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Leverage away your wedge: An analysis of banks' impact on output*

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Abstract

In this paper we present a general equilibrium model where heterogeneous agents endogenously choose whether to become workers, consumers or entrepreneurs in order to analyze how limits on the leverage of banks affect real output. In our model tighter limits on the leverage of banks cause an increase in the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits. A higher spread results in two types of distortions: First, firms with the same productivity will have different size. Second, productive firms will cease to exist, while nonproductive ones will enter. These distortions result in lower production.

JEL Classification: E44, G21, G28

Keywords: Banking leverage, misallocation

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Apalancando la cuña: un análisis del impacto de los bancos en la producción

Enoch Hill David Perez-Reyna

Resumen

En este artículo presentamos un modelo de equilibrio general donde agentes heterogéneos escogen entre ser trabajadores, consumidores o empresarios para analizar cómo los límites en el apalancamiento de los bancos afecta la producción real. En nuestro modelo límites mas estrictos en el apalancamiento de los bancos causa un aumento en el margen de intermediación de los bancos. Un margen más amplio tiene como consecuencia dos distorsiones: primero, empresas con la misma productividad tienen tamaño diferente. Segundo, empresas productivas dejan de existir, mientras que empresas no productivas entran. Estas distorsiones resultan en una menor producción.

JEL Classification: E44, G21, G28

Palabras clave: apalancamiento de los bancos, distorsiones

1 Introduction

The banking sector doesn't produce a tangible product but it is clear that it has a tangible effect on the real economy. In this paper we develop a model which allows us to analyze what these effects are and the channel through which these effects are transmitted. Using our model we find that the leverage of banks has both direct and indirect effects on occupational choice, and also indirectly effects the distribution of firm sizes, and real output. The primary channel through which these effects are transmitted is through the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits which we will henceforth refer to as the margin of intermediation. We will consider a model without risk so we will only focus on the downside of having limits on the leverage of banks.

In our model firms need to pay for their workers before they produce, in the spirit of Arellano et al. (2012). To do this, they can either use their own assets or can take out loans from the banking sector. The primary result from our model is that as banks become less leveraged (i.e. the ratio between deposits over equity decreases), the resulting margin of intermediation in the general equilibrium increases. This margin is responsible for two types of distortion relative to a model with an unconstrained banking sector: First, firms with the same productivity will hire different amounts of workers depending on the assets of the firm. Second, skilled unwealthy consumers will choose to work rather than become entrepreneurs, while nonskilled wealthy consumers will choose to manage firms. In addition, lower leverage in the banking sector will require more bankers to satisfy the demand for loans and deposits. Each of these factors results in a reduction in real output.

An appealing feature of our model is its clear and intuitive characterization of occupational choice among consumers. We allow heterogeneity in consumers along two dimensions; namely, wealth and skill. Rich unskilled consumers choose to become bankers while unwealthy unskilled consumers become workers and skilled consumers choose to manage firms as entrepreneurs. In the parameterization where banks are infinitely leveraged, real allocations are not dependant upon wealth and the model collapses to the model in Lucas (1978) where wealth only affects consumption but has no effect on occupational choice or real output. In this case skilled consumers choose to become entrepreneurs while unskilled consumers choose to work; and the marginal productivity across firms is constant and firms are perfectly assortative in size along the skill of entrepreneurs.

As the leverage of the banking sector decreases, the margin of intermediation increases. As a consequence, the wealth of consumers begin to have real effects on occupational choices of consumers and the hiring decisions of firms, since this causes the cost of the marginal worker to differ based on the wealth of the entrepreneur. Essentially, wealthy entrepreneurs

face a lower marginal cost per worker than unwealthy entrepreneurs, which causes firm size to vary across the wealth of entrepreneurs. For two entrepreneurs with the same skill level, the wealthier entrepreneur will hire more workers than the less wealthy entrepreneur. If the skill of the entrepreneurs is sufficiently small, it is possible that the unwealthy consumer will prefer to become a worker rather than manage a firm and face the higher interest rate on loans required to hire workers. Additionally, it is possible that wealthy consumers, who would have worked in the scenario where banks are infinitely leveraged, are incented to become entrepreneurs due to the reduction in the return of their assets. Even though their skill at managing workers is low relative to the rest of the entrepreneurs, they can obtain a higher return from using their wealth to hire workers rather than investing their assets with banks.

We relate our model to the misallocation literature. Banerjee and Moll (2010), Buera et al. (2011) and Midrigan and Xu (2014), among others, analyze the role of misallocation on the productivity of an economy. In these models, misallocation arises mostly due to financial constraints. In our case, it is the leverage of the financial intermediaries that causes the misallocation.

Our model is also related to the literature relating financial development and efficiency with production. Levine (2005) offers a comprehensive literature review of this field. In Greenwood et al. (2010) and Greenwood et al. (2013) costly state verification causes a difference in the marginal product of capital and its user cost. In our model it is the leverage of the banking sector that causes different consumers to face different interest rates.

Erosa (2001) is probably the most similar model to ours. In his model intermediation costs cause an exogenous margin of intermediation. In our case this margin of intermediation arises endogenously from the leverage of the banking sector. Erosa (2001) also analyzes occupational choice, although in his setup the only heterogeneity of consumers is in age, as his model is dynamic. We explore an additional occupational choice; namely, becoming a banker.

This paper is organized as follows: Section 2 presents the model; Section 3 characterizes the solution to the model; Section 4 presents a benchmark model without limits on the leverage of banks; Section 5 shows the main results; Finally, Section 6 concludes.

2 Model

We consider a two period model. Consumers are heterogeneously endowed with skill and wealth. At the beginning of the first period they choose whether to become workers, entrepreneurs or bankers. Workers receive their wages in the first period and save to consume

in the second period. Entrepreneurs manage firms and need to pay their workers in the first period. Nonetheless, the firms they manage produce in the second period. Therefore they might need to borrow from banks in order to pay their wage bills. Bankers set up a bank in the first period and receive the profits from the bank in the second period. Banks take deposits from consumers and lend to firms. They face an exogenous limit on their leverage.

