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Factor shares, inequality and capital flows

Hernando Zuleta *

Abstract

We present an endogenous growth model where innovations are factor saving and model the choice of technologies in an Overlapping Generations framework. Markets are competitive and factor prices are determined by marginal productivity of factors; therefore, the income share of reproducible factors increases with the stage of development. Beyond the standard results of this type of model we find that (i) without bequests long run growth is not possible, (ii) if the economy presents long run growth then intra generation inequality may last forever but if the economy does not present long run growth then in steady state there is no intra generation inequality (iii) when the economy is open, the pattern of capital flows depends not only on the relative abundance of factors but also on the technologies and, for this reason, capital may not flow from rich to poor economies (iv) consistently, capital flows may not help to break poverty traps.

JEL Classification: O11, O31, O33, O40, F20.

Keywords: inequality, capital markets, endogenous growth, capital income share, technology.

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Participación factorial, desigualdad y flujos de capital

Hernando Zuleta

Resumen

Se presenta un modelo de crecimiento endógeno donde las innovaciones son ahorradoras de factor y modela la elección de tecnologías en un marco de generaciones solapadas. Los mercados son competitivos y los precios de los factores están determinados por su productividad marginal; por lo tanto, la proporción del ingreso que se destina a remunerar factores reproducibles aumenta con el nivel de desarrollo. Más allá de los resultados típicos de este tipo de modelo se encuentra que (i) sin herencias el crecimiento de largo plazo no es posible, (ii) si la economía presenta crecimiento a largo plazo entonces desigualdad intra-generación puede durar para siempre pero si la economía no presenta crecimiento a largo plazo entonces en estado estacionario no hay desigualdad intra-generación (iii) cuando la economía está abierta, la dirección de los flujos de capitales no sólo depende de la abundancia relativa de los factores, sino también de las tecnologías y, por esta razón, el capital puede no fluir de países ricos a países pobres (iv) los flujos de capital no siempre ayudan a romper las trampas de la pobreza.

Palabras Clave: desigualdad, mercado de capitales, crecimiento endógeno, participación del capital, tecnología

Códigos JEL: O11, O31, O33, O40, F20.

I am grateful to Veneta Andonova, Giuseppe Bertola, Michele Boldrin, Tim Kehoe, Albert Marcet, Jose Vicente Rodríguez Mora, the editor Pietro Peretto and two anonymous referees for comments and suggestions. The usual caveat applies.

1 Introduction.

The role of capital markets in economic development is a central issue in macroeconomics. The standard neoclassical view suggests that these markets lead to the efficient allocation of savings by facilitating the flows of capital and, consequently, accelerating economic development in poor economies. According to this view, financial flows are equalizing forces, which bring a faster convergence across households and countries. However, some poor countries hardly receive private credit from abroad and there is a wide literature related to the question posted by Lucas (1990) “Why doesn’t capital flow from rich to poor countries?”

In this paper we present a biased innovations model based on Zuleta (2008a) to study the effects of investment flows on inequality both between and within countries. For simplicity, we consider only two factors in the model, one reproducible K and one not reproducible L . For concreteness, we refer to these factors as capital and labor respectively. We assume that innovations are labor saving and capital using, technologies can be changed paying a cost and technological change takes place only if the benefits are larger than the costs. Finally, we consider two types of heterogeneity in the model: First, old agents are capital owners and young agents are workers, and second, there are two types of capital owners differentiated by the amount of capital they own. In this setting, wealth inequality is transmitted from one generation to the other through bequests.

In our framework, agents live two periods. In the first period they work, consume and save. In the second period they use their savings to build capital and create or adopt technologies, produce, consume, leave a bequest to their children, and die. Finally, we assume that each firm operates with one plant and one technology.¹ This is a simplifying assumption that does not affect the main results of the model.

In models of factor saving innovations, factor shares are endogenous and, in equilibrium, they depend on the factor abundance of the economy. This fact implies that

¹Zuleta (2008a) and Peretto and Seater (2013) show that when there is a set of available technologies, it may optimal for the firms to work with two plants. However, if capital owners have to take care of administration and supervision and they are not willing to associate with other capital owners then each capital owner must work with only one plant and, consequently, one technology.

the process of economic growth is accompanied by a re-distribution of income in favor of capital owners and against workers. The change in the functional distribution of income has no consequences in a context of homogeneous agents. However, when we consider a model where workers and capital owners are different people with different savings rates the distributive issue becomes more relevant.

Because of the complementarity between factor abundance and factor shares, models of factor saving innovations often predict the existence of a poverty trap, namely, poor economies converge to a steady state (no long run growth). The other side of the story is that, because of this complementarity, for rich economies the capital share converges from below to a positive number equal or lower than one. Finally, long run growth can only be positive if the capital income share converges to one, that is, when the production function converges to an AK .²³

In this setting, there are two main results regarding inequality and investment flows: (i) when agents are heterogeneous within generations, the existence of capital markets and labor mobility eliminate intra generation inequality in the long run if the economy converges to a steady state (no long run growth). However, if there is long run growth then inequality is persistent. (ii) When we allow for international capital mobility, capital does not necessarily flow from rich to poor economies and, for this reason, capital flows may not generate convergence.

Result (i) is explained by the behavior of factor income shares. The income of the young consumer has two components: wage and bequest. Workers are homogeneous so there is only one wage for each generation. Capital stocks are different for different capital owners and, for this reason, capital income is higher for rich capital owners. As a result, bequests are higher for the descendants of rich capital owners. In the long run there are two possible scenarios:

- If there is long run growth then the labor income share decreases constantly and collapses to zero so the only source of income is the bequest. The marginal

²One of the main results of Zuleta (2008a) is that for rich economies the production function converges to an AK . This result is only one specific case in our framework. The analogous result here is that for rich economies the capital share converges from below to a positive number equal or lower than one.

³Other models where capital intensity is endogenous are found in Jones and Manuelli (1992), Klump and De la Grandville (2000), and Zuleta (2004) but such endogeneity is independent of the producer's decisions.

product of capital converges to a constant and, consequently, the interest rate is constant in the long run. For this reason, the capital income is persistently higher for the rich capital owners.

