner's Dilemma (PD). Based on this game and its restrictions in terms of payoffs, the other couple of cooperation dilemmas relevant in this work will be derived. The restriction held in all games is called the "suboptimality condition" (Frohlich and Oppenheimer, 1996; Rapoport and Chammah, 1965), which guarantees that the ranking of the four attainable payoffs remains unaltered. The other two conditions describe the ranking of the outcomes in terms of social welfare. An outcome is more desired than other outcome in terms of social welfare if the sum of all players' payoff is higher. In the standard PD, the social welfare is maximized under mutual cooperation, and the mathematical form of this expression is known as the "externality larger than internality" condition (Frohlich and Oppenheimer, 1996; Rapoport and Chammah, 1965). Also, in the PD the social welfare is minimized under mutual defection. Despite that it involves another "externality larger than internality" condition, the expression that describes this condition is usually neglected.

It's not a coincidence that the definition of different cooperation dilemmas must be supported in the analysis of the asymmetric outcomes of the games, defined as the outcomes where a player defects and the other cooperates, leading to payoff differences. In PD, these outcomes are easily neglected because they represent the intermediate outcomes in terms of social welfare. If a considerable increase of the benefits received by the defector when the other player cooperates (or a similar increase in the cost for the player that cooperates despite their beliefs about its counterpart desire to defect) affects the ranking of the social outcomes, the potential redistributive inequality of the game must be incorporated in the analysis and determine its effect on individuals' decisions.

Alexis de Tocqueville, in his well-known *Democracy in America* (1835, 1840) describes a paradox around the tolerance for inequality: in democratic times, the thriving passion for equality leads to a higher inequality accepted by everyone. This passion is structured on individualism, a feeling that boosts the pursuit of personal wealth, the importance of public opinion in the perceptions and decisions of the economic agents and the increasing competition between subjects of the same social status as a substitute of the competition between social classes. The promulgation of equality, in relative terms, in democratic times strengthens individualism and makes competition fiercer among equals. This competition substitutes the comparison across different wealth levels, and upper levels of wealth become a benchmark describing the rewards obtained by hardworking. The wage, as a competitive result of the labor market, nullifies the perception of absolute inequality and neutralizes the revolutionary character of the labor's price (Hurtado, 2011). Working class individuals compare themselves with other workers of their same level, and class struggle is substituted by a benchmark of social mobility determined by the individuals' levels of effort. This favors common beliefs about the correlation between success and effort and, paradoxically, leads to higher tolerance to the increasing levels of inequality.

In terms of a cooperation dilemma, the legitimization of effort represents the positive result that mutual cooperation is better than mutual defection. However, one of the goals of this work is remark the link between the loss of interest in the effects of absolute inequality and the lack of concern for the asymmetric outcomes in a cooperation dilemma. Using three different cooperation dilemmas, I remark the relevance of the outcomes with unequal payoffs in the decisions of cooperation. Also, it is shown how modifications in the differences between payoffs lead to these two different games where the asymmetric outcomes correspond to the maximization or minimization of social welfare.

A couple of studies concerning social mobility and inequality have been developed. Alesina and Angeletos (2005), and Benabou and Tirole (2006) use the convictions about the correlation between effort and wealth to explain how the agents' preferences for fairness and the belief in a just world can, in aggregated way, determine the level of redistributive taxes desired. In the former it is assumed that exist different self-fulfilling beliefs of the same reality, while in the latter it is assumed that the divergence is due to different ideologies. Benabou and Tirole set two different equilibriums: "American" and "European". The "American" equilibrium is characterized by a dominant belief in the correlation success-effort, a considerable tolerance for inequality and a low level of redistributive taxes. On the other side, the "European" equilibrium is described by opposed characteristics to the "American" and hence is associated with higher levels of redistributive taxes.

The promulgation of relative equality in democratic times strengthens individualism in the "American" as in the "European" equilibrium. In both cases the absolute inequality constitute itself a central argument in the promotion of their convictions, although in opposite directions. On the one hand, the "American" equilibrium legitimizes inequality as the result of effort and promotes the pursuit of wealth through hard-

working, converting the greed in a socially accepted characteristic of individuals instead of being a potential source of coordination and cooperation failures. On the other hand, the “European” equilibrium recognizes the existence of poverty traps and the low level of social mobility. Being aware of this problem, a society can enhance the conditions received in the “worst case” outcome through higher redistributive taxes (Bowles, 2011). Raising the earnings of the worst outcome is a way to reduce the fear of being exploited when individuals make an effort to cooperate.

This work explores the dimensions of greed and fear through three different cooperation dilemmas. First, describing a classification and origins of the games derived from the PD; once the game’s taxonomy is clear, the work is focused on the differences found across payoff structures using analytical tools from CGT and EGT.

Related literature can be divided in two groups: the works dedicated to the analysis of PD variations where the trade-off between equality and efficiency exists, and the works focused in the evolution of cooperation in the PD using EGT. Kuhn and Moresi (1995) study the variation of PD in which the highest social surplus is achieved through asymmetric cooperation. They suggested that in a repeated game the alternation between cooperation and defection is an equilibrium that maximizes payoffs for both players. Frohlich and Oppenheimer (1996) part from the same variation of the standard PD and provide an analysis in the case of a public goods game, the multiple agents’ version of the PD game. They analyze the implications of a modified version of the game where the optimal rate of contribution is below unity. Nowak (1990, 2006a, 2006b) and Sigmund and Nowak (1999) study the evolution of cooperation under the PD incentives using tools provided by EGT, which allow to find a set of stochastic and deterministic strategies that sustain mutual cooperation over time.

Study the role of payoffs’ differences in the cooperation decisions is relevant given the experimental evidence. Different works show how individual behavior in the PD differ considerably from the *homo economicus* predictions (Sally, 1995; Ahn *et al.*, 2001), giving importance to the out of equilibrium outcomes, hence, to the dimensions of fear and greed.

In Section 2 are defined the restrictions to categorize a PD and the other cooperation dilemmas. This description is followed by an analysis of the three resulting games in the cases of complete contracts and repeated games using classical game theory. Section 3 starts with a short introduction to evolutionary game theory, followed by the existent results for the PD in this framework. In this section is also presented a comparison between the most unequal cooperation dilemmas. Section 4 describes the algorithm and the results from the simulation of the evolution of cooperation using replicator dynamics for the three games. Finally, Section 5 presents the conclusions and recommendations for future works.

2. Revisiting the Prisoner's Dilemma

2.1. The standard Prisoner's Dilemma

Prisoner's Dilemma (PD) is characterized by having a unique Nash equilibrium which is not Pareto-optimal. Players result trapped in this equilibrium due to a coordination failure given that, seeking the maximization of their individual profit, they end up in an undesirable outcome. Table 1 show the strategic form representation of the game in the case of symmetric players, where a couple of agents must decide between cooperation and defection. Given the expression (1) it can be easily verified that defection is a dominant strategy for both players, therefore is the only Nash equilibrium.

Table 1. Prisoner's Dilemma in its strategic form

P1 / P2	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

$$T > R > P > S \quad (1)$$

$$2R > T + S \quad (2)$$

Expressions (1) and (2) describe the characteristic incentives in a PD. Payoffs reported in condition (1) are defined according to the convention adopted in Rapoport & Chammah (1965), and as was early mentioned, is known as the “suboptimality condition.” T corresponds to the temptation to defect when the other player cooperates, R corresponds to the reward received by both players in case of mutual cooperation, the payoff P is received as the punishment when neither of them cooperates; and finally, S is defined as the sucker's payoff received when the player cooperates and its counterpart does not. The order of these payoffs evidence why cooperation is dominated by defection: The temptation to defect is higher than the reward for cooperate, and the payoff received as a punishment for mutual defection is higher than the sucker's payoff.