We assume that there is a unit measure of consumers who maximize utility by choosing to be an entrepreneur, a worker or a banker. Each consumer is endowed with a skill level z and some wealth a . We assume that z and a are drawn from a distribution with positive support that we will denote by $G(z \times a)$. A consumer's decision is characterized by z and a , so we will denote consumers by the realizations of these random variables. Consider consumer (z, a) . He solves the following problem

$$u(z, a) = \max_{\mathcal{O} \in \{\mathcal{W}, \mathcal{E}, \mathcal{B}\}} \{u^{\mathcal{W}}(a), u^{\mathcal{E}}(z, a), u^{\mathcal{B}}(a)\}, \quad (1)$$

where $u^{\mathcal{W}}(a)$ denotes the utility derived from becoming a worker, $u^{\mathcal{E}}(z, a)$ is the utility from becoming an entrepreneur, and $u^{\mathcal{B}}(a)$ is the utility from becoming a banker.

2.1 Workers

Denote the set of workers by $\mathcal{W}$. In period 1 workers use their wages w and wealth to consume and to save, s . In the second period the worker's income is given by the return on savings, r^D . The utility of being a worker with wealth a is given by (2).

$$\begin{aligned} u^{\mathcal{W}}(a) &= \max_s \ln c_1 + \beta \ln c_2 \\ c_1 &= w + a - s \\ c_2 &= (1 + r^D)s. \end{aligned} \quad (2)$$

2.2 Entrepreneurs

Denote the set of entrepreneurs by $\mathcal{E}$. In the first period entrepreneurs use their wealth to consume, to pay for the workers l they hire, and they can save or borrow. In the second period entrepreneurs consume the production of the firm. If they borrowed in the first period, they repay their debt at an interest rate of r^L . If they saved, they get a return of r^D on their savings. The utility of being an entrepreneur with skill z and wealth a is given by (3).

$$u^{\mathcal{E}}(z, a) = \max_{s, l} \ln c_1 + \beta \ln c_2 \quad (3)$$

$$c_1 = a - wl - s$$

$$c_2 = zl^\alpha + (1 + r^L)s\mathbf{1}_{\{s < 0\}} \\ + (1 + r^D)s\mathbf{1}_{\{s \geq 0\}}.$$

Each entrepreneurs belongs to one of three types: Entrepreneurs that borrow to pay for their workers, $s < 0$, entrepreneurs that have enough wealth to pay for workers and deposit the difference, $s > 0$, and entrepreneurs that spend all their available wealth to hire workers, $s = 0$. We will denote by $\mathcal{E}_L$ the set of entrepreneurs that borrow, by $\mathcal{E}_D$ the set of entrepreneurs that save and by $\mathcal{E}_O$ the rest of entrepreneurs.

2.3 Bankers

Denote the set of bankers by $\mathcal{B}$. In the first period bankers consume part of their wealth. The rest of their wealth, s , is used as equity for the bank they manage. In the second period bankers consume the profits from that bank. The utility of a banker with wealth a is given by (4).

$$u^{\mathcal{B}}(a) = \max_s \ln c_1 + \beta \ln c_2 \quad (4)$$

$$c_1 = a - s$$

$$c_2 = \pi^B(s).$$

The profits of a bank with equity s are given by (5)

$$\pi^B(s) \equiv \max_{L, D} (1 + r^L)L - (1 + r^D)D \quad (5)$$

$$\text{s. t. } D + s = L$$

$$\frac{D}{s} \leq \lambda.$$

The first constraint in (5) is the balance sheet constraint of the bank: The bank lends its equity and the deposits it takes. The second constraint implies that there is a limit on how many resources a bank can intermediate. Specifically the limit is on how many deposits a bank can take per unit of equity. This limit is exogenous and we denote it by λ .

We now define an equilibrium for this economy:

Definition 1. *An equilibrium for this economy is allocations $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$, $x^{\mathcal{E}}(z, a) \equiv \{l(z, a), s^{\mathcal{E}}(z, a)\}$, $x^{\mathcal{B}}(a) \equiv \{s^{\mathcal{B}}(a)\}$ and $x^{\mathcal{B}}(s) \equiv \{L^{\mathcal{B}}(s), D^{\mathcal{B}}(s)\}$, prices $p \equiv \{w, r^L, r^D\}$ and sets $\mathcal{W}$, $\mathcal{B}$ and $\mathcal{E}$ such that*

1. $\mathcal{W}$, $\mathcal{B}$ and $\mathcal{E}$ are such that $\mathcal{O}(z, a)$ is a solution to (1) for all (z, a) ;
2. given p , $x^{\mathcal{W}}(a)$ is a solution to (2);
3. given p , $x^{\mathcal{E}}(z, a)$ is a solution to (3);
4. given p , $x^{\mathcal{B}}(a)$ is a solution to (4);
5. given p , $x^{\mathcal{B}}(s)$ is a solution to (5);
6. and markets clear:

(a) Deposits:

$$\int_{\mathcal{W}} s^{\mathcal{W}}(a) dG(z \times a) + \int_{\mathcal{E}_D} s^{\mathcal{E}}(z, a) dG(z \times a) = \int_{\mathcal{B}} D^{\mathcal{B}}(s^{\mathcal{B}}(a)) dG(z \times a);$$

(b) loans:

$$\int_{\mathcal{E}_L} s^{\mathcal{E}}(z, a) dG(z \times a) + \int_{\mathcal{B}} L^{\mathcal{B}}(s^{\mathcal{B}}(a)) dG(z \times a) = 0;$$

(c) labor:

$$\int_{\mathcal{W}} dG(z \times a) = \int_{\mathcal{E}} l(z, a) dG(z \times a);$$

(d) goods:

$$\begin{aligned} \int c_1(z, a) dG(z \times a) &= \int a dG(z \times a) \\ \int c_2(z, a) dG(z \times a) &= \int_{\mathcal{E}} z l(z, a)^{\alpha} dG(z \times a). \end{aligned}$$

3 Characterizing the model

We will first prove that in equilibrium the interest rate of loans is greater than the interest rate on deposits. This implies that there is an incentive to manage a bank, rather to deposit in one. Additionally, the profits from banks are linear in the wealth that is used to run them.