- If the economy converges to a steady state then the labor income share is positive and constant in the long run. Under such circumstances any increase in the capital labor ratio of the economy generates an increase in wages and a decrease in the interest rate. As the economy grows the bequest becomes a smaller part of the young agent's income. This redistribution of income in favor of labor and against capital reduces income differences between young agents and, for this reason, reduces inequality in savings and capital.

Result (ii) responds to the fact that rich economies are capital abundant and, for this reason, in these economies firms choose capital intensive technologies, while poor economies are labor abundant and firms choose labor intensive technologies. As a result, the elasticity of income with respect to capital is higher in rich economies. Therefore, the marginal product of capital (MPK) is not a monotonous function of the capital labor ratio. Moreover, there can be equalization of MPK with different capital labor ratios and different income levels.

The role of bequests is crucial for the possibility of long run growth and for the dynamics of income distribution. Given that old agents do not save, the only way in which an increase in capital income can affect savings is through bequests. Therefore, if there are no bequests any redistribution of income in favor of capital and against labor ends up reducing savings and, for this reason, there is no long run growth without bequests. Similarly, given that labor is homogeneous there is only one wage for each generation so the only way in which income differences can persist from generation to generation is through bequests and, for this reason, without bequests if there is inequality then it disappears after one period.

The paper at hand is related to Zeira (2007) who presents a theoretical contribution linking factor saving innovations, globalization and income distribution. However, regarding the global economy his emphasis is on international trade and not on capital flows. The model predicts that in developed countries technical progress and trade liberalization increase inequality, while in less developed countries technical progress

increases inequality but trade liberalization reduces it. Interestingly, the model also predicts that technical progress increases the share of trade only in developed countries. Though the type of technological change we consider is similar to that of Zeira (2007) the problems we study are different. Our main question is how capital markets affect income distribution within and between economies.

The relation between income distribution and growth has been studied thoroughly.⁴ The novelty of the paper at hand comes from the fact that technological change affects the marginal productivity of factors and, as a result, factor income shares and income distribution.

Studies of capital using and labor saving innovations constitute a substantial literature. Some examples include Zeira (1998), Acemoglu (2002), Boldrin and Levine (2002), Zuleta (2008a) and Peretto and Seater (2013). However, in this literature it is often assumed that agents are homogeneous and the main focus is not the issue of income distribution.⁵

This paper is also related to Bertola (1993 and 1996). He finds that, in a standard model with infinitely-lived agents, there is a positive relation between capital income share and economic growth (Bertola, 1993), but in an Overlapping Generations Model, a relationship of the opposite sign may appear (Bertola, 1996). We build on Bertola's argument by making endogenous the capital income share. Since we assume competitive markets, capital income share is determined by the capital intensity of the technology.

The paper is organized in 5 sections. In the next section we present some evidence in order to motivate the model. In section three we present the basic OLG model and the results. In the fourth section we consider two types of old agents, study the effects of capital and labor mobility under this setting and use these results to analyze the possible effects of international capital mobility. Section five concludes.

⁴See Persson and Tabellini (1994), Galor and Tsiddon (1997), Galor and Moav (2000) and Hassler and Rodríguez Cordero (2000), among others.

⁵Zuleta and Young (2013) present a two-sector (manufacturing and services) model of factor saving innovations. However, they also consider homogeneous agents.

2 Some Evidence about Factor Shares and Economic Growth.

As we state before, the models of capital using and labor saving innovations predict a positive relation between the income share of reproducible factors (the elasticity of output with respect to reproducible factors) and income per capita. This prediction is supported by the following pieces of evidence:

1. In the field of empirical economic growth, Durlauf and Johnson (1995), Duffy and Papageorgiou (2000) and Zuleta (2008b) find that as economies grow, their technologies become more intensive in reproducible factors, that is, the elasticity of output with respect to reproducible factors is higher in rich economies.
2. With regard to the behavior of factor income shares, we know that:
 - In USA, the share of land in Net National Product has a negative trend (see Rhee, 1991) and, regarding cross section data, the income share of natural capital is negatively correlated with income per capita (Caselli and Feyrer, 2007)⁶.
 - Over the past 60 years, the US relative supply of skilled work has increased rapidly. However, there has not been a downward trend for the returns to college education. On the contrary, over this period, the college premium has increased. Moreover, according to Gottshalk (1997), real wages of unskilled labor significantly fell during the last few decades of the 20th century. Finally, according to the calculations made by Krueger (1999) in the US the raw labor share decreased from 0.12 to 0.06 between 1980 and 1999.
 - It has been argued that labor income share does not decrease or increase with development (Gollin, 2002). However, the standard measure of labor income share includes skilled and unskilled labor income share, that is, it includes human capital. In the same way, the standard measure of capital

⁶Sturgill (2012) confirms this result and finds also a positive correlation between human capital's share and income per capita.

income share includes land income share. Therefore, given the behavior of unskilled labor and land income share, it seems that the income share of non-reproducible factors (land and unskilled labor) has decreased, while the income share of reproducible factors has increased during the 20th century.

3. Since the late 70's a dramatic decrease in labor income share has occurred in the OECD countries (see Blanchard, 1998, Bentolila and Saint Paul, 2003, Bils and Zuehlke, 2003, and Maliranta, 2012, and Zuleta, 2012). In Figure 1 we report the change in labor shares for a group of OECD countries between 1975 and 2000. The labor income share decreased for the majority of the countries.⁷ In figure 2A we plot the change in labor shares and the growth rate of GDP per worker (taken from the Penn World Data Tables). Though the negative correlation is apparent, New Zealand and Netherlands are clear outliers.⁺ This fact may be explained by the behavior of hours worked in these countries. To control for changes in hours worked, in figure 2B we plot the change in labor shares and the growth rate of GDP per hours worked. Unfortunately, there is no data available for all the countries in the sample.⁸ However, there is information for Netherlands, one of the outliers. The negative correlation is apparent and there are no clear outliers.

⁷The data was taken from Finnoff and Jayadeff (2006). They use the United Nations data set to construct labor shares.

⁸There is no data for Canada, Iceland, New Zealand, Norway, Switzerland and USA.