Expression (2), known as the “externality larger than internality” condition, guarantees that mutual cooperation aligns efficiency and equality, given that both players share the same payoff while they maximize the social welfare. Furthermore, this restriction ties up the level of inequality attainable in a PD. One of the cooperation dilemmas studied in this work results when this condition is no longer held in a former PD.

The graphical representation of a 2x2 game is useful for the understanding of the payoff transformation obtained after the manipulation of the outcomes' inequality, as well as the resulting variations in the aggregated welfare attainable through the four

different outcomes. Each axis corresponds to the payoff of a player, while the four potential outcomes define the vertices in a convex hull that contains all the possible joint payoffs available using mixed strategies.

Figure 1 displays the graphical representation of the standard PD. The points marked as (D,D) and (C,C) corresponds to the Nash equilibrium and the socially efficient outcome respectively, while (C,D) and (D,C) are associated with the unequal outcomes of the game. The distance between the origin and each point indicates the social welfare obtained in each one of the outcomes, proving the superiority of mutual cooperation over the rest of results from the game in terms of the sum of the players' payoffs. The dotted line in Figure 1 connects the asymmetric outcomes in the PD game; while this line lies below the point that corresponds to mutual cooperation, condition (2) is guaranteed.

Figure 1. Prisoner's Dilemma graphical representation

If the dotted line lies above the mutual cooperation point, condition (2) is no longer fulfilled and the new outcomes that maximize social welfare are the asymmetric outcomes (C,D) and (D,C). As a consequence, we must expect the existence of another rule that constraints the dotted line from being below the (D,D) point. Condition (3) defines this constraint and guarantees that in a standard PD the outcome with the lowest level of social welfare corresponds to the case of mutual defection. This condition has not been mentioned previously in the literature despite its relevance in the configuration of the direction of inequality in the PD game. This expression is analogous to the inequality shown in (2), and also defines an “externality larger than internality” condition.

$$2P < T + S \quad (3)$$

2.2. Variations from the standard Prisoner's Dilemma through greed and fear

The modifications proposed in this work vary the difference between payoffs in opposite directions, which were previously defined in literature as the dimensions of greed and fear (Komorita *et al.* 1980, Rapoport and Chammah, 1965). The greed is defined as the benefit that can be obtained by a player when abandons mutual cooperation, and can be measured as the difference between the temptation (T) and reward (R) payoffs. The fear is the cost in which a players incur if he tries to leave mutual defection and the other player does not, and corresponds to the difference between the payoff received as punishment (P) and the sucker's payoff (S). Ahn *et. al* (2001) conducted an experiment where they vary the level of inequality through the dimensions of greed and fear, as potential mechanisms to explain the cooperation rate in the game. However, the effect of inequality was below the expected. This fact can be explained by the restrictions over the levels of inequality set in the experiment, in other words, the constraints set by expressions (2) and (3).

The cooperation dilemmas derived from PD corresponds to modifications in the distances between payoffs, raising the dimensions of greed and fear, separately, until the point in which conditions (2) or (3) are no longer held. The one-shot equilibrium of the game is the same for all three games under these new configurations of the dilemma, while the apparition of greater differences between the payoffs in assymetric outcomes derives in a new ranking of the outcomes in terms of social welfare.

If the dimension of greed is increased enough condition (2) is no longer held. As a consequence now we have, which means that the outcomes related to asymmetric cooperation generate a higher level of social welfare than mutual cooperation. Graphically, this variation implies that the payoff hull is concave above the outcome (C,C). This transformation implies that the set of mixed strategies is expanded and the expected payoff of the game is maximized through a mixed strategy that does not correspond to mutual cooperation.

On the other side, a considerable rise in the dimension of fear implies the violation of condition (3), thus we now have that $2P > T + S$. This expression indicates that mutual defection is no longer the worst outcome in terms of social efficiency, because it has been replaced by the pair of cases of asymmetric cooperation. This transformation also implies a loss of convexity from the payoff hull given that the dotted line is below the point (D,D). Once more, the set of mixed strategies is expanded, but in this case the new attainable strategies do not lead to payoffs' maximization. In contrast, mixed strategies could lead to lower levels of social welfare in terms of expected payoffs than the usually obtained under mutual defection.

Figure 2 shows graphically the results of the two transformations just described. In the left panel appears the cooperation dilemma defined as "Cooperation Under Fear" (CUF), given that was obtained after the increase of the difference between reward and temptation. Consequently, is defined the other cooperation dilemma as

“Cooperation Under Greed” (CUG), whose graphical representation is displayed in the right panel of the same figure. The standard Prisoner’s Dilemma is shown in the center of Figure 2, and is used as the baseline for the comparison of the effects of payoffs’ inequality.

Figure 2. Graphical representation of Prisoner’s Dilemma variations

Table 2 summarizes the classification of standard PD and its variations, focusing on the characterization of the restrictions associated to the attainable social welfare of the different outcomes as a function of the magnitude and direction of inequality. Standard PD is set as the baseline to analyze the modifications induced by the increase in the dimensions of fear or greed. The resulting variations are susceptible to less equitable outcomes, allowing a depth analysis of how fear and greed can affect the decision of cooperation of the players.

Table 2. Classification of Prisoner’s Dilemma and its variations

	$2R > T + S$ Mutual cooperation maximizes social welfare	$2P < T + S$ Mutual defection minimizes social welfare	Loss of convexity
CUF (cooperation under fear)	Yes	No	Below (D,D)
Prisoner’s Dilemma	Yes	Yes	Convex hull
CUG (cooperation under greed)	No	Yes	Above (C, C)

2.3. Inequality and cooperation under complete contracts

A complete contract looks for an explicit and verifiable *ex-ante* agreement that enforce the *ex-post* distribution for all the possible resulting states after the players’ interaction. Despite the fact that it is an idealized case, its analysis is relevant given that the solutions obtained under these conditions are useful as benchmark in the design of institutional mechanisms that will reduce the problems under incomplete contracts.

Given a couple of agents facing in a cooperation dilemma (PD, CUF or CUG), the proposed contract consist in a transfer from one to the other player, defined as a fraction ρ of the payoff received identical for both individuals. The objective of this transfer is a reduction of the difference between unequal payoffs. If this reduction is high enough, the ranking of the payoffs of the game is altered, modifying the Nash equilibria. In Table 3 is shown the modified payoff matrix including the proposed transfer.

Table 3. Modified payoffs due to the transfer between players

P1 / P2	Cooperate	Defect
Cooperate	R, R	$(1-\rho)S + \rho T$, $(1-\rho)T + \rho S$
Defect	$(1-\rho)T + \rho S$, $(1-\rho)S + \rho T$	P, P

The main result under complete contracts is that the magnitude and direction of inequality affects the critical transfer level $\hat{\rho}$ that redefines the order of incentives, and as a consequence, the modification of the equilibria for the three cooperation dilemmas. The modification of the “suboptimality condition” described in expression (1) appears once is reached at least one of the two conditions described in (4) and (5). The first expression corresponds to a reduction in the temptation payoff to a level where the reward payoff is preferable. The second condition increases the sucker’s payoff until a level where this payoff is preferable than the punishment payoff.

$$R > (1 - \hat{\rho})T + \hat{\rho}S \quad (4)$$

$$P < \hat{\rho}T + (1 - \hat{\rho})S \quad (5)$$

The results for each one of the three dilemmas differ as a consequence of the binding condition once is reached $\hat{\rho}$. In the CUF case the first condition reached is (4) because the fear dimension ($P-S$) is larger than the greed dimension ($T-R$). On the opposite side, in the CUG game the condition that defines the critical transfer $\hat{\rho}$ is the expression (5). Finally, in a symmetric standard PD game both conditions are simultaneously reached; aligning the payoff incentives towards the cooperation.