Lemma 1. *In equilibrium $r^L \geq r^D > -1$ and $r^L = r^D$ only as $\lambda \rightarrow \infty$. Furthermore, the profits of a bank with equity s can be written as $\pi^B(s) = (1 + r^B)s$ with*

$$r^B = r^L + \lambda(r^L - r^D);$$

$r^B \geq r^D$ and $r^B = r^D$ only as $\lambda \rightarrow \infty$. Additionally, loan supply and deposit demand are given by

$$D^d(s) = \lambda s \tag{6}$$

$$L^s(s) = (1 + \lambda)s.$$

Proof. First notice that if $r^D \leq -1$, then banks will demand an infinite amount of deposits. If $r^L < r^D$ then banks will not supply loans since the cost of deposits is higher than the revenue they can get from lending, so it must be the case that $r^L \geq r^D$. Now, the bank is risk neutral. Therefore the second constraint in (5) binds. The supply of loans follows from the balance sheet constraint ($D(s) + s = L(s)$). This proves (6).

Plugging (6) into the objective function of (5) yields $\pi^B(s) = (1 + r^B)s$, where $r^B \equiv r^L + \lambda(r^L - r^D)$. Now, notice that the only way to have a finite r^B as $\lambda \rightarrow \infty$ is if $r^L = r^D$. Additionally, r^B can also be written as $r^B = r^D + (1 + \lambda)(r^L - r^D)$. $r^L \geq r^D$ implies that $r^B \geq r^D$, with equality only as $\lambda \rightarrow \infty$. $\square$

Now we will prove that both workers and bankers save in the first period. The reason for this is that these consumers have no source of income in the second period. Lemma 2 characterizes the solution of (2) and (4).

Lemma 2. *The solution of (2) is*

$$s^W(a) = \frac{\beta}{1 + \beta}(w + a). \tag{7}$$

The solution of (4) is

$$s^B(a) = \frac{\beta}{1 + \beta}a. \tag{8}$$

Proof. The first order condition of (2) is

$$\frac{1}{w + a - s} = \frac{\beta}{s}.$$

Lemma 1 implies that the first order condition of (4) can be written as

$$\frac{1}{a-s} = \frac{\beta}{s}.$$

Solving for s in (7) and (8) yields the result. $\square$

Finally we will characterize the solution of (3). Since entrepreneurs that borrow and entrepreneurs that save face a different interest rate, there will be misallocation: Firms with the same productivity have different sizes depending on the wealth of the entrepreneur that manages them. The misallocation depends on the margin of intermediation.

Lemma 3. *The solution of (3) is*

$$l(z, a) = \begin{cases} \left(\frac{\alpha z}{(1+r^L)w} \right)^{\frac{1}{1-\alpha}} & \text{if } a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \\ \frac{\alpha\beta}{1+\alpha\beta} \frac{a}{w} & \text{if } \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) \\ \left(\frac{\alpha z}{(1+r^D)w} \right)^{\frac{1}{1-\alpha}} & \text{if } a \geq \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) \end{cases}$$

$$s^{\mathcal{E}}(z, a) = \begin{cases} \frac{\beta}{1+\beta} a - \frac{1+\alpha\beta}{1+\beta} \left(\frac{z}{1+r^L} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} & \text{if } a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \\ 0 & \text{if } \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) \\ \frac{\beta}{1+\beta} a - \frac{1+\alpha\beta}{1+\beta} \left(\frac{z}{1+r^D} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} & \text{if } a \geq \delta_{\mathcal{E}_D, \mathcal{E}_O}(z), \end{cases}$$

where

$$\begin{aligned} \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) &\equiv \frac{1+\alpha\beta}{\beta} \left(\frac{z}{1+r^D} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \\ \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) &\equiv \frac{1+\alpha\beta}{\beta} \left(\frac{z}{1+r^L} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z)$ will borrow, entrepreneurs with $a \geq \delta_{\mathcal{E}_D, \mathcal{E}_O}(z)$ will save and entrepreneurs with $\delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z)$ will spend all their available wealth in paying for workers.

Proof. If $s \neq 0$ the first order conditions of (3) can be written as

$$\begin{aligned} \frac{w}{a-wl-s} &= \frac{\beta\alpha z l^{\alpha-1}}{z l^{\alpha} + (1+r)s} \\ \frac{1}{a-wl-s} &= \frac{\beta(1+r)}{z l^{\alpha} + (1+r)s}, \end{aligned}$$

where $r = r^D$ if $s > 0$ and $r = r^L$ if $s < 0$. If $s = 0$ first order conditions of (3) can be written as

$$\frac{w}{a - wl} = \frac{\alpha\beta}{l}.$$

The proof of the Lemma follows from solving for s and l . The expressions for $\delta_{\mathcal{E}_D, \mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O, \mathcal{E}_L}(z)$ follow from analyzing when $s > 0$ or $s < 0$. $\square$

For the moment fix w . Then there are two effects on entrepreneurs of having a positive margin of intermediation: The larger this margin is, the bigger the range in firm sizes for consumers with the same skill z across the spectrum of wealth a .

Wealthy entrepreneurs will have enough wealth to pay for their workers and save the difference. Due to this, the marginal cost of an employee will depend on r^D . On the other hand, unwealthy entrepreneurs need to borrow to pay for their workers, so the marginal cost of an employee will depend on r^L . Figure 1a highlights this point.

Additionally, a higher margin of intermediation implies that the difference in wealth between an entrepreneur that is able to save and an entrepreneur that borrows is higher. In other words, holding w constant, a higher margin of intermediation implies a larger difference between $\delta_{\mathcal{E}_D, \mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O, \mathcal{E}_L}(z)$. As a consequence less entrepreneurs use banks as financial intermediaries, since more entrepreneurs use all their available wealth to hire workers. See Figure 1b for a graphical representation of this point.

Figure 1: Entrepreneur with skill z

Corollary 1 shows misallocation in a slightly different way. Let $r(z, a)$ be the marginal return of hiring $l(z, a)$ workers. This return will be decreasing in the wealth of the en-

trepreneur. $r(z, a)$ is the opportunity cost of using wealth for hiring workers. If an unwealthy entrepreneur uses one extra dollar to hire workers, he is borrowing more and therefore is spending r^L . On the other hand, an extra unit of wealth that a wealthy entrepreneur spends on hiring workers could be used to get a return of r^D if it was used instead to save in a bank.