Figure 1

Change in Labor Shares (1975-2000)
Source: United Nations and Finnoff and Jayadeff (2006)

Figure 2A

Change In Factor Shares and Growth of GDP per Worker
Source: United Nations and Finnoff and Jayadeff (2006)
Penn World Data Tables

Figure 2B

In summary, we observe that the income share of non-reproducible factors (land and unskilled labor) has decreased, while the income share of reproducible factors has increased during the 20th century. We also observe a positive correlation between income per capital and capital share. However, we cannot claim that the share of income paid to capital is approaching to 100% because we do not know if the increasing trend is going to persist forever. For this reason, we consider a general cost function for the adoption of new technologies. With the general cost function we get the result of an increasing capital share. However, the long run value for this variable depends of the specific functional form assumed.

As stated before, we consider only two factors in the model, one reproducible K and one not reproducible L . To do such an aggregation we need to make the following simplifying assumptions: (i) Young people do not own human (or physical) capital but they can acquire some for the next period by reducing consumption. (ii) Old people own reproducible factors (physical and human capital). Therefore, old people may

work but their labor income comes completely from human capital. In other words, the worker decides if he/she wants to become skilled but can only use the skills next period (when old). Additionally, raw labor cannot be used by the elderly. (iii) Young people receive land as a bequest, use it, and transfer it to the next generation

Land cannot be consumed or reproduced, so it must be transferred from old to young people. For simplicity, we assume that young people receive land as a bequest. However, land is a constant factor, as it is the case with the variable L and must be transferred from one generation to the other regardless of the preferences of the consumers. For this reason, we do not explicitly model land bequests and, from now on, we use the term bequests to denote transfers of income from one generation to the other.

3 The Model.

In this section we present an OLG model where agents can leave bequests. For simplicity, we assume a logarithmic utility function, so current savings are completely determined by current levels of capital and technology.

Savings are used to build or buy capital goods and these goods can be of different qualities. Technology is embodied in capital goods, and technological changes are *labor-saving* and *capital-using*. In particular, we consider a Cobb-Douglas production function ($Y = AK^\alpha L^{1-\alpha}$) and assume that technologies are differentiated by their capital intensity α . Any technology has a positive cost which depends on the desired α . This cost is paid by the old people before the production process.

The capital used in the production process is a share $\delta(\alpha)$ of their savings. That is, if an agent devotes s units of output to build a stock of capital of quality α , the amount of capital of quality α is equal to $\delta(\alpha)s$. We also assume the existence of a primitive technology (α_0), whose cost is equal to the price of the consumption good ($\delta(\alpha_0) = 1$). The function $\delta(\alpha)$ can be understood as the productivity of research and development ($R\&D$). As α increases, the productivity of $R\&D$ falls (i.e. $\delta'(\alpha) < 0$) and a larger sacrifice of resources is required to attain better technologies (higher levels of α). In other words, as technology advances, improving technology gets harder; as α gets bigger a given amount of savings s yields a smaller quantity of capital of quality α . The intuition for this reduction in the quantity of capital is that savings go to $R\&D$ and capital production. Therefore, if $R\&D$ is more expensive there are fewer savings

available for producing capital. Finally, the primitive technology (α_0) does not require *R&D* so savings can be fully converted in to capital of quality α_0 .

We are aware of the fact that the process of creation of new technologies is not deterministic as we are assuming. However, the aim of the paper is not to explain the process of creation of new technologies but to explain (i) why some societies adopt capital intensive techniques of production while others use labor intensive technologies and (ii) what are the effects of these differences in terms of functional distribution of income and economic growth.

The share of savings devoted to produce the final good ($\delta(\alpha)$) is assumed to be a continuous function with the following properties:

$$(i) 0 \leq \delta(\alpha) \leq 1; (ii) \delta(\alpha_0) = 1; (iii) \delta'(\alpha) < 0; (iv) \delta''(\alpha) \leq 0$$

where α_0 is the primitive technology and $\delta'(\cdot)$ is the first derivative of $\delta(\cdot)$ with respect to α . Therefore, the cost of new technologies is higher for more *capital-intensive* technologies. The assumption that the cost of α depends positively on its value implies that, at any given time, capital goods that embody labor saving technologies cost more than capital goods that embody labor intensive technologies.

3.1 Consumption, savings and bequests

The representative consumer lives two periods and her utility depends on the consumption when young (c), the consumption when old (d) and an inter-generational transfer or bequest (b). We also assume zero population growth and a logarithmic utility function, which combines the three arguments (c , d and b). The income of a young consumer is given by the wage w plus the transfer that⁹she gets from her parents. We assume full depreciation, so the return to savings is the interest rate r . The young consumer takes α as given, so the problem for the consumer is the following:

$$\max \{ \log(c_t) + \beta (\log(d_{t+1}) + \gamma \log(b_{t+1})) \} \text{ s.t. } w_t + b_t = c_t + \frac{d_{t+1} + b_{t+1}}{r_{t+1}} \quad (1)$$

where β is the discount factor. From this maximization problem we find:

$$c_t = \frac{d_{t+1}}{\beta r_{t+1}} \quad (2)$$

$$b_{t+1} = \gamma d_{t+1} \quad (3)$$

and

$$s_t = (w_t + b_t) \left(\frac{\beta(1+\gamma)}{1+\beta(1+\gamma)} \right) \quad (4)$$

Since the elasticity of substitution between present and future consumption is equal to one, savings do not depend on the interest rate. This is a standard result in Overlapping Generations Models (see Auerbach and Kotlikoff, 1995). However, two new elements appear. First, α is not a parameter but a variable, and, second, the stock of capital depends not only on savings, but also on the technology. Therefore, factor prices depend not only on factor abundance but also on the technology.

3.2 Technology, factor prices and savings

For simplicity, we assume that the population is equal to 1 for each generation, so total output is Ak_t^α .

Agents decide savings when young, so when they are old they take savings as given. We also assume perfect competition so capital owners take factor prices as given. In this context the elderly choose technology α_t , and labor l_t in order to maximize their income taking into account $k_t = \delta(\alpha_t)s_{t-1}$.

$$\max_{\alpha_t, l_t} \left[A(\delta(\alpha_t)s_{t-1})^{\alpha_t} l_t^{1-\alpha_t} - w_t l_t \right] \quad (5)$$

Therefore, factor prices are determined by savings and technology,

$$w_t = (1 - \alpha_t) A(\delta(\alpha_t)s_{t-1})^{\alpha_t} \quad \text{and} \quad r_t = \alpha_t A(\delta(\alpha_t)s_{t-1})^{\alpha_t-1} \quad (6)$$

The interest rate and the wage are given by the factor's marginal productivity. So α is equal to capital income share and $(1 - \alpha)$ is equal to labor income share.