For the three dilemmas, CUF, PD and CUG, the maximization of social welfare is attained through at least one of the equilibria in the transformed game. In the PD the coordination failures are eliminated, and the only Nash equilibrium of the modified game corresponds to the point (C,C), which is Pareto-optimal and maximizes the social welfare at the same time.

In the game under fear (CUF), after the modification we have two Nash equilibria: The first one matches the mutual cooperation outcome (C,C), which is Pareto-optimal and guarantees a higher and equal payoff to both players. The second equilibria still corresponds to mutual defection (D,D). This result implies that once is reached the transfer level $\hat{\rho}$, the possibility of leaving mutual defection depends on the beliefs about other strategy and their ability to coordinate.

In the game under greed (CUG), the implementation of a contract with a transfer level of at least $\hat{\rho}$ results in a game with two Nash equilibria, where both players try to not match the other's decision. In this game both equilibria are symmetric and Pareto-optimal; and even if they maximize the combined payoff of the players, these outcomes are also associated to a prevalent inequality between agents.

Figure 3 shows graphically the implementation of the transfer contract for the three games, as well as the apparition of new equilibria. In each image the initial set of payoffs is represented by the light and straight lines, while the set of payoffs corresponding to the game transformation is depicted by the dark and dashed lines. The new equilibria, marked in Figure 3 with the black dots, appear once the transfer level $\hat{\rho}$ is reached.

The explanation for the appearance of the new equilibria requires the introduction of the concepts of conflict and common interest in a game (Bowles, 2003). The conflict in a game implies that each unit of the monetary payoff won by one player represent the same loss by the other player, and correspond to a trajectory from northwest to southeast in the graphical representation of the players' payoffs. On the other hand, the common interest in the game appears when both players can increase their payoff jointly, and corresponds to a trajectory from southwest to northeast.

Figure 3. Modification of the games after the transfer agreement

The transfers proposed in the complete-contract state of the world are reducing the game's conflict dimension, leading to an indirect increase in the magnitude of the common interest dimension. The *ex-post* redistribution reduces the incentives for defecting, transferring a share of the net payoffs when outcomes are asymmetric.

Consider the matrix payoffs displayed in Table 4 and a complete contract based on a transfer of the 30% of the material benefit received ($\rho = 0.30$).

Table 4. Payoffs previous to the transformation under complete contracts

CUF			PD			CUG		
P1 / P2	C	D	P1 / P2	C	D	P1 / P2	C	D
C	8,8	-6,10	C	8,8	2,10	C	8,8	2,18
D	10,-6	4,4	D	10,2	4,4	D	18,2	4,4

The modified version of the matrix payoffs for each game given the transfer is shown in Table 5. The rearrangement of incentives leads to a new set of Nash equilibria according to the predictions mentioned before and can be easily verified. Calculating $\hat{\rho}$ for each game using conditions (4) and (5) we obtain $\hat{\rho}_{DPM} = 0.125$, $\hat{\rho}_{DPA} = 0.125$ and $\hat{\rho}_{DP} = 0.25$, which means that higher levels of inequality require lower transfer fractions from the payoff holding constant the difference between the symmetric outcomes; and that $\hat{\rho}$ does not depend on the direction of inequality, just its magnitude.

Table 5. Payoff matrices modified after the transfer with

Modified CUF			Modified PD			Modified CUG		
P1 / P2	C	D	P1 / P2	C	D	P1 / P2	C	D
C	8,8	-1.2,5.2	C	8,8	4.4,7.6	C	8,8	6.8, 13.2
D	5.2,-1.2	4,4	D	7.6,4.4	4,4	D	13.2,6.8	4,4

Increasing incentives for defection, as in CUF and CUG, block the possibility that transfers under complete contracts lead to mutual cooperation as the unique equilibrium of the game. Besides, the direction of redistributive inequality also defines the rearrangement of incentives, affecting the equilibria obtained in presence of a transfer above $\hat{\rho}$. Unfortunately complete and incomplete contracts coexist, and the latter are more relevant in an uncountable number of situations where is not possible to enforce *ex-ante* agreements, leaving almost intact the possibility of being trapped in coordination failures.

2.4. Repeated cooperation dilemmas

In this subsection is shown the effect of inequality over the discount factor δ that is required to sustain the intertemporal cooperation in the infinitely repeated game. The dynamic analysis of cooperation dilemmas introduces the notion of reputation and the apparition of intertemporal threats that derives in the well-known “folk theorem” and its multiple equilibria, including mutual cooperation. This strategy is supported by retaliation as long as the temporal horizon is considerably extended and the impatience factor is low enough (Vega-Redondo, 2003).

To preserve the intertemporal cooperation as equilibrium is needed that the expected payoff of this strategy will be higher than the expected benefits from other strategies,

like the mentioned below. Equations (6) to (8) indicate the present value of the payoffs received for playing three different strategies, which are useful in the analysis of the dynamic game and the effects of inequality. Appendix I shows the process to obtain the payoffs displayed in these equations. $\pi_i^{C\infty}$ corresponds to the payoff obtained by intertemporal cooperation, $\pi_i^{C\tau}$ is the payoff of an alternative strategy where an agent cooperates until the period τ and defects from then on, and $E[\pi_i^{Alternate}]$ is the expected payoff by an alternation between cooperation and defection each round.

$$\pi_i^{C\infty} = \frac{R}{(1-\delta)} \quad (6)$$

$$\pi_i^{C\tau} = \left(\frac{1-\delta^\tau}{1-\delta}\right)R + \delta^\tau T + \left(\frac{\delta^{\tau+1}}{1-\delta}\right)P; \text{ for } \tau > 0 \quad (7)$$

$$E[\pi_i^{Alternate}] = \left(\frac{T+S}{2}\right)\left(\frac{1}{1-\delta}\right) \quad (8)$$

The comparison between the expected payoffs yields a couple of results related to the differences between payoffs across cooperation dilemmas. First, comparing equations (6) and (7) shows that an increase in the greed dimension has as a consequence an increase in discount factor δ required to sustain cooperation indefinitely. Expression (9) is obtained (See Appendix I), showing that higher values of $(T-R)$ implies a higher discount factor.

$$\delta > \frac{T-R}{T-P} \quad (9)$$

This expression does not depend on parameter S associated to the sucker's payoff. This result implies that the discount factor is not affected by modifications in the dimension of fear. This makes sense if we remind that the mechanism that maintains cooperation over time is the threat of retaliation of switching to defection, hence, the incentives promoted under this strategy are focused in avoiding the abandon of cooperation and they correspond to the greed dimension.

The second result is related to the sustainability and plausibility of the alternating cooperation and defection in the CUG, and is obtained through the comparison of equations (6) and (8). As is shown in the Appendix I, this result corresponds to the formalization of a statement pointed by Kuhn and Moresi (2008): the intertemporal payoff maximization in a CUG game is obtained when both players alternate coordinately between the two pure strategies. However, this joint strategy requires the assumption that both players are indifferent between receiving the sucker's payoff (S) and the temptation payoff (T) in the first round of the game, which in many cases may result implausible given the impossibility to enforce this agreement when the counterpart must take the suckers' payoff.

2.5. Relevance and direction of the analysis

In this section was presented an analysis that incorporates the relevance of unequal outcomes in three different cooperation dilemmas using analytical tools derived from

classical game theory. The analysis started with a characterization of the standard PD and its variations based on the magnitude and direction of payoff differences; next, was studied the transformation of the three games under complete contracts that implies an *ex-post* transfer between payers; finally, the comparison between intertemporal strategies yielded two results that describes the effect of inequality in repeated games.

It is necessary the incorporation of tools that allows examining the transition to equilibrium, introducing a dynamic analysis to the problem from another framework. This new approach will introduce the variation in time of the adoption of different strategies and their convergence, using a new set of tools from evolutionary game theory. The implementation of the replicator dynamics will be useful in the comparison between the three games, as will be shown in the next sections.