Corollary 1. *Let $r(z, a) \equiv \frac{z\alpha l(z, a)^{\alpha-1}}{w} - 1$. Then*

$$r(z, a) = \begin{cases} r^L & \text{if } a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \\ z \left(\frac{\alpha}{w}\right)^\alpha \left(\frac{1+\alpha\beta}{\beta a}\right)^{1-\alpha} - 1 & \text{if } \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) \\ r^D & \text{if } a \geq \delta_{\mathcal{E}_D, \mathcal{E}_O}(z). \end{cases}$$

$r(z, a)$ is continuous and decreasing in a .

With the results shown in Lemmas 2 and 3 we are able to determine the occupational choice of consumers. In Appendix A we determine explicitly the boundaries in skill and wealth that determine the occupational choice of consumers as a function of the prices in this economy. That allows us to fully characterize each consumer, taking prices as given.

From Lemma 3 we conclude that within the set of entrepreneurs, wealthy entrepreneurs (high a) will be able to save and unwealthy entrepreneurs (low a) will need to borrow. In general we find that consumers with low wealth a and low skill z will choose to become workers. The fact that these consumers have low skill makes it better for them to work than to set up a firm. Additionally, their low wealth makes it optimal for them to get an extra source of income in the first period. The only way to do this is by becoming a worker.

On the other hand, consumers with low skill and high wealth will become bankers. Similar to workers, having a low skill level is a deterrent from becoming an entrepreneur. Nonetheless, the high level of wealth makes it better for these consumers to set up a bank, rather than to become workers, since $r^B > r^D$ in equilibrium. Finally, entrepreneurs will be consumers with high skill. As shown in Lemma 3, the level of wealth will affect the size of the firm that they manage.

It is worth mentioning that the occupational choice of consumers depends on their wealth. Figure 2 shows graphically the different occupation choices of consumers in (z, a) space.

4 Model with perfectly efficient banking sector

Contrast the model characterized in Section 3 with a model where the margin of intermediation is 0. In this model there will only be an interest rate r . As mentioned in Lemma 1, this can be achieved in the limit as λ approaches infinity. Recall from Lemma 1 that in this

Figure 2: Types of consumers depending on skill and wealth

$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

case $r^B = r^L = r^D$, so only a consumer with infinite wealth will be willing to be a banker and this bank will be infinitely leveraged and have 0 profits. We can interpret this case as a model where consumers do not need a financial intermediary to get wealth from consumers that are willing to save to borrowing entrepreneurs. In this case consumers choose whether to become entrepreneurs or workers. The utility of a consumer endowed with skill level z and wealth a is given by

$$u(z, a) = \max_{\mathcal{O} \in \{\mathcal{W}, \mathcal{E}\}} \{u^{\mathcal{W}}(a), u^{\mathcal{E}}(z, a)\}, \quad (9)$$

where

$$u^{\mathcal{W}}(a) = \max_s \ln c_1 + \beta \ln c_2 \quad (10)$$

$$c_1 = w + a - s$$

$$c_2 = (1 + r)s.$$

and

$$u^{\mathcal{E}}(z, a) = \max_{s, l} \ln c_1 + \beta \ln c_2 \quad (11)$$

$$c_1 = a - wl - s$$

$$c_2 = zl^\alpha + (1 + r)s.$$

In this case, entrepreneurs face the same interest rate, regardless if they borrow or save. Workers, as before, will decide to save since they don't have any source of income in the second period. An equilibrium for this economy is defined as follows.

Definition 2. *An equilibrium for this economy is allocations $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$ and $x^{\mathcal{E}}(z, a) \equiv \{l(z), s^{\mathcal{E}}(z, a)\}$, prices $p \equiv \{w, r\}$ and sets $\mathcal{W}$ and $\mathcal{E}$ such that*

1. $\mathcal{W}$ and $\mathcal{E}$ are such that $\mathcal{O}(z, a)$ is a solution to (9) for all (z, a) ;
2. given p , $x^{\mathcal{W}}(a)$ is a solution to (10);
3. given p , $x^{\mathcal{E}}(z, a)$ is a solution to (11);
4. and markets clear:

(a) *Savings:*

$$\int_{\mathcal{W}} s^{\mathcal{W}}(a) dG(z \times a) + \int_{\mathcal{E}} s^{\mathcal{E}}(z, a) dG(z \times a) = 0;$$

(b) *labor:*

$$\int_{\mathcal{W}} dG(z \times a) = \int_{\mathcal{E}} l(z) dG(z \times a);$$

(c) *goods:*

$$\begin{aligned} \int c_1(z, a) dG(z \times a) &= \int a dG(z \times a) \\ \int c_2(z, a) dG(z \times a) &= \int_{\mathcal{E}} z l(z)^{\alpha} dG(z \times a). \end{aligned}$$

Lemmas 4 and 5 characterize the solution to (10) and (11).

Lemma 4. *The solution of (10) is*

$$s^{\mathcal{W}}(a) = \frac{\beta}{1 + \beta}(w + a).$$

Proof. The first order condition of (10) is

$$\frac{1}{w + a - s} = \frac{\beta}{s}.$$

Solving for s yields the result. □

Lemma 5. *The solution of (11) is*

$$l(z) = \left(\frac{\alpha z}{(1+r)w} \right)^{\frac{1}{1-\alpha}}.$$

$$s^{\mathcal{E}}(z, a) = \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta} \left(\frac{z}{1+r} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}}.$$

Let

$$\delta_{\mathcal{E}_D, \mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta} \left(\frac{z}{1+r} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs will save if $a \geq \delta_{\mathcal{E}_D, \mathcal{E}_L}(z)$ and borrow otherwise.

Proof. The first order conditions of (11) can be written as

$$\begin{aligned} \frac{w}{a - wl - s} &= \frac{\beta \alpha z l^{\alpha-1}}{z l^{\alpha} + (1+r)s} \\ \frac{1}{a - wl - s} &= \frac{\beta(1+r)}{z l^{\alpha} + (1+r)s}, \end{aligned}$$

The proof of the Lemma follows from solving for s and l . The expression for $\delta_{\mathcal{E}_D, \mathcal{E}_L}(z)$ follows from analyzing when $s > 0$ or $s < 0$. $\square$

Notice that in this case the size of the firms does not depend on the wealth of the entrepreneur. With the results shown in Lemmas 4 and 5 we are able to determine the occupational choice of consumers. In Appendix B we determine the explicit boundaries between the two occupational choices of consumers as functions of w and r .