Similarly, the optimal level of α_t is given by the following equality:

$$A(\delta(\cdot)s_{t-1})^{\alpha_t} \left(\ln(\delta(\cdot)s_{t-1}) + \delta'(\cdot) \frac{\alpha_t}{\delta(\cdot)} \right) = 0 \quad (7)$$

Note that after paying the cost of technology, the capital labor ratio must be greater than one, that is, $\delta(\cdot)s_{t-1} > 1$, otherwise it is preferable to keep the primitive technology. Rearranging equation 7,

$$\ln s_{t-1} = -\delta'(\cdot) \frac{\alpha_t}{\delta(\cdot)} - \ln \delta(\cdot) \quad (8)$$

Deriving equation 8, it is possible to find the relation between the savings, s , and technology, α ,

$$\frac{\partial \ln s_{t-1}}{\partial \alpha_t} = -\delta'(\cdot) \left(\frac{2}{\delta(\cdot)} \right) + \frac{\alpha_t}{\delta(\cdot)} \left((\delta'(\cdot))^2 \frac{1}{\delta(\cdot)} - \delta''(\cdot) \right) \quad (9)$$

Both the first and the second terms of equation 9 are non-negative, so there exists a positive relation between α_t and s_{t-1} .

From equations 8 and 9, we can define the function $s^{**}(\alpha) = \frac{e}{\delta(\cdot)} - \delta'(\cdot) \frac{\alpha}{\delta(\cdot)}$ and relate s and α . Depending on the shape of the function $\delta'(\cdot)$ there may exist a minimum level of savings greater than one, such that an increase in α is profitable. Therefore, $s_{t-1} > 1$ is a necessary but may not be a sufficient condition for investment in higher qualities of capital to be profitable. A sufficient condition is $s_{t-1} > s^{**}(\alpha_0)$.

We can summarize the previous results in Proposition 1:

Proposition 1 (i) $s_{t-1} > 1$ is a necessary condition for an increase in capital intensity (and capital income share) to be profitable. (ii) $s_{t-1} > s^{**}(\alpha_{t-1})$ is a sufficient condition for an increase in capital intensity (and capital income share) to be profitable (iii). In capital abundant economies ($s_{t-1} > s^{**}(\alpha_0)$), there exists a positive relation between α_t and s_{t-1} .

On the one hand, given the technology, the production function is concave and satisfies the Inada conditions, in particular $\lim_{s \rightarrow 0} \frac{\partial y}{\partial s} = \infty$. On the other hand, the marginal productivity of a unit of savings invested in new technologies positively depends on the amount of savings. Therefore, for low savings levels it is better not to invest in capital intensive technologies. Now, if the savings level is high enough to make technological changes profitable, then there exists a positive relation between savings and technologies because the gains derived from adopting capital intensive technologies positively depend on the level of savings.

Figure 3

In Figure 3, we plot α against s using the function $s^{**}(\alpha) = \frac{e^{-\delta'(\cdot)\frac{\alpha}{\delta(\cdot)}}}{\delta(\cdot)}$. The first thing to note is that α is constant for saving levels below $s^{**}(\alpha_0)$. Second, above $s^{**}(\alpha_0)$ there exist a positive relation between α and s . Third, in the long run α converges to a number, $\alpha_{\max}$, from below. Depending on the functional form of $\delta(\alpha)$, $\alpha_{\max}$ may be lower or equal to one. It is straight forward that $\lim_{\alpha \rightarrow 1} \delta(\alpha) > 0$ is a necessary condition for $\alpha_{\max}$ to be equal to one.

The intuition for this result is clear: In economies where capital is relatively abundant it is better to use capital more intensively than labor while in economies where labor is abundant it is better to use labor in a more intensive way.

3.2.1 Savings

The output must be divided between young people (w) and old people (rk). Old people consume and leave a bequest ($rk = b + d$). Therefore,

$$A(\delta(\alpha_t)s_{t-1})^{\alpha_t} = w_t + b_t + d_t \quad (10)$$

Combining with equations 3 and 6, and rearranging, yields

$$b_t = \left(\frac{\gamma}{1+\gamma} \right) \alpha_t A(\delta(\alpha_t)s_{t-1})^{\alpha_t} \quad (11)$$

so equation 4 can be written as

$$s_t = A(\delta(\alpha_t)s_{t-1})^{\alpha_t} (1 + \gamma - \alpha_t) \left(\frac{\beta}{1 + \beta(1 + \gamma)} \right) \quad (12)$$

Therefore, the savings level at time t is fully determined by the state variables of time $t - 1$. This result, which depends on the assumed utility function, greatly simplifies the analysis. Indeed, since in the optimum technology is a function of savings, technology at time t is also fully determined by the state variables at time $t - 1$.

We can find a steady state level of savings given the technology:

$$s_{ss}(\alpha) = \left(\frac{(1 + \gamma - \alpha) \beta A \delta(\cdot)^\alpha}{1 + \beta(1 + \gamma)} \right)^{\frac{1}{1-\alpha}} \quad (13)$$

Now, to have an idea of the shape of $s_{ss}(\alpha)$ we take logs and derivatives,

$$\frac{\partial \ln s_{ss}}{\partial \alpha} = \frac{1}{1 - \alpha} \left(\ln s_{ss} + \left(\alpha \frac{\delta'(\cdot)}{\delta(\cdot)} - \frac{1}{1 + \gamma - \alpha} \right) \right) \quad (14)$$

Note that the term $\left(\alpha \frac{\delta'(\cdot)}{\delta(\cdot)} - \frac{1}{1 + \gamma - \alpha} \right)$ is always negative because $\delta'(\cdot) < 0$ and $\alpha < 1$. So for low levels of s_{ss} the slope is negative, but for high levels of s_{ss} the slope of $s_{ss}(\alpha)$ is positive. Therefore, both the level of s_{ss} for a given α and the slope of $s_{ss}(\alpha)$ depend of Total Factor Productivity, A , preference for bequest, γ , the discount factor, β , and the cost of technologies, $\delta(\cdot)$.