3. Inequality in cooperation dilemmas under the evolutionary approach

3.1. A brief introduction to Evolutionary Game Theory

Maynard Smith and Price (1973) took game theory from economy to biology to develop a mathematical tool that allows modeling interactions between species in such a way that the fitness, or reproductive success, of each strategy depends on the frequency with which all the strategies are played. Years later the evolutionary game theory (EGT) was incorporated in the analysis of economic problems, providing models of cultural and institutional evolution for populations. EGT tools allows introducing new models to analyze institutional transitions and the endogeneity of preferences to explain the dynamics of different social behaviors like the apparition of property rights, patterns of segregation and the extraction of common-pool resources (Boyd and Richerson, 1985; Sethi and Somanathan, 1998; Gintis, 2000; Bowles, 2003).

There are three key differences between classical game theory (CGT) and evolutionary game theory (EGT): First, in CGT the players are endowed with a set of strategies from where they choose rationally, while in EGT the set of strategies is defined as the cultural or institutional alternatives that can be chosen or inherited by individuals. It means that EGT considers agents are able to develop, imitate or modify behaviors according to the perception of the relative payoff in terms of the available strategies. Also implies that the decisions are not based exclusively on their rationality. Secondly, evolutionary stable strategies (ESS) replace the Nash Equilibrium as the principal concept of equilibrium in this framework. A strategy is evolutionarily stable if given an invasion of a reduced group of individuals with a different strategy, denominated mutants; the incumbent agents can repel the new strategy. It is important to clarify that all Nash equilibrium is ESS, but not the converse. Finally, EGT requires a dynamic background where is assumed that interactions between members of the population are random (in each round they interact with a different agent), and the strategy chosen is defined genetically or inherited culturally, instead of being based on the previous experience in the game or in a highly rational analysis (Gintis, 2000).

The analysis of dynamic evolution of strategies under the EGT facilitates the transition from individual actions to the diffusion of behaviors in the society (Gintis, 2000). In this framework the modeling is based on replicator dynamics, where the share of the total population that plays a given strategy varies over time and is described by a set of differential equations. The gradient of the behaviors dynamics is given by the comparison of the fitness of a given strategy with the average fitness of all the strategies. Strategies with a higher fitness have a greater chance to spread in the population and overthrow weaker strategies (Nowak, 2006a).

The description of evolution of strategies is given by the replicator equation shown in equation (10). The variation in time of individuals attached to the strategy i , defined as $\dot{x}_i$, depends of the fraction of individuals x_i , of the strategy's fitness $f_i(\vec{x})$ and of the average fitness of all strategies denoted as ϕ . In other words, the replicator equation points out that if the fitness of a strategy is higher than the average fitness, there will be an increase over time in the fraction of the population associated to this strategy.

$$\dot{x}_i = x_i [f_i(\vec{x}) - \phi] \quad (10)$$

The fitness function for the strategy i is shown in equation (11). It corresponds to the expected payoff by facing each one of the j strategies (included the i strategy itself), which is denoted as a_j ; the probability of interact with an agent that represents strategy j is given by the fraction of population x_j associated to the strategy. Equation (12) describes the average fitness ϕ of the whole population, where the fitness of all the strategies involved is weighted by the fraction of population x_j .

$$f_i(\vec{x}) = a_1 x_1 + a_2 x_2 + \dots + a_i x_i + \dots + a_N x_N \quad (11)$$

$$\phi = \sum_{i=1}^N x_i f_i(x_1, x_2, \dots, x_N) \quad (12)$$

3.2. Inequality and the rate of extinction of cooperation

The replicator equation was employed to analyze the effect of inequality on cooperation dilemmas and then compare the results obtained by Nowak (2006a) for the standard PD game with the most unequal games, CUF and CUG. The theoretical development (Nowak 1990, 2006a, 2006b; Sigmund and Nowak, 1993) has been focused in the mechanisms that allow the evolution of cooperation via the PD game. Previous results included rules that determine the apparition of cooperation (Nowak, 2006b) and the effectiveness of retaliation imposing over the mutual defection problem. The contribution of this part of the work is to extend the PD analysis to the other cooperation dilemmas in a dynamic level.

The replicator equation requires the definition of a payoff matrix that includes the benefits of an agent under all the possible interactions. Assuming symmetrical players that interact randomly, the corresponding payoff matrix is displayed in Figure 4. Here, the agent's strategies correspond to the rows while the strategies that he might face

correspond to the columns. Based on this matrix we can derive the functions of individual and average fitness required in the replicator equation.

Figure 4. Payoff matrix for Prisoner's Dilemma game and its variations

		C	D
C	[	R	S
D		T	P

The set of differential equations that describes the dynamic of replication is composed by $N-1$ independent equations, where N is the number of strategies defined for the system. The remaining equation is dependent because the total population is normalized to unity and supposed constant over time. In this case we just have a couple of strategies so we can describe the evolution of cooperation with just one differential equation.

If we define x as the fraction of agents that cooperate and $(1-x)$ as the fraction of agents that defect, the dynamic evolution of this pair of strategies is described by equation (13). The fitness of cooperation and the average fitness in the population are represented by equations (14) and (15) respectively. The fitness of cooperation implies that with probability x the agent will receive the reward payoff R while with probability $(1-x)$ he will receive the sucker's payoff S . The other equation represents the average fitness, which is weighted according to the proportion of agents in each strategy.

$$\dot{x} = x(f_x(x) - \phi) \quad (13)$$

$$f_x(x) = (Rx + S(1 - x)) \quad (14)$$

$$\begin{aligned} \phi &= x(Rx + S(1 - x)) + (1 - x)(Tx + P(1 - x)) \\ \phi &= Rx^2 + S(x - x^2) + T(x - x^2) + P(1 - x)^2 \end{aligned} \quad (15)$$

Replacing equations (14) and (15) into equation (13) we obtain the expression below, which describes the variation in time of the proportion of individuals cooperating. If we define $\dot{x} = f(x)$, where $f(x)$ corresponds to the right side in equation (16), it can be guaranteed the existence of this expression if $f(x)$ is continuous and double differentiable (Gintis, 2000), conditions validated for the replicator equation. In addition, for each set of initial conditions (the initial fraction of cooperators and defectors) exists a unique path that is the solution of the differential equation (Boyce y DiPrima, 2002).

$$\begin{aligned} \dot{x} &= x\{Rx(1 - x) + S(1 - x) - (S + T)x(1 - x) - P(1 - x)^2\} \\ \dot{x} &= x(1 - x)\{(R + P - S - T)x + (S - P)\} \\ \dot{x} &= x(1 - x)\{(R - T)x + (S - P)(1 - x)\} \end{aligned} \quad (16)$$

It is easy to check that equation (16) is always negative, and as a consequence the extinction of cooperation is guaranteed. Given that x is a fraction of the population, the first two factors are positive and the sign of equation depends only of the term in brackets. The definition of incentives in the cooperation dilemmas implies that the payoff associated to temptation is higher than the reward ($R-T < 0$) and the benefits received in the punishment are higher than the sucker's payoff ($S-P < 0$), vanishing the fraction of cooperators in time and yielding a unique equilibrium in $x=0$, independently of the initial composition of the system.

The introduction of the three cooperation dilemmas in this analysis shows that the equilibrium of the game is the same for CUF, PD and CUG; but that they differ in the rate of convergence to defection, which result higher when asymmetric payoffs are most unequal. The equilibrium is the same for the three games because CUG and CUF shares with the standard PD the "suboptimality condition" defined in expression (1). However, under the most unequal games cooperation disappears faster by the augmented effects of the dimensions of greed ($R-T$) and fear ($S-P$) respectively, as we can see in equation (16). Once the difference between payoffs is raised in any of these directions, the variation in time of cooperation becomes more negative, speeding up the convergence to the equilibrium. Is important to note that for a given level of inequality, the extinction of cooperation is faster under the dimension of fear than under the dimension of greed because the former term is multiplied by the fraction of defectors in the system, whose proportion in the system is growing over time; while the latter is multiplied by the fraction of cooperators, who are disappearing periodically.