Since every consumer faces the same interest rate, the boundary that determines the occupational choice between workers and entrepreneurs will not depend on the level of wealth. Similar to Lucas (1978), the occupation choice depends exclusively on skill level. Less skilled consumers will become workers since the consumption they get from setting up a firm would be lower than consumption from working. Figure 3 shows a graphical characterization of consumers in the (z, a) space.

We solve this model by stating and solving an equivalent Social Planner Problem (See Appendix C).

Figure 3: Types of consumers depending on skill and wealth

$\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

5 Results

We first prove a lemma that allows us to characterize labor remuneration in the model. We then provide an example which allows us to get a closed form solution. Finally we show some numerical results to highlight the main results of our model.

5.1 Labor remuneration is constant

Lemma 6 proves that labor remuneration in the model is constant.

Lemma 6. *Let A be the total amount of wealth in the economy and denote the mass of workers by $M_{\mathcal{W}}$. Then*

$$wM_{\mathcal{W}} = \alpha\beta A.$$

Proof. See Appendix D. □

We give an overview of the proof of Lemma 6. Consider first the case where wages are paid in the same period as when production takes place. Given the production function of the firms in our model, it holds that $wM_{\mathcal{W}} = \alpha Y$, where Y denotes total production. Now, in our model wages are paid in period 1, while production takes place in period 2. Therefore the marginal cost of labor depends on interest rates. that is,

$$(1 + \tilde{r})wM_{\mathcal{W}} = \alpha Y, \tag{12}$$

where $\tilde{r}$ is an average interest rate of the economy.¹ Additionally, in our model total consumption in the first period is given by the total amount of wealth in the economy, A , while total consumption in the second period equals total production, Y . Finally, the fact that consumers have log utility implies the following aggregate Euler equation

$$Y = \beta(1 + \tilde{r})A. \quad (13)$$

Plugging in (12) into (13) yields the result in the Lemma. Lemma 6 implies that the effect of productivity affects wages indirectly through the measure of workers. In other words, more productive economies will have higher wages since the measure of workers will be smaller.

5.2 Example

We consider a particular distribution that allows us to find a close form solution to the model. Given constraints on parameters, we are able to abstract from changes in occupational choice to focus on the main source of distortion; namely, the difference in size by firms with the same productivity. Finally we are able to derive an analytical solution for total output and show that it is increasing in λ since output decreases with the margin of intermediation. The distribution we consider is specified in Definition 3.

Definition 3. *Let $\tilde{G}(\cdot)$ be the following distribution on z and a :*

1. *z takes values z_1 and z_2 , with weights δ_z and $1 - \delta_z$, respectively. We assume $z_1 < z_2$.*
2. *For $z = z_1$: a takes values a_1 and a_2 , with weights δ_a and $1 - \delta_a$, respectively. We assume $a_1 < a_2$.*
3. *For $z = z_2$: Continuum of values of a distributed according to a uniform distribution between 0 and $\bar{a}$.*

Proposition 1 states the equilibrium prices and occupational choices in this economy. The specific set of assumptions on parameters, as well as the proof of the proposition, can be found in Appendix D.

¹Recall from the corollary to Lemma 3 that each type of entrepreneur faces a different interest rate in this economy.

Proposition 1. *Given $\tilde{G}(\cdot)$,*

$$\begin{aligned}
w &= \alpha\beta \frac{\delta_a \delta_z a_1 + (1 - \delta_a) \delta_z a_2 + (1 - \delta_z) \frac{\bar{a}}{2}}{\delta_a \delta_z} \\
1 + r^L &= \frac{z_2}{\beta} (1 + \alpha\beta)^{1-\alpha} \left(\frac{\delta_a \delta_z}{\delta_a \delta_z a_1 + (1 - \delta_a) \delta_z a_2 + (1 - \delta_z) \frac{\bar{a}}{2}} \right)^\alpha \left(\frac{1 - \delta_z}{2\bar{a}(1 + \lambda)(1 - \delta_a) \delta_z a_2} \right)^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta} (1 + \alpha\beta)^{1-\alpha} \left(\frac{\delta_a \delta_z}{\delta_a \delta_z a_1 + (1 - \delta_a) \delta_z a_2 + (1 - \delta_z) \frac{\bar{a}}{2}} \right)^\alpha \\
&\quad \times \left(\frac{1}{1 - \left(\frac{2}{\bar{a}(1 - \delta_z)} \right)^{\frac{1}{2}} ((\lambda - \alpha\beta)(1 - \delta_a) \delta_z a_2 - \alpha\beta(1 - \delta_z) \frac{\bar{a}}{2} - (1 + \alpha\beta) \delta_a \delta_z a_1)^{\frac{1}{2}}} \right)^{1-\alpha}
\end{aligned}$$

and occupational choices

$$\begin{aligned}
\mathcal{W} &= (z_1, a_1) \\
\mathcal{B} &= (z_1, a_2) \\
\mathcal{E}_D &= \{(z_2, a) : \delta_{\mathcal{E}_D, \mathcal{E}_O}(z_2) \leq a \leq \bar{a}\} \\
\mathcal{E}_O &= \{(z_2, a) : \delta_{\mathcal{E}_O, \mathcal{E}_L}(z_2) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z_2)\} \\
\mathcal{E}_L &= \{(z_2, a) : 0 \leq a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z_2)\}
\end{aligned}$$

are an equilibrium in this economy, where $\delta_{\mathcal{E}_D, \mathcal{E}_O}(z)$, and $\delta_{\mathcal{E}_O, \mathcal{E}_L}(z)$ are as defined in Lemma 3.

Proof. See Appendix D. □

The fact that the margin of intermediation is decreasing in λ is a corollary to Proposition 1.

Corollary 2. $r^L - r^D$ is decreasing in λ .

Additionally, we prove that the differences in sizes for firms decreases as λ increases.

Corollary 3. *The dispersion in firm size is decreasing in λ .*

Proof. The dispersion in sizes of firms is increasing in $r^L - r^D$. □

Furthermore, from Proposition 1 we are able to derive an expression for total output in this economy. Proposition 2 proves that total production is increasing in λ , since it is decreasing in the margin of intermediation.