Finally, from equation 13 it follows that (i) if $\frac{\gamma \beta A \delta(\cdot)^\alpha}{1 + \beta(1 + \gamma)} < 1$, then the function $s_{ss}(\alpha)$ is bounded from above and (ii) $\frac{\gamma \beta A \delta(\cdot)^\alpha}{1 + \beta(1 + \gamma)} > 1$ for any α is a necessary condition for the economy to have long-run growth. We present this result in Proposition 2.

Proposition 2 $\lim_{\alpha \rightarrow 1} \delta(\alpha) > \frac{1 + \beta(1 + \gamma)}{A \gamma \beta}$ is a necessary condition for the economy to present long-run growth.

Therefore, bequests must be positive ($\gamma > 0$) and TFP high, otherwise long-run growth is not possible.⁹ Additionally, for the economy to reach long-run growth, the cost of capital goods must be bounded ($\lim_{\alpha \rightarrow 1} \delta(\alpha) > 0$).

We explain the intuition behind Proposition 2 in the following lines:

First, savings at time $t + 1$ are the product of the savings rate and the output at time t . So given the savings rate, the higher the TFP, the higher the savings level. For any initial conditions, if TFP is low, savings are low, the technology is labor intensive and long-run growth is not possible.

⁹This result was previously obtained by Boldrin (1992) and Jones and Manuelli (1992) in different setups.

Second, since output is divided between workers and capital owners and the latter do not save, the savings rate of the economy depends on the labor income share. If the technology continuously grows more capital intensive then the labor income share decreases period by period. If bequests are zero, then the savings rate decreases as the technology becomes more capital intensive and converges to zero as α goes to one. Therefore, if there are no bequests long-run growth is not possible.

Third, if $\lim_{\alpha \rightarrow 1} \delta(\alpha) = 0$ then the cost of capital goods goes to infinity as α goes to one, so the incentives to adopt more capital intensive technology disappear.

Finally, note that from equation 12 it follows that if there is long run growth then the rate of growth converges to a constant, $\lim_{\alpha \rightarrow 1} \frac{s_t}{s_{t-1}} = \lim_{\alpha \rightarrow 1} \delta(\alpha) \left(\frac{A\gamma\beta}{1+\beta(1+\gamma)} \right)$. In other words, in the long run the growth rate of the economy is constant.

3.3 Steady state and long-run growth

Until now we have described the behavior of capital abundant economies, where agents have incentives to adopt technical changes and the behavior of labor abundant economies, where there are no incentives for technical changes. In this section we identify the conditions under which an economy can achieve long-run growth and characterize the steady state, whenever it exists.

To characterize the steady state we can use the functions $s^{**}(\alpha)$, which shows the relation between technology and savings that old agents choose, and $s_{ss}(\alpha)$, which shows the steady state level of savings given the technology. Using these functions it is possible to find a steady state level of technology, α_{ss} , defined by $s_{ss}(\alpha_{ss}) = s^{**}(\alpha_{ss})$. Note that, if $s = s^{**}(\alpha_{ss})$, then the chosen technology is α_{ss} and if the technology is α_{ss} then the steady state level of savings is $s_{ss}(\alpha_{ss})$. Therefore, α_{ss} is a steady state technology.

In figures 4 and 5 we plot the functions $s^{**}(\alpha)$ and $s_{ss}(\alpha)$. In figure 4 we assume that the slope of $s_{ss}(\alpha)$ is negative (low levels of A , γ and β). In this case the two functions cross only once, so there is one steady state technology (α_{ss}) and one steady state savings level ($s_{ss}(\alpha_{ss})$). Note also that the steady state is stable, that is, if $s > s_{ss}(\alpha_{ss})$, then the growth rate of savings is negative and if $s < s_{ss}(\alpha_{ss})$, then the growth rate of savings is positive.

In figure 5 we assume that the slope of $s_{ss}(\alpha)$ is positive and converges to infinity when α goes to one (high levels of A , γ and β and $\lim_{\alpha \rightarrow 1} \delta(\alpha) > \frac{1}{A} \frac{1+\beta(1+\gamma)}{\gamma\beta}$). In this case, there may be two steady states, one stable, α_{ss} , and one unstable, α^* . However, we may also have no steady state. Indeed, any increase in TFP moves up the function $s_{ss}(\alpha)$, augmenting α_{ss} and reducing α^* . Therefore, there exists a level of TFP, A^{**} , such that $\alpha_{ss} = \alpha^*$. From the definition of A^{**} it follows that if $A > A^{**}$, there is no steady state.

Figure 4

Figure 5

Proposition 3 summarizes the previous results.

Proposition 3 *If $\lim_{\alpha \rightarrow 1} \delta(\alpha) > \frac{1}{A} \frac{1+\beta(1+\gamma)}{\gamma\beta}$ and $A < A^{**}$, then there exists a level of technology, α^* , such that: (i) $s_{ss}(\alpha^*) = s^{**}(\alpha^*)$, (ii) if under the initial conditions the agents of the economy choose a technology α such that $\alpha < \alpha^*$, the economy converges to a steady state and (iii) if under the initial conditions the agents of the economy choose a technology α such that $\alpha > \alpha^*$, the economy has long-run growth.*

Thus, a minimum level of savings, $s^{**}(\alpha^*)$ is needed to achieve long-run growth.

The productivity of new technologies depends on the savings level and the productivity of savings depends on the capital intensity of the technology. Therefore, if savings are small it is not worth it to invest in capital intensive technologies and if the technology is labor intensive the effect of savings on output is small. Thus, if the initial conditions are such that the technology is labor intensive and the savings level is small, then the economy can be in a poverty trap. However, if the initial conditions are such that the savings level is high, then it is optimal to adopt capital intensive technologies and the effect of savings on output is big. Thus, if the initial conditions are such that the technology is capital intensive and the savings level is high, the economy presents long-run growth.

Propositions 1, 2 and 3 tell us that capital abundant economies have incentives to increase α and for these economies any increase in α generates increases in output and savings, yielding a virtuous cycle of technological change and economic growth. This process of economic growth may last forever if TFP and bequests are high and if the cost of capital goods is bounded. The other side of the story is that when TFP and bequests are not high enough, poor economies are trapped in a steady state.