The next step in the analysis under the replicator dynamic is the verification of the evolutionary stability of defection given a variation in the levels of inequality of the game. The defection is stable if given an invasion of cooperators, a small fraction of unconditional cooperators that appear in the system, the predominantly population of defectors will repel the mutants after a short time (Nowak, 2006a). Given that defection is the only pure Nash equilibrium for all cooperation dilemmas, it is an evolutionarily stable strategy in the three games (Gintis, 2000) and the proof is shown below. An invasion of agents can be considered effective if the payoff of the entrant strategy is higher than the payoff of the incumbent one. In this case, if the expected payoff received for cooperation overcomes the expected payoff of defection as expression (17) suggests.

$$\begin{aligned} \pi_C &> \pi_{NC} \\ (1-\varepsilon)S + \varepsilon R &> (1-\varepsilon)P + \varepsilon T \\ (1-\varepsilon)(S-P) + \varepsilon(R-T) &> 0 \end{aligned} \tag{17}$$

According to condition (1), the incentives structures in a PD game implies that the expression shown above can't be achieved for any size of invasion $\varepsilon \in [0,1]$ in any cooperation dilemma, because the left side of inequality is always negative. The effect of inequality is similar to its effect over the convergence to defection: The

relative proportion between defectors and cooperators derives in a larger effect of the dimension of fear because is multiplied by $(1-\varepsilon)$, which is bigger than ε .

3.3. The evolution of retaliation

Given the impossibility to sustain unconditional cooperation over time, the strategies based on the intertemporal threat of retaliation have resulted more plausible as an explanation in the evolution of cooperation. This subsection starts with a summary of the evolution of strategies related to conditional cooperation in the standard PD, and is followed by another contribution of this work: the analysis of these strategies in the CUF and CUG cooperation dilemmas, exploring the relationship between retaliation and the unequal outcomes when these are increased.

The strategy that initially overthrown defection was proposed by Rapoport (1965) and is called GRIM, which suggests to cooperate while the other player follows the same strategy; but once the opponent defects in one round, do not cooperate again. The “tit-for-tat” (TFT) strategy is defined as “start with cooperation and play the same that the other played last round”, which means that players are not trapped in defection after a mistake because a second mistake can put them back in a path of mutual cooperation. The generous “tit-for-tat” (GTFT) appears as an alternative, introducing a stochastic rate of forgiveness that depends of the parameters of the game, leading to a positive probability of maintain cooperation in case that a player faces a mistake from the opponent (Nowak, 1990). Finally, a strategy that incorporates a deterministic correction of mistakes defeats its predecessors, and is known as “win-stay, lose-shift” (WSLS). Under WSLS the player repeat its decision from the last round if he has received the payoffs associated to reward R or temptation T , and switch its strategy if the last payoff received was the penalty P or the sucker’s payoff S (Nowak, 2006a). However, the analysis of inequality under this last strategy is out of the reach of this work.

We will start the analysis with GRIM, which suggest cooperate while the other cooperates too. Its stability requires that the expected number of rounds, $\bar{m}$, follows condition (18). This restriction does not depend on parameter S , which means that the minimum number of rounds required to sustain cooperation is not affected by an increase in the fear dimension, hence there are no differences between PD and CUF.

$$\bar{m} > \frac{T-P}{R-P} \quad (18)$$

On the other hand, an increase in the greed dimension has a positive effect in the number of rounds required to use GRIM. Adding and subtracting the parameter R from expression (18) leads to condition (19), where the payoff differences associated to greed defined as $(T-R)$, which raises the minimum value of $\bar{m}$

$$\bar{m} > 1 + \frac{T-R}{R-P} \quad (19)$$

The analysis for the TFT strategy is very similar to the analysis previously displayed in subsection 2.4, and the contribution can be shortly described: the expected payoff differs from equation (7) only in the benefits from period τ (instead of T , in other words, the same strategy viewed from the cooperator), and as a consequence the effect of an increase in inequality derives in the requirement of more patient individuals to maintain cooperation indefinitely.

The analysis of GTFT requires the definition of reactive strategies: they correspond to strategies where the contemporary decision does not take into account the previous movement of himself, just the previous movement of the opponent. Lets define p as the probability of cooperate if the other player cooperated the last round, and q as the probability to cooperate if the contender defected in his previous movement. q can also be defined as a stochastic rate of forgiveness. In GTFT, p and q are described by the pair of equations described in condition (20) (Nowak, 2006a).

$$p = 1; q = \min \left\{ 1 - \frac{T-R}{R-S}, \frac{R-P}{T-P} \right\} \quad (20)$$

With the incorporation of the most unequal cooperation dilemmas to the analysis of GTFT, we found that in the case of CUG this strategy is not attainable, while in CUF is always defined a positive stochastic rate of forgiveness. In the CUG game we have that the first argument of the minimum function that describes q can be rearranged and written as $(2R-T+S)/(R-S)$. This expression is always negative in the CUG dilemma, proving that does not exist a positive value of q in this variation of the standard game. On the other hand, in a CUF game the rate of forgiveness is equal or superior to the obtained in the corresponding standard PD (same values of T, R and but a higher value of S). In Appendix II is shown that incentives of CUF lead to condition (21), and as a consequence q is always $(R-P)/(T-P)$ in this game.

$$1 - \frac{T-R}{R-S} > \frac{R-P}{T-P} \quad (21)$$

The differences between the rate of forgiveness for the CUG and the CUF can be explained based on the different baseline outcome of these games. While an increase in the dimension of greed give incentives to abandon mutual cooperation, an augmentation in the dimension of fear reduces the incentives to leave mutual defection. Given that the purpose of the GTFT strategy is the sustainability of mutual cooperation, the increasing incentives to leave this strategy results severely punished due to the rise in the greed level, measured as $(T-R)$. On the other side, GTFT promotes cooperation in a CUF to attenuate the effects of an increase in the fear dimension, trying to lead the players to mutual cooperation. In other words, if the cost of leaving the social optimum in a CUF is higher than the social cost of the abandon of mutual cooperation in CUG, the need to return to the path of cooperation can explain the higher rate of forgiveness in the game under fear.

4. Computational modeling of Replicator Dynamics

4.1. The simulation of cooperation dilemmas

In this section is presented a set of simulations of the ecology of strategies with different rates of cooperation. The manipulation of the incentives of the game allows exploring the differences across cooperation dilemmas. The exercise performed is an adaptation of the “in silico evolution” reported by Nowak (2006a), where is defined a group of strategies that only differs in the probability of cooperation that interact for a given period of time using the payoff matrix that corresponds to a standard PD. The result is a subgroup of strategies that succeed over the others. The contribution in this part of the analysis lies in the introduction of different payoff structures, exploring several games across the three cooperation dilemmas.

The simulations were performed using one hundred randomly generated strategies H_i , where the rate of cooperation is randomly drawn from an uniform distribution between zero and one. All the strategies share the same initial fraction of population $x_i=0.01$. The evolution was modeled using the replicator dynamics, with random interactions independent from the spatial component of the game (the probability to interact with the nearest neighbor or with the farthest individual is the same) and with a probability of cooperation independent of the game history. The independence of the game history was supported in our interest over the direct effect of payoff differences in asymmetric outcomes, not in the retaliation. The variation in the adoption of strategies was based exclusively in the replicator equations derived from the system, and the evolution in time of the different strategies was calculated using the Euler’s method for differential equations.