Proposition 2. *Let*

$$\mathcal{C}_L \equiv \frac{1 - \delta_z}{2\bar{a}(1 + \lambda)(1 - \delta_a)\delta_z a_2}$$

$$\mathcal{C}_D \equiv \frac{1}{1 - \left(\frac{2}{\bar{a}(1 - \delta_z)}\right)^{\frac{1}{2}} \left((\lambda - \alpha\beta)(1 - \delta_a)\delta_z a_2 - \alpha\beta(1 - \delta_z)\frac{\bar{a}}{2} - (1 + \alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}}.$$

Then total output in this economy is equal to

$$Y = z_2 \frac{1 - \delta_z}{\bar{a}} \left(\frac{1}{1 + \alpha\beta}\right)^\alpha \left(\frac{M_W}{A}\right)^\alpha \left[\bar{a} \left(\frac{1}{\mathcal{C}_D}\right)^\alpha - \frac{\alpha}{1 + \alpha} \left[\left(\frac{1}{\mathcal{C}_D}\right)^{1+\alpha} - \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}} \right] \right]$$

Furthermore, an increase in λ decreases the last term in Y , which is a function of $r^L - r^D$.

Proof. See Appendix D. □

5.3 Numerical solutions

We now solve the model numerically by assuming that z and a are drawn from independent uniform distributions. We also set $\alpha = 0.7$ and $\beta = 0.96$.² We analyze what happens as λ increases. We find that $r^L - r^D$ is decreasing in λ . Additional to the effect this has on the dispersion on the sizes of firms with the same productivity, we also find that there is an effect on occupational choice; namely, we observe that unskilled consumers who choose to become entrepreneurs in economies with low levels of λ , will choose to become workers as λ increases. Also, skilled consumers who became workers for low levels of λ , choose to manage firms for higher limits on the leverage of banks.

As λ increases, the margin of intermediation goes down, which implies lower misallocation: The heterogeneity in size among firms with same productivity decreases. Figure 4 shows what happens to misallocation as λ increases. Figure 4a shows the effect on firm size for an entrepreneur under different values of λ . As $\lambda \rightarrow \infty$ misallocation disappears. Firms with the same productivity will have the same size, as the continuous line shows. For finite values of λ , there will be misallocation.

Now, recall that the kinks in firm size correspond to values of a such that $a = \delta_{\mathcal{E}_O, \mathcal{E}_L}$ and $a = \delta_{\mathcal{E}_D, \mathcal{E}_O}$. A higher value of λ implies a smaller difference between $\delta_{\mathcal{E}_O, \mathcal{E}_L}$ and $\delta_{\mathcal{E}_D, \mathcal{E}_O}$. A consequence of a higher λ is that more entrepreneurs use banks as financial intermediaries, since less entrepreneurs use all their available wealth to hire workers. Figure 4b highlights this point: higher levels of λ imply less entrepreneurs that neither borrow nor save.

²The results we show hold qualitatively for various parameterizations.

Figure 4: Entrepreneur with skill z

Furthermore, as λ increases another distortion diminishes: Skilled consumers choose to manage firms, while unskilled consumers become workers. That is, consumers that choose to become entrepreneurs when $\lambda < \infty$, choose to become workers when $\lambda \rightarrow \infty$. Similarly, consumers that choose to become workers when $\lambda < \infty$, decide to become entrepreneurs when $\lambda \rightarrow \infty$.

To understand the source of this distortion, Figure 6 shows the occupational choices of consumers in the (z, a) space for different values of λ . As λ increases the threshold in wealth above which consumers prefer to become bankers over workers goes up. That is, less workers become bankers. The main reason for why this occurs is that the spread between r^B and r^D decreases, which is largely a consequence of the decrease in the margin of intermediation.

More importantly, as λ increases, the slope of the the boundary that determines the occupation choice between workers and entrepreneurs increases. That is, the decision between becoming a worker or an entrepreneur becomes less dependent on the level of wealth of the consumer than on its skill. Given that the margin of intermediation is decreasing in λ , Lemma 7 shows this by proving that in an economy with $\lambda < \infty$ an unwealthy consumer needs to have a higher skill than a wealthy consumer in order to become an entrepreneur. Furthermore, this difference is increasing in the margin of intermediation. We prove the lemma for a particular case in which the minimum value that a attains is 0. Nonetheless the result holds for any general distribution with positive support.

Lemma 7. *A consumer with wealth above $a_1 = \frac{1+\alpha\beta}{\beta(1-\alpha)}w$ will become an entrepreneur as long*

as $z \geq z_1$ where

$$z_1 \equiv \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1+r^D).$$

A consumer with no wealth ($a = 0$) will become an entrepreneur as long as $z \geq z_2$ where

$$z_2 \equiv \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1+r^L) \left(\frac{1+r^D}{1+r^L} \right)^{\frac{\beta(1-\alpha)}{1+\beta}}.$$

$z_2 > z_1$ as long as $r^L - r^D > 0$. Furthermore, $z_2 - z_1$ is decreasing in λ .

Proof. See Appendix D. □

Figure 6 shows the effect of the distortion on occupation choice. Figure 6a shows consumers who choose to become entrepreneurs in an economy without limits on leverage ($\lambda \rightarrow \infty$) but in the model with $\lambda < \infty$ choose to become workers. That is, without limits on leverage these consumers choose to manage firms since they are skilled consumers. However, these consumers have low wealth. If they chose to become entrepreneurs when $\lambda < \infty$, they would need to finance a large portion of their wage bill by borrowing. Therefore, in an economy with $\lambda < \infty$ they are better off by becoming workers.

On the other hand, Figure 6b shows the consumers that would choose to become entrepreneurs in an economy with limits on leverage ($\lambda < \infty$). These consumers are wealthy enough to be able to finance a significant share of their wage bill without needing to borrow. As λ increases, these consumers are better off becoming workers, since they are not skilled enough.

The increased distortions mentioned before cause production to be lower for low levels of λ . Figure 7 shows how production is increasing in λ . In this figure we compare the production in an economy with a given level of λ relative to the production of an economy with $\lambda = 9$. With the current parametrization we are able to explain up to 56% differences in production.

We also analyze what happens as we change other parameters in the model. In particular, we analyze what happens as we change the ratio of wealth to skill in the model. That is, let $\bar{a}$ be the supremum of the support of the distribution for wealth and let $\bar{z}$ be the equivalent for skill. We analyze changes in $\frac{\bar{a}}{\bar{z}}$ for a given value of λ and holding other parameters constant. We find that distortions of having limits on leverage decrease: As this ratio increases, consumers are wealthier in the first period, relative to the second period. This implies that interest rates will be lower in equilibrium since there is more wealth that is going to be saved and less will be borrowed. Furthermore, banks become bigger, which causes the margin of intermediation to be lower.