In summary, we have presented an OLG model, where any technology can be adopted paying a cost. Markets are competitive and innovations affect factors income share. As a result, the elasticity of output with respect to reproducible factors and the income share of such factors (physical and human capital) depend on the factor abundance of the economies. Under this framework, depending on the initial conditions, in some economies the production function converges to AK, while in other economies, long-run growth is zero.

3.4 Dynamics of the model

So far we have presented the model, characterized the steady state and described the possible equilibrium paths including long-run growth and a poverty trap. In this section we combine the production function, the savings rate and the relation between technology and savings to illustrate the dynamics of the model in a graphical way. For shortness, we concentrate on the case where long run growth is possible.

We use the inverse of the function $s^{**}(\alpha)$ to express technology, α , as a function of savings, $\alpha(s)$.

Given the Cobb-Douglas production function, if $s_{t-1} < s^{**}(\alpha_0)$ then

$$\frac{\partial Y}{\partial s_{t-1}} = \alpha_0 A (s_{t-1})^{\alpha_0-1} \quad \text{and} \quad \frac{\partial^2 Y}{\partial s^2} = (\alpha_0 - 1) A (s_{t-1})^{\alpha_0-1} < 0$$

namely the function is concave in savings. However, if $s_{t-1} > s^{**}(\alpha_0)$, then

$$\frac{\partial Y}{\partial s} = A (\delta(\cdot) s_{t-1})^{\alpha(\cdot)} \left(\frac{\alpha(\cdot)}{s_{t-1}} + \alpha'(\cdot) \left(\alpha(\cdot) \frac{\delta'(\cdot)}{\delta(\cdot)} + \ln(\delta(\cdot) s_{t-1}) \right) \right) \quad (15)$$

From equation 8 it follows that $\alpha(\cdot) \frac{\delta'(\cdot)}{\delta(\cdot)} = \ln(\delta(\cdot) s_{t-1})$, so

$$\begin{aligned} \frac{\partial Y}{\partial s_{t-1}} &= \alpha(\cdot) A (\delta(\cdot) s_{t-1})^{\alpha(\cdot)-1} \delta(\cdot) \\ \text{and } \lim_{\alpha \rightarrow 1} \frac{\partial Y}{\partial s_{t-1}} &= A \lim_{\alpha \rightarrow 1} \delta(\cdot) \end{aligned} \quad (16)$$

Thus, if $\lim_{\alpha \rightarrow 1} \delta(\cdot) > 0$, then in the long-run, the production function converges to a linear function of type AK . Therefore, we can plot production and savings in a graph (see figure 6). The bold line is the production function. It is concave for low levels of savings and as the savings grow, the slope of the function gets closer to a constant ($A \lim_{\alpha \rightarrow 1} \delta(\cdot)$).

Figure 6

The savings rate is given by $(1 + \gamma - \alpha_t) \left(\frac{\beta}{1 + \beta(1 + \gamma)} \right)$, so if $s_{t-1} < s^{**}(\alpha_0)$, it is constant and if $s_{t-1} > s^{**}(\alpha_0)$ it decreases as α grows (see figure 7) and converges to $\left(\frac{\gamma\beta}{1 + \beta(1 + \gamma)} \right)$ as α goes to one.

Figure 7

Finally, given the behavior of the savings rate, we can plot the level of savings at time $t + 1$ as a function of the level of savings at time t (see figure 8). When savings at time t are smaller than $s^{**}(\alpha_0)$, savings are a constant proportion of output. When $s_{t-1} > s^{**}(\alpha_0)$, the savings rate decreases with economic growth. However, the savings rate is bounded from below and in the limit it is equal to $\frac{\gamma\beta}{1+\beta(1+\gamma)}$, so as savings converge to infinity, the savings rate converge to a constant and the growth rate of savings becomes equal to the growth rate of output. That is, the slope of the savings function becomes constant in the long-run. The dashed line in figure 8 is the 45° line, so when the savings function is above the dashed line, the growth rate of savings is positive and when it is below the dashed line, the growth rate of savings is negative.

Figure 8

In figure 8 we observe that depending on the initial level of savings, in some economies the production function converges to AK, while in other economies, long-run growth is zero.

4 Inequality and Capital Flows

4.1 Inequality Within Generations

Until now we have considered a framework where agents are homogeneous within generations. However, income inequality is also an intra-generational phenomenon. For this reason, in this section we consider two types of old people differentiated only by their wealth and explore the interactions between growth and income distribution. In this setting, inequality is transmitted from one generation to the other through bequests. We begin the analysis assuming that each old person owns a firm and, given the amount of savings, chooses capital and technology.

In the absence of innovations the marginal productivity of capital is negatively related to the capital labor ratio. In this setting, if firms have different capital labor ratios then capital markets operate until the point where marginal productivity of capital is the same in all firms. Given that the capital labor ratio must be the same in every firm, wages must be identical for all the workers.

Now, suppose that the savings of a capital owner ($s_{R,t}$) are such that this agent has incentives to increase the capital share. If this is the case then there are two possible scenarios.

Scenario 1: Capital owners use the capital market and then choose the technology.

Scenario 2: Capital owners choose technology and then use the capital market.

In an economy without frictions Scenario 1 would be the relevant one. However, if there are firms with limited access to credit then Scenario 2 becomes relevant. The literature on credit constraints is vast and many authors claim that less wealthy households are more likely to be credit-constrained (see for example Hayashi, 1985 and Zeldes, 1989). Similarly, empirical tests support the hypothesis that small firms tend to be disadvantaged relative to their larger counterparts in terms of access to finance (see Audretsch and Elston, 1994 and Beck, Demirgüç-Kunt and Maksimovic, 2005). Therefore, following the literature on credit constraints we can argue that firms, in general, begin small and as a result have limited access to capital markets. Consequently they have to produce and gain a certain size before becoming eligible for receiving loans. In these circumstances, firms must choose the technology before having access to capital markets. Because technologies are embodied in capital goods and many times are irreversible, firms are likely to keep the technologies they chose before having access to capital markets.