A short description of the algorithm implemented is described next, while a detailed explanation can be found in Appendix III. First, the number of periods in the simulation was defined and the n strategies were randomly generated. Secondly, the payoff matrix P was calculated using the probability of cooperation defined for each strategy; each element in the matrix represents the expected payoff for strategy i when faces the strategy j as is shown in equation (22). In this expression the parameters T , R , P and S corresponds to the payoffs in Figure 4 (defined for the row player), while H_k corresponds to the probability of cooperation in strategy k for $k = \{i, j\}$.

$$P(i, j) = R(H_i H_j) + S(H_i(1 - H_j)) + T(1 - H_i)H_j + P(1 - H_i)(1 - H_j) \quad (22)$$

The next step corresponds to the fitness calculation for each strategy and later the average fitness. Let’s define the vector X of size $(n \times 1)$ that contains the fraction of the population that plays each strategy. Using this vector and the matrix P defined in (22) is obtained a new vector that contains the fitness of each strategy as is shown in equation (23). Once is obtained the vector f it can be obtained the average fitness using equation (24).

$$f = X'P' \quad (23)$$

$$\Phi = fX \quad (24)$$

The numerical approximation that describes the system of differential equations is shown in expressions (25) and (26). The first one determines the variation in time of the fraction of individuals in each strategy. The second one corresponds to the updating rule in the fraction of each strategy, where δ is a parameter that makes smooth the evolution of the system.

$$\Delta X(i) = X_t(i)(f(i) - \Phi) \quad (25)$$

$$X_{t+1}(i) = X_t(i) + \delta \Delta X(i) \quad (26)$$

4.2. The convergence to defection

Simulations were set for the three cooperation dilemmas to analyze the dynamic evolution of strategies with different levels of fear and greed. The results allow verifying the differences in the rate of convergence to equilibrium previously obtained theoretically. In the left column in Figure 5 are shown the results of the simulation of the CUF, PD and CUG based on the payoff matrices displayed in the right column if the same figure. The classical PD is used as baseline, and the CUF and CUG appear once the inequality is increased in eight units through the dimensions of fear ($P-S$) and greed ($T-R$) respectively.

Figure 5. Differences between games in the convergence to defection

P1 / P2	C	D
C	8,8	2,10
D	10,2	4,4

PD

P1 / P2	C	D
C	8,8	2,18
D	18,2	4,4

CUG

The three images in Figure 5 represent the evolution in time of the fraction of population that adopts the different strategies available. The strategies were grouped in quartiles according to its probability of cooperation for a better interpretation of the trajectories and to reduce the variation between repetitions given the random character of the strategies. The first quartile groups the 25 strategies with the highest probability of cooperation (pointed line), in the fourth quartile are aggregated the 25 strategies with the lowest probability of cooperation (straight line), and the remaining 50 strategies are divided into the second and third quartiles (dashed lines). The horizontal axis represents the period of time and the vertical axis the fraction of population that has adopted each group of strategies.

The results shown in Figure 5 prove that higher levels of inequality speed up the convergence to defection. The straight line, which corresponds to the strategies with the lowest probability of cooperation, converges faster to unity (the whole population) in the CUG and the CUF than in the standard PD. Similarly, the dotted line approaches zero more rapidly in the unequal variations of the game, ratifying from the opposite angle the extinction of cooperation. These results support the theoretical discussion in Section 3, where was initially suggested the effect of inequality over the convergence to equilibrium.

In Section 3 is also remarked a difference in the rate of convergence between CUG and CUF: an increase in the dimension of fear is associated with a faster extinction

of cooperation because the term $(P-S)$ is multiplied by the growing population of defectors. The results of the simulation confirm the difference between the most unequal cooperation games: the straight line reaches unity faster in the game under fear than under greed. The explanation is given by the proliferation of strategies that weight more the term $(P-S)$ than the difference $(T-R)$ in the fitness calculation.

4.3. Inequality and the average rate of cooperation

The average rate of cooperation (ARC) was defined as a measure of the evolution of cooperation across repetitions of the simulation, given the stochastic character of the strategies implemented. The ARC is defined as the average probability of cooperation in the system weighted by the fraction of individuals in each strategy for a period of time, as is shown in equation (27).

$$ARC = \sum_i S_i X_i \quad (27)$$

The initial value of ARC is 0.5 for the three payoff matrices because it only depends of the random distribution of the probability of cooperation and the initial composition of the population. The ARC was measured again after a hundred periods, when this measure has been reduced about ten times from its starting value for the PD, game used as baseline.

The effect of an increase in difference between payoffs in the cooperation dilemmas was studied using simulations for nine different payoff matrices with 3000 repetitions for each parameter configuration. Simulation results are shown in Figure 6. The baseline is represented by the point where the increase in inequality is zero, and the dimensions of fear and greed were raised until they reaches eight units respectively, where the payoff matrices matches the previously displayed in Figure 5 for the CUF and CUG games.

Figure 6 confirms that an increase in inequality has a greater effect over the cooperation under the dimension of fear than under the dimension of greed. When inequality reaches ten units in greed or fear according to the simulated game, the difference in the ARC between this couple of configurations is remarkable: its value in the CUF is 0.389 times the value in CUG.

Figure 6. Reduction of cooperation with an augmentation in inequality

In order to verify that the effects found are caused by inequality *per se* and not just by an increase in the distance between payoffs, we introduce the difference between the payoffs for reward and punishment ($R-P$), known as the cooperator's gain (Ahn *et al.*, 2001). This distance between the symmetric payoffs can be interpreted as a measure of the incentives to cooperate, given that an increase in the cooperator's gain can improve the payoff of both players simultaneously.

A new set of simulations was implemented to prove that the effect of the variation in the cooperator's gain is null, and as a consequence is the inequality and not the distance between payoffs the mechanism that accelerates the extinction of cooperation. The three payoff matrices defined in Figure 5 were set as baselines, and the difference ($R-P$) was modified from two to six units, maintaining fixed the distances ($T-R$) and ($P-S$). In this simulation were made 3,000 repetitions for each matrix payoff, for a total of 45,000 repetitions. Figure 7 shows that variations in the value of cooperator's gain have a null effect over the *ARC*: its value is maintained over 0.054 in the standard PD, close to 0.039 in the CUG case and in 0.015 in the CUF. This result confirms that the effect of the dimensions of fear and greed is due to inequality, not to net differences between the attainable payoffs.

Figure 7. Cooperator's gain effect on ARC

In terms of policy recommendations, this result suggests that an effort to promote cooperation focused in the reduction of inequality could be more effective than an increase in the difference between the payoffs of the reward and the punishment, in other words, the perception of a reduction of inequality could bring better results in terms of cooperation than an increase in the benefits when the players pass from mutual defection to mutual cooperation.

4.4. Evolutionary stability of defection

The evolutionary stability of cooperation was tested using simulations with the introduction of mutants periodically at a fixed rate. Results show that the trajectories towards mutual defection recover their original path. The number of periods between invasions was established in forty, decision based on the trade-off between the fast extinction of cooperation and the time required for the system to return to its original trajectory.

The algorithm for the insertion of invasions in the system is described as follows: first, the size of the invasion was set in 5% of the total population, partitioned in five new strategies with an initial fraction of $x=0.01$ each, identical to the starting fraction of the original strategies. Secondly, the fraction of the rest of strategies was reduced in 5% in order to maintain the total population normalized in unity and keep constant the proportion between the current strategies. In third place was defined the probability of cooperation of the invaders. Our case of interest is an invasion of cooperators, and the probability of cooperation of these new strategies corresponds to a random draw

from a uniform distribution between 0.95 and 1.00. Finally, the entrant strategies were included and the system develops until the next invasion 40 periods later.