6 Conclusion

This paper analyzes the relationship between the leverage of banks and real allocations. Economies with banks that have a lower leverage experience a higher margin of intermediation, which affects both the occupational choices of consumers and the distribution of firm sizes across the wealth of individuals. When the margin of intermediation is large, wealthy entrepreneurs can hire workers for a significantly lower cost relative to unwealthy entrepreneurs. This variation in marginal costs of employees translates into heterogeneity in firm sizes across the spectrum of wealth for otherwise identical firms. Occupational choice is also distorted: consumers with substantial skill but sufficiently small wealth may be dissuaded from managing firms due to the large costs of taking out loans, whereas wealthy but less skilled consumers may manage firms due to their relative advantage in inexpensive financing and the low opportunity cost of using those funds to hire workers rather than depositing. Future work will focus on analyzing the tradeoff of having limits on the leverage of banks in an economy with risk.

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Figure 5: Boundaries for different values of λ

$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

Figure 6: Changes in occupational choice

(a) Skilled consumers become entrepreneurs... (b) ... and unskilled consumers become workers

$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

Figure 7: Production relative to economy with $\lambda = 9$

Appendix A Characterizing the boundaries that determine the occupational choices

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w , r^D and r^L . This characterization allows us to analyze how changes in λ affect production. Consumers will make an occupational choice depending on the utility they can achieve from that occupation (see (1)). Lemma 8 characterizes the utility that consumers get depending on their occupational choice.

Lemma 8. *The utility of consumer (z, a) is*

$$u(z, a) = \begin{cases} (1 + \beta) \ln \frac{a+w}{1+\beta} + \beta \ln \beta(1 + r^D) & \text{if } (z, a) \in \mathcal{W} \\ (1 + \beta) \ln \frac{a}{1+\beta} + \beta \ln \beta(1 + r^B) & \text{if } (z, a) \in \mathcal{B} \\ (1 + \beta) \ln \left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta} \left(\frac{z}{1+r^L} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \right) + \beta \ln \beta(1 + r^L) & \text{if } (z, a) \in \mathcal{E}_L \\ (1 + \beta) \ln \left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta} \left(\frac{z}{1+r^D} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \right) + \beta \ln \beta(1 + r^D) & \text{if } (z, a) \in \mathcal{E}_D \\ \ln \frac{a}{1+\alpha\beta} + \beta \ln z \left(\frac{\alpha\beta}{1+\alpha\beta} \frac{a}{w} \right)^\alpha & \text{if } (z, a) \in \mathcal{E}_O. \end{cases}$$

Proof. In this model $c_2(z, a) = \beta(1 + r(z, a))c_1(z, a)$ for all consumers (z, a) , where $r(z, a) = r^D$ for $(z, a) \in \mathcal{W}$, $r(z, a) = r^B$ for $(z, a) \in \mathcal{B}$ and it is defined in Corollary 1 of Lemma 3 for $(z, a) \in \mathcal{E}$. Therefore

$$u(z, a) = (1 + \beta) \ln c_1(z, a) + \beta \ln \beta(1 + r(z, a)).$$

The proof of the lemma follows then from Lemmas 2 and 3. □

We now compare the utility derived from two occupations at a time. Lemmas 9 to 15 show the boundaries that arise from these comparisons. The proof of these lemmas follows from Lemma 8. It is worth noticing that some boundaries only depend on the level of wealth (the choice between being a worker and a banker), or exclusively on the skill level (the choice between being a worker and an entrepreneur that saves). The rest of the boundaries depend on a combination of both wealth and skill level.

Lemma 9. *Let*

$$a_{\mathcal{W}, \mathcal{B}} \equiv \frac{w}{\left(\frac{1+r^B}{1+r^D} \right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq a_{\mathcal{W}, \mathcal{B}}$ then $(z, a) \notin \mathcal{W}$ and if $a < a_{\mathcal{W}, \mathcal{B}}$ then $(z, a) \notin \mathcal{B}$.

Lemma 10. *Let*

$$z_{\mathcal{W}, \mathcal{E}_D} = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha w(1 + r^D).$$

If $z < z_{\mathcal{W}, \mathcal{E}_D}$ then $(z, a) \notin \mathcal{E}_D$ and if $z \geq z_{\mathcal{W}, \mathcal{E}_D}$ then $(z, a) \notin \mathcal{W}$.

Lemma 11. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_O}(a) = \beta^{1-\alpha}(1+r^D) \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{a+w}{1+\beta}\right)^{\frac{1+\beta}{\beta}} \left(\frac{1+\alpha\beta}{a}\right)^{\frac{1+\alpha\beta}{\beta}}.$$

If $z < \delta_{\mathcal{W},\mathcal{E}_O}(a)$ then $(z, a) \notin \mathcal{E}_O$ and if $z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a)$ then $(z, a) \notin \mathcal{W}$.

Lemma 12. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_L}(z) = \frac{w - (1-\alpha) \left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} \left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a < \delta_{\mathcal{W},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{E}_L$ and if $a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{W}$.

Lemma 13. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_D}(z) = \frac{(1-\alpha) \left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_D}(z)$ then $(z, a) \notin \mathcal{E}_D$ and if $a < \delta_{\mathcal{B},\mathcal{E}_D}(z)$ then $(z, a) \notin \mathcal{B}$.

Lemma 14. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_O}(z) = \frac{1}{\beta} \frac{(1+\beta)^{\frac{1+\beta}{\beta(1-\alpha)}}}{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta(1-\alpha)}}} \left(\frac{z}{1+r^B}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_O}(z)$ then $(z, a) \notin \mathcal{E}_O$ and if $a < \delta_{\mathcal{B},\mathcal{E}_O}(z)$ then $(z, a) \notin \mathcal{B}$.

Lemma 15. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_L}(z) = \frac{(1-\alpha) \left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{E}_L$ and if $a < \delta_{\mathcal{B},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{B}$.