4.1.1 Scenario 1.

If capital owners use the capital market first then when they choose technology the capital labor ratio is the same for all firms and, for this reason, the chosen technology is the same. In this setting, using equation 4 we can find the savings of the rich ($s_{R,t}$) and poor ($s_{P,t}$) workers are respectively,

$$s_{R,t} = ((1 - \alpha_t)Ak_t^{\alpha_t} + \left(\frac{\gamma}{1 + \gamma}\right)\alpha_t s_{R,t-1}Ak_t^{\alpha_t-1}) \left(\frac{\beta(1 + \gamma)}{1 + \beta(1 + \gamma)}\right) \quad (17)$$

$$s_{P,t} = ((1 - \alpha_t)Ak_t^{\alpha_t} + \left(\frac{\gamma}{1 + \gamma}\right)\alpha_t s_{P,t-1}Ak_t^{\alpha_t-1}) \left(\frac{\beta(1 + \gamma)}{1 + \beta(1 + \gamma)}\right) \quad (18)$$

from where

$$s_{R,t} - s_{P,t} = (s_{R,t-1} - s_{P,t-1}) \left(\gamma \frac{\alpha_t \beta}{1 + \beta(1 + \gamma)}\right) Ak_t^{\alpha_t-1} \quad (19)$$

and

$$\lim_{\alpha \rightarrow 1} (s_{R,t} - s_{P,t}) = (s_{R,t-1} - s_{P,t-1}) \left(\frac{\gamma \beta A}{1 + \beta(1 + \gamma)}\right) \quad (20)$$

If a steady state exists then it is characterized by $k^* = \left(\frac{(1 - \alpha_t + \gamma)\beta A}{1 + \beta(1 + \gamma)}\right)^{\frac{1}{1 - \alpha}}$. Therefore, the difference in savings between rich and poor is $s_{R,t} - s_{P,t} = (s_{R,t-1} - s_{P,t-1}) \left(\frac{\gamma \alpha}{1 - \alpha + \gamma}\right)$.

Given that $\frac{\gamma \alpha}{1 - \alpha + \gamma} < 1$ it follows that if there is a steady state then in the long run $s_{R,t} - s_{P,t} < (s_{R,t-1} - s_{P,t-1})$ for any $s_{R,t-1} > s_{P,t-1}$. In other words, if a steady state exists then the differences in income decrease period by period and converge to zero in the long run.

From equations 19 and 20 we see that if $\frac{\gamma \beta A}{1 + \beta(1 + \gamma)} < 1$ then intra-generation income inequality decreases with time and disappear in the long run. On the other hand, if the capital share converges to one and $\frac{\gamma \beta A}{1 + \beta(1 + \gamma)} > 1$ then income differences increase with time. Finally, from proposition 2 it follows that $\frac{\gamma \beta A}{1 + \beta(1 + \gamma)} \geq 1$ is a necessary condition for long run growth. Therefore, we can formalize the results of this subsection in proposition 4.

Proposition 4 *In the long run: (i) If the capital share is smaller than one, then there is no intra generation inequality. (ii) If the capital share converges to one then there is inequality in the long run.*

The income of the young consumer has two components: wage and bequest. Bequests are higher for the descendants of rich capital owners while wages are equal for all workers.

If the capital share converges to one then the labor income collapses to zero in the long run so the only source of income is the bequest. The marginal product of capital converges to a constant and, consequently, the interest rate is constant in the long run and, for this reason, the capital income is persistently higher for the rich capital owners.

If the economy does not present long run growth then the labor income share is positive and constant in the long run. Therefore, any increase in the capital labor ratio of the economy generates an increase in wages and a decrease in the interest rate. So, as the economy grows the bequest becomes a smaller part of the young agent's income. This redistribution of income in favor of labor and against capital reduces income differences between young agents and, for this reason, reduces inequality in savings and capital.

4.1.2 Scenario 2:

From equations 6 and 9 it follows that capital owners with different savings may choose different technologies. If this is the case and capital owners choose technology before using the capital market then there are two types of firms with different capital labor ratios and different technologies.

If firms choose different technologies then factors move until their marginal productivity is the same in all firms. From equation 6 it follows that the capital labor ratio in the two firms must be such that:

$$\frac{k'}{k} = \frac{\alpha'}{1 - \alpha'} \frac{1 - \alpha}{\alpha} \quad (21)$$

Where k' and α' are the capital labor ratio and the capital share of the firm owned by the rich old agent and k and α the capital labor ratio and the capital share of the firm owned by the poor old agent.

Equation 21 guarantees that the marginal productivity of both factors is the same

for every firm. Now, combining equations 21 and 6 we get,

$$k' = \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}} \quad (22)$$

$$k = \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha'}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha'}{\alpha'-\alpha}} \quad (23)$$

Therefore, the capital labor ratio for the two firms is completely determined by the technologies. Note that for the labor abundant firm the capital labor ratio is smaller than one and, for this reason, capital owners don't have incentives to increase the capital share. Note also that equations 22 and 23 only hold if the capital labor ratio of the economy is lower than k' and higher than k . Otherwise, all factors are allocated to one type of firm. Indeed, given the average capital labor ratio k_T , k' and k must satisfy

$$k_T = \lambda k + (1-\lambda)k' \quad (24)$$

where λ is the share of labor allocated to the firms with low capital labor ratio. Labor is mobile so λ is endogenous and

$$\lambda_{eq} = \frac{\left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}} - k_T}{\left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}} - \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha'}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha'}{\alpha'-\alpha}}} \quad (25)$$

Where λ_{eq} is the equilibrium level of λ when labor is fully mobile.

Finally, note that given that there are two types of firms and one does not change the technology, the average capital labor ratio (k_T) is given by $\frac{\delta(\alpha')s_{Rt} + s_{Pt}}{2}$.

The implications of equations 22 to 25 are summarized in proposition 5.

Proposition 5 *When there are two types of firm and two different technologies α and α' such that $\alpha' > \alpha$:*

1. *If $\frac{\delta(\alpha')s_{Rt} + s_{Pt}}{2} \geq \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}}$ then only technology α' is used and firms with low levels of capital disappear.*
2. *If $\frac{\delta(\alpha')s_{Rt} + s_{Pt}}{2} \leq \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha'}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha'}{\alpha'-\alpha}}$ then only technology α is used.*
3. *If $s_{Pt} > \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha'}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha'}{\alpha'-\alpha}}$ then capital flows from poor to rich old agents.*
4. *If $\delta(\alpha')s_{Rt} > \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}}$ then capital flows from rich to poor old agents.*

In this scenario capital markets work but given that firms owned by rich agents use more capital intensive technologies, in equilibrium, they also choose a higher capital labor ratio. Therefore, the standard result that capital flows to the firms where it is relatively scarce does not necessarily hold.