The results of the introduction of mutations in PD and its variations are shown in Figure 8. The three panels expose the evolution of the strategies for CUF, PD and CUG during 200 periods; time corresponding to four invasions. As in Figure 5, the vertical axis represents the fraction of population immersed in each strategy and the horizontal axis the periods of time. Now each line represents a single strategy, because the aggregation in quartiles does not allow identifying clearly the effect of the invasions. The intensity in the color of the lines describes the probability of cooperation: the darkest lines are associated to the strategies with the lowest probability and the lighter lines represent the strategies with higher chance of cooperation. It can be observed that after each invasion appears a temporary fall in the fraction of the strategies, but after a reduced number of periods the system recovers its original path. This behavior confirms the evolutionary stability of defection in all three games, independently of the level of inequality.

To check the robustness of the results related to the stability the simulations were repeated with three different modifications: the period of time between invasions, the total size of the mutation introduced in the system and the type of mutants included. The frequency of invasions was varied from one invasion each 30 periods to one each 100 periods; finding that less distance between invasions slows down the convergence to defection, but this fact is explained by a higher amount of exogenously introduced cooperators that delays defection. The size of the invasion was increased to 10% of the total population, and the result was a slight reduction in the speed

of convergence, but does not derive in a modification of the trajectory. Finally, to check the effect of the type of invaders its probability of cooperation was drew from a uniform distribution between 0.475 and 0.525; one more time, the evolution of the system was not affected in a different that with cooperative mutants.

5. Conclusions

The inclusion of higher differences between the attainable payoffs in a cooperation dilemma gave origin to a couple of variations from the standard PD which is subject to considerable levels of inequality. These variations appear due to an increase in the dimensions of greed and fear, originating the games defined as Cooperation Under Fear (CUF) and Cooperation Under Greed (CUG).

The classification across cooperation dilemmas presented in this work is based on a set of restrictions associated to the attainable differences between the players' payoffs. With the different payoff structures clearly defined, were explored the implications of the redistributive inequality represented in the fear and greed dimensions. These dimensions, combined with prosocial preferences related to fairness, can provide explanations to the existence of outcomes not predicted by the Nash equilibrium in cooperation dilemmas.

The relationship between the standard PD, the CUF and the CUG was analyzed under the frameworks of classic game theory and evolutionary game theory, finding in both cases the relevance of the magnitude and the direction of the payoffs' differences. The first part of this work explores the differences under complete contracts, where an agreement of an *ex-post* redistribution modifies the payoffs of the three games in such a different way that new and different equilibria appear for each dilemma. Also was studied the sustainability of mutual cooperation in a repeated game, finding that an augmentation in the dimension of greed requires more patient individuals. The alternation between the two strategies maximizes the expected payoff for both players in the CUG, but as was shown, the assumptions over the ability to coordinate and over the willingness to accept the sucker's payoff at the beginning of the game are very restrictive.

In the second part of the work, under the evolutionary framework, the equilibrium in the three games derives into the extinction of cooperators, but the rate of convergence depends on the level and the direction of inequality in the game. The results of the simulations ratify the theoretical results previously mentioned. The average rate of cooperation (*ARC*) was defined as a measure of the evolution of cooperation that allows comparisons between simulations with variations in the parameters. The effect of inequality in the early extinction of cooperation was ratified based on more than 60,000 simulations; furthermore, using the variation in the cooperator's gain (*R-P*) was discarded the possible explanation that the drop of cooperation was due to the distance between payoffs as an effect of inequality *per se*. This result implies that institutional mechanisms designed for the promotion of cooperation could be more effective if are focused in the reduction of the differences between payoffs from

both parties rather than in the promotion of benefits received in the migration from defection to cooperation.

In the comparison of the existent results related to conditional cooperation across the three dilemmas, the main result is related to the generous tit-for-tat strategy (GTFT): In a CUG does not exist a stochastic rate of forgiveness after the defection of the opponent in the last round, while in the CUF the probability of forgiveness is higher (or at least equal) than in the PD. This result is consequent with the objective of the GTFT strategy, which tries to generate enough incentives to avoid deviations from mutual cooperation.

The effect of inequality in the decision of cooperation remains considerably unexplored. Two recommendations for further research correspond to computational simulations using reactive strategies and the design and conduction of laboratory economic experiments. In the field of simulations, the inclusion of strategies that responds to the opponent's history of game is relevant in the exploration of inequality's role over retaliation. In economic experiments, the exploration of fear and greed under the three games, CUG, PD and CUF, will result useful in the exploration of how the perception of inequality of economic agents influences their cooperation decisions.

References

- Ahn, T., Ostrom, E., Schmidt, D., Shupp, R., & Walker, J. (2001). Cooperation in PD games: Fear, greed and history of play. *Public Choice* .
- Alesina, A., & Angeletos, G.-M. (2005). Fairness and Redistribution. *The American Economic Review* , 960-980.
- Axelrod, R. (1984). *The Evolution of Cooperation*. New York: Basic Books.
- Axelrod, R., & Hamilton, W. (1981). The Evolution of Cooperation. *Science*, Vol. 211 .
- Bénabou, R., & Tirole, J. (2006). Belief in a Just World and Redistributive Politics. *The Quarterly Journal of Economics* .
- Bowles, S. (2011). Is Liberal Society a Parasite on Tradition? *Philosophy and Public Affairs* , 46-81.
- Bowles, S. (2003). *Microeconomics: Behavior, Institutions and Evolution*. Princeton University Press.
- Boyce, W. E., & DiPrima, R. C. (2002). *Elementary Differential Equations and Boundary Value Problems*. New York: Wiley Custom Services.
- Frohlich, N., & Oppenheimer, J. (1996). When Is Universal Contribution Best for the Group? Characterizing Optimality in the Prisoners' Dilemma. *The Journal of Conflict Resolution*, Vol. 40, No. 3 , 502-516.
- Gintis, H. (2000). *Game Theory Evolving*. Princeton University Press.
- Hurtado, J. (2011). Tocqueville on Poverty in Industrial Democracies. *Documentos CEDE* .
- Komorita, S., Sweeney, J., & Kravitz, D. A. (1980). Cooperative Choice in the N-Person Dilemma Situation. *Journal of Personality and Social Psychology* , 504-516.
- Kuhn, S., & Moresi, S. (2008). Pure and Utilitarian Prisoner's Dilemmas. *Economics and Philosophy*, 11. , 333-343.
- Maynard Smith, J., & Price, G. R. (1973). The Logic of Animal Conflict. *Nature* .
- Nowak, M. (2006). *Evolutionary dynamics: exploring the equations of life*. Cambridge: Harvard University Press.

- Nowak, M. (2006). Five Rules for the Evolution of Cooperation. *Science*, Vol 314 , 1560-1563.
- Nowak, M. (1990, V. 38). Stochastic Strategies in the Prisoner's Dilemma. *Theoretical Population Biology* .
- Rapoport, A., & Chammah, A. (1965). *Prisoner's Dilemma*. University of Michigan Press.
- Sally, D. (1995). Conversation and Cooperation in Social Dilemmas: A Meta-Analysis of Experiments from 1958 to 1992. *RATIONALITY AND SOCIETY*, Vol. 7, No.1 , 85-92.
- Sethi, R., & Somanathan, E. (1996). The Evolution of Social Norms in Common Property Resource Use. *The American Economic Review* .
- Sigmund, K., & Nowak, M. (1999). Evolutionary game theory. *Current Biology* .
- Sugden, R. (1989). Spontaneous Order. *Journal of Economic Perspectives* , 85-97.
- Tocqueville, A. d. (1969 [1835-1840]). *Democracy in America* (J. Mayer Ed. y G. Lawrence Trans.). Garden City: Anchor Books.
- Tucker, A. (2001). A Two-Person Dilemma. In E. Rasmusen, *Readings in Games and Information* (pp. 7-8). Wiley-Blackwell.
- Vega-Redondo, F. (2003). *Economics and the Theory of Games*. Cambridge: Cambridge University Press.