Additionally to the boundaries shown in the previous lemmas, there are two other boundaries we need to take into account: $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$. As mentioned in Lemma 3,

$$\begin{aligned} \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) &\equiv \frac{1+\alpha\beta}{\beta} \left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} \\ \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) &\equiv \frac{1+\alpha\beta}{\beta} \left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ will borrow, entrepreneurs with $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will save, and entrepreneurs with $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will spend all their available wealth in paying for workers. It is worth mentioning that these boundaries do not depend on utility,

but rather on feasibility: If an entrepreneur borrows ($s^E(z, a) < 0$) then this entrepreneur cannot choose to save.

Proposition 3 characterizes the sets of consumers. There are three cases: If the spread between the return of setting up a bank, r^B , and the interest rate on deposits, r^D is sufficiently low, then there will be three types of entrepreneurs. On the other extreme, if the spread between r^B and the interest rate on loans, r^L is sufficiently large, in equilibrium there will only be entrepreneurs that borrow.

Proposition 3. *If in equilibrium $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$, then there will be the three types of entrepreneurs. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}\mathcal{B} &= [\underline{z}, z_{\mathcal{W}, \mathcal{E}_D}) \times [a_{\mathcal{W}, \mathcal{B}}, \bar{a}] \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{E}_D} \text{ and } a \geq \delta_{\mathcal{B}, \mathcal{E}_D}(z)\} \\ \mathcal{W} &= [\underline{z}, z_{\mathcal{W}, \mathcal{E}_D}) \times [\underline{a}, a_{\mathcal{W}, \mathcal{B}}] \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{E}_D}, a \geq \underline{a}, z < z_{\mathcal{W}, \mathcal{E}_O, \mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W}, \mathcal{E}_O}(a)\} \\ &\quad \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{E}_O, \mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W}, \mathcal{E}_L}(z)\} \\ \mathcal{E}_D &= \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{E}_D} \text{ and } \delta_{\mathcal{E}_D, \mathcal{E}_O}(z) \leq a < \delta_{\mathcal{B}, \mathcal{E}_D}(z)\} \\ \mathcal{E}_O &= \{(z, a) : z \geq \delta_{\mathcal{W}, \mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D, \mathcal{E}_O}(z)\} \\ \mathcal{E}_L &= \{(z, a) : a \geq \delta_{\mathcal{W}, \mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z)\},\end{aligned}$$

where

$$z_{\mathcal{W}, \mathcal{E}_O, \mathcal{E}_L} \equiv \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{\beta}{(1+\beta) \left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)} \right)^{1-\alpha} w(1+r^L).$$

If in equilibrium $\frac{1+r^B}{1+r^L} \leq \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^D}$, then there will be no entrepreneurs that deposit. In this case the occupational choice of consumers is characterized by the following sets:

$$\begin{aligned}\mathcal{B} &= [\underline{z}, z_{\mathcal{W}, \mathcal{B}, \mathcal{E}_O}) \times [a_{\mathcal{W}, \mathcal{B}}, \bar{a}] \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{B}, \mathcal{E}_O} \text{ and } a \geq \delta_{\mathcal{B}, \mathcal{E}_O}(z)\} \\ \mathcal{W} &= [\underline{z}, z_{\mathcal{W}, \mathcal{B}, \mathcal{E}_O}) \times [\underline{a}, a_{\mathcal{W}, \mathcal{B}}] \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{B}, \mathcal{E}_O}, a \geq \underline{a}, z < z_{\mathcal{W}, \mathcal{E}_O, \mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W}, \mathcal{E}_O}(a)\} \\ &\quad \bigcup \{(z, a) : z \geq z_{\mathcal{W}, \mathcal{E}_O, \mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W}, \mathcal{E}_L}(z)\} \\ \mathcal{E}_O &= \{(z, a) : z \geq \delta_{\mathcal{W}, \mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O, \mathcal{E}_L}(z) \leq a < \delta_{\mathcal{B}, \mathcal{E}_O}(z)\} \\ \mathcal{E}_L &= \{(z, a) : a \geq \delta_{\mathcal{W}, \mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O, \mathcal{E}_L}(z)\},\end{aligned}$$

where

$$z_{\mathcal{W}, \mathcal{B}, \mathcal{E}_O} \equiv \frac{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}} \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{\beta}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1} \right)^{1-\alpha} w(1+r^B).$$

If in equilibrium $\frac{1+r^B}{1+r^L} > \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}}$, then there will only be entrepreneurs that borrow. In this case the occupational choice of consumers is characterized by the following sets:

$$\begin{aligned}\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [a_{\mathcal{W},\mathcal{B}}, \bar{a}] \bigcup \{(z, a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)\} \\ \mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z, a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\ \mathcal{E}_L &= \{(z, a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{B},\mathcal{E}_L}(z)\}.\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}} - 1}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}\right)^{1-\alpha} w(1+r^L).$$

Proof. The strategy to prove the proposition relies on the fact that, depending on the values of $\frac{1+r^B}{1+r^D}$ and $\frac{1+r^B}{1+r^L}$, some types of entrepreneurs will not exist. This will be a consequence of the slope of some boundaries.

First, recall from Lemma 1 that $r^B > r^L > r^D$ in equilibrium as long as $\lambda < \infty$. Let

$$a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O} = \frac{1+\alpha\beta}{\beta(1-\alpha)} w.$$

Notice that

$$\begin{aligned}\delta_{\mathcal{B},\mathcal{E}_D}(z_{\mathcal{W},\mathcal{E}_D}) &= a_{\mathcal{W},\mathcal{B}} \\ \delta_{\mathcal{W},\mathcal{E}_O}(a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}) &= z_{\mathcal{W},\mathcal{E}_D} \\ \delta_{\mathcal{E}_D,\mathcal{E}_O}(z_{\mathcal{W},\mathcal{E}_D}) &= a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}.\end{aligned}$$

As long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ there will exist entrepreneurs that save in equilibrium. $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ and $\delta_{\mathcal{B},\mathcal{E}_D}(z) \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$. Finally, $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) > \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ since $r^L > r^D$, so if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$ there will be entrepreneurs that neither borrow nor deposit.

Now consider the case where there are no entrepreneurs that save in equilibrium. Notice that

$$\begin{aligned}\delta_{\mathcal{W},\mathcal{E}_O}(a_{\mathcal{W},\mathcal{B}}) &= z_{\mathcal