Now, given that wages are the same for all workers and there is only one interest rate and Proposition 4 holds in this Scenario.

4.2 Capital Flows

4.2.1 Capital and Labor are Mobile

A world with two different countries and perfect factor mobility is, for the purpose of the model, very similar to the case of two types of old people. If we redefine the variables k and α as the capital labor ratio and the capital share of the labor abundant economy (poor economy) and k' and α' as the capital labor ratio and the capital share of the capital abundant economy (rich economy) then we can use the results of the previous subsection in order to analyze the possible effect of capital flows.

From the results of Scenario 1 we find that if capital flows are fast, that is, if they happen before technological changes, then after capital flows (i) capital per worker and wages are the same in the two economies, (ii) capital income may be different in the two countries, (ii) there is convergence in growth rates but not necessarily in income levels. In other words, capital movement may not eliminate income differences among citizens of different countries.

From Scenario 2 we find that if capital flows are slower than technological changes then different countries choose different technologies. In this setting, capital may flow in any direction.

We illustrate this in figure 9. Average capital labor ratio is in the horizontal axis and savings per worker of the poor economy in the vertical axis. To save notation we define $k_{\alpha'} = \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha}{\alpha'-\alpha}}$, $k_{\alpha} = \left(\frac{\alpha}{\alpha'}\right)^{\frac{\alpha'}{\alpha'-\alpha}} \left(\frac{1-\alpha}{1-\alpha'}\right)^{\frac{1-\alpha'}{\alpha'-\alpha}}$ and $k_T = \frac{\delta(\alpha')s_{Rt}+s_{Pt}}{2}$.

First of all note that the white area is not feasible because the capital per worker of the poor economy must be smaller than the average capital per worker. Second, given the technologies, when the average capital labor ratio is small (area A), factors flow to the poor economy and only the less capital intensive technology (α) is used. When the

average capital labor ratio is big (area D), factors flow to the rich economy and only the most capital intensive technology (α') is used. Third, when the average capital labor ratio is between k_α and $k_{\alpha'}$ the rich economy uses the capital intensive technology and the poor economy uses the labor intensive technology. In this case, if the savings per worker of the poor economy are small (area B) then capital flows from the rich to the poor economy and if the savings per worker of the poor economy are big (area C) then capital flows from the poor to the rich economy.

Finally, note that a technological change in the capital abundant country (increase in α') increases the size of the area C+B so, given the supply of factors, it increase the chances of using the two technologies.

Figure 9

4.2.2 Income Differences, GDP and GNP

Note that capital flows affect Gross Domestic Product (*GDP*) and Gross National Product (*GNP*) in different ways. *GDP* includes the capital income associated to capital goods within the geographical limits of the country, while *GNP* includes the capital income associated to capital goods owned by citizens of the country regardless of the location of such capital goods:

$$GDP_{pw} = rk + w \text{ and } GNP_{pw} = rs + w.$$

In scenario 1 there is factor price equalization and the technology is the same for both economies. Therefore, both capital labor ratio and *GDP* per worker are equalized. However the wealth of the agents differs and, for this reason, *GNP* per worker is higher in the rich economy. In other words, in scenario 1, capital flows can eliminate differences in *GDP* per worker but not differences in *GNP* per worker.

In scenario 2 technologies are different. Therefore, even though there is equalization of the ratio of factor prices, the capital labor ratio in the poor economy is smaller as well as *GDP* per worker.

Casselli and Feyrer (2007) calculate the Marginal Productivity of Capital (MPK) for a broad set of countries and find that “The return from investing in capital is no higher in poor countries than in rich countries. This means that one can rationalize virtually all of the cross-country variation in capital per worker without appealing to international capital-market frictions.” They also quantify the output losses due to the MPK observed differences and find that: “If we were to reallocate capital across countries so as to equalize MPKs, the corresponding change in world output would be negligible.”

The empirical results obtained by Caselli and Feyrer (2007) are consistent with the predictions of the model: Poor economies are likely to have lower capital shares and equalizing MPKs cannot be expected to get rid of income differences.

4.2.3 What if labor is not internationally mobile?

Until now, we have assumed that both factors are internationally mobile. However, many times labor is not perfectly mobile. If there is no labor mobility then λ is constant. From equation 25 it follows that if the population is distributed between the two countries in such a way that $\lambda > \lambda_{eq}$ then, in equilibrium, the capital labor ratio of the poor economy is lower than k_α . In other words, given the world capital labor ratio and the available technologies, the lack of labor mobility reduces the capital labor ratio and the wage of the poor economy. This implies that the lack of labor mobility decreases the chance for capital flows to eliminate poverty traps.

5 Conclusions

Traditionally, growth theory considers innovations as changes in TFP. Here we consider the possibility of changing the factor intensity of the technology. Using this framework, we find that capital abundant countries have incentives to make labor saving innovations, while labor abundant economies do not have incentives to innovate.

If TFP and bequests are high enough, a raise in capital share (α) increases the future capital labor ratio. As the capital labor ratio increases, α grows too. Thus, for initially capital abundant economies, both the capital labor ratio and the capital share grows. Depending on the cost of new technologies the production function of these economies may converge to AK, so they can have permanent growth even without changes in TFP. If bequests and TFP are small, there is no possibility of long-run growth because technological change reduces wages and capital accumulation.

For economies where the capital labor ratio is low, there are no incentives to innovate. Nevertheless, if the steady state capital labor ratio is high enough, at some moment the incentives to change technology appear.²⁴ Depending on the TFP and bequests, these economies can be trapped in a steady state or can reach a capital labor ratio high enough to start making innovations.

When agents are heterogeneous within generations, the existence of capital markets and labor mobility may eliminate intra generation inequality in the long run if the economy converges to a steady state (no long run growth). However, if there is long run inequality is persistent.

Finally, when we allow for international capital mobility, capital does not necessarily flow from rich to poor economies and, for this reason, capital flows may not generate convergence.

The model we have presented here is a simplification that helps to identify some of the implication of capital using and labor saving innovations. For future research it can be interesting to analyze the effects of these innovations in a setting where assets are randomly distributed among agents and workers can be capital owners. Other extension can be related to the fiscal policy implications of this type of technological change.

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