Appendix I. Analysis of CUF and CUG in repeated games

The results shown in subsection 2.4 are based in the comparison of intertemporal benefits of a group of relevant strategies in the preservation of cooperation. The net present value is defined as π_i and sums the discounted payoffs of each round W_i^t . For simplicity I will assume an infinite horizon, but similar results can be obtained with a finite horizon of time.

$$\pi_i = \sum_{t=1}^{\infty} \delta^{t-1} W_i^t$$

The first strategy described is intertemporal cooperation for both players, whose payoff is

$$\begin{aligned} \pi_i^{C\infty} &= R \sum_{t=1}^{\infty} \delta^{t-1} \\ \pi_i^{C\infty} &= \frac{R}{(1-\delta)} \end{aligned} \quad (6)$$

The second strategy to specify consists in mutual cooperation until a period τ and from $\tau + 1$ deviate to mutual defection. Assuming that $\tau > 0$, the corresponding payoff for this strategy is

$$\begin{aligned} \pi_i^{C\tau} &= \sum_{t=1}^{\tau} \delta^{t-1} (R) + \delta^{\tau} (T) + \sum_{t=\tau+1}^{\infty} \delta^{t-1} (P) \\ \pi_i^{C\tau} &= \left(\frac{1-\delta^{\tau}}{1-\delta} \right) R + \delta^{\tau} T + \left(\frac{\delta^{\tau+1}}{1-\delta} \right) P \end{aligned} \quad (7)$$

A third strategy, of special interest in the CUG game, consists in the coordinate alternation between cooperation and defection. We must define its expected payoff based in the benefits received in the strategy when the player starts cooperating and the opposite, where he starts defecting. The payoff for who starts with cooperation is

$$\begin{aligned} \pi_i^{C-D} &= T \sum_{t=1}^{\infty} \delta^{2t-1} + S \sum_{t=1}^{\infty} \delta^{2t-2} \\ \pi_i^{C-D} &= T \sum_{t=1}^{\infty} \delta^{2t-1} + S \sum_{t=1}^{\infty} \delta^{2t-2} = \left(\frac{\delta}{1-\delta^2} \right) T + \left(\frac{1}{1-\delta^2} \right) S \end{aligned}$$

And the payoff received by the player that begin defecting is

$$\pi_i^{D-C} = \left(\frac{\delta}{1-\delta^2} \right) S + \left(\frac{1}{1-\delta^2} \right) T$$

Assuming that, *ex-ante*, both players are indifferent between starts cooperating or defecting, they have the next expected payoff

$$\begin{aligned}
E[\pi_i^{Alternate}] &= \frac{1}{2}\pi_i^{C-NC} + \frac{1}{2}\pi_i^{NC-C} = \left(\frac{T+S}{2}\right)\left(\frac{1+\delta}{1-\delta^2}\right) \\
E[\pi_i^{Alternate}] &= \left(\frac{T+S}{2}\right)\left(\frac{1}{1-\delta}\right) \quad (8)
\end{aligned}$$

With the definition of equations (6) to (8) is possible to make a comparison of incentives between strategies. First, using equations (6) and (7) is possible to analyze how the required discount factor to sustain cooperation is affected by the level of inequality. We have

$$\begin{aligned}
\pi_i^{C\infty} &> \pi_i^{C\tau} \\
\frac{1}{(1-\delta)}R &> \left(\frac{1-\delta^\tau}{1-\delta}\right)R + \delta^\tau T + \left(\frac{\delta^{\tau+1}}{1-\delta}\right)P \\
\delta^\tau R - \delta^\tau T + \delta^{\tau+1}T - \delta^{\tau+1}P &> 0 \\
\delta &> \frac{T-R}{T-P} \quad (9)
\end{aligned}$$

This expression is independent of the parameter S , which implies that there is no difference between CUF and the standard PD. On the other hand, an increase in the dimension of greed ($T-R$) implies that the implementation of intertemporal cooperation requires individuals with a higher valuation of its future payoffs.

The comparison between equations (6) and (8) confirms that payoff alternation is a potential strategy under the incentives of a CUG if the assumptions mentioned before are plausible.

$$\begin{aligned}
E[\pi_i^{Alternate}] &> \pi_i^{C\infty} \\
\left(\frac{T+S}{2}\right)\left(\frac{1}{1-\delta}\right) &> \frac{R}{(1-\delta)} \\
T + S &> 2R
\end{aligned}$$

The expression shown above is validated only in a CUG, because it implies that the social welfare is maximized under an unequal outcome, not under mutual cooperation. The repetitive character of the game allows an equalitarian distribution of the benefits that were jointly maximized.

Appendix II. Variations in GTFT in the DPM game

To prove that in a CUF the minimum in equation (20) always corresponds to its second argument we start from the inequality

$$1 - \frac{T - R}{R - S} > \frac{R - P}{T - P}$$

Subtracting a unit to both sides we have

$$-\frac{T - R}{R - S} > \frac{R - P}{T - P} - 1$$
$$\frac{R - T}{R - S} > \frac{R - T}{T - P}$$

Dividing by

$$\frac{1}{R - S} < \frac{1}{T - P}$$

$$T - P < R - S$$

$$R + P > T + S$$

The restriction associated to the social welfare in a CUF implies that $2P > T + S$, and the arrangement of incentives of any game with a PD structure leads to $R > P$, then

$$R + P > 2P > T + S$$

Proving that in a CUF the probability of forgiveness is defined as

$$q = \frac{R - P}{T - P}$$

Appendix III. Matlab code for replicator dynamics

```
%% Simulation of a system of strategies under PD incentives

%% 1. Generate the strategies

% n is the number of strategies and t the amount of periods

n = 100; t = 100;

% Define a "step" for the actualization in the differential equations

h = 0.1;

% Generate a nx1 vector with each probability of cooperation

A = rand(n,1); A = sort(A);

%% 2. Set values of the parameters for PD, CUF or CUG

T = 10 ; R = 8 ; P = 4 ; S = -6;

%% 3. Obtain the payoff matrix

for i = 1:n % Each strategy defined in a row...

    for j = 1:n % Facing each strategy

        B(i,j)=R*(A(i)*A(j))+S*((A(i))*(1-A(j)))+...

            T*((1-(A(i)))*(A(j)))+P*((1-(A(i)))*(1-A(j)));

    end

end

%% 4. Replicator Dynamics

% Initial frequency of each strategy

X = ones(n,1)/n ;
```

```

% Fitness for each strategy

f = X'*B';

% Average fitness

phi = f*X ;

% Generate a matrix that stores the fractions of each period

Z = zeros(n,t);

% Define initial fractions

for i= 1:n

Z(i,1)= X(i);

end

% Replicator equation

for j = 1:t-1

for i = 1:n

DeltaZ(i)= X(i)*(f(i)-phi);

Z(i,j+1)= Z(i,j) + h*DeltaZ(i);

end

% Recalculate f and phi

for i = 1:n

X(i)= Z(i,j+1);

end

```

```

f = X'*B';

phi = f*X;

end

%% 5. Results

% Store in a vector the simulation's final result

for i= 1:n

Xfinal(i)= Z(i,t);

end

Output = [A Xfinal'];

Output = sortrows(Output);

% Aggregate strategies in quartiles

StratQuart = zeros(4,t);

for i=1:t

for j=1:25

StratQuart(1,i)=Z(j,i)+ StratQuart(1,i);

StratQuart(2,i)=Z(25+j,i)+ StratQuart(2,i);

StratQuart(3,i)=Z(50+j,i)+ StratQuart(3,i);

StratQuart(4,i)=Z(75+j,i)+ StratQuart(4,i);

end

end

Quartile1 = zeros(1,t); Quartile2 = zeros(1,t);

```

```
Quartile3 = zeros(1,t); Quartile4 = zeros(1,t);  
  
for i=1:t  
  
    Quartile1(i)=StratQuart(4,i); Quartile2(i)=StratQuart(3,i);  
  
    Quartile3(i)=StratQuart(2,i); Quartile4(i)=StratQuart(1,i);  
  
end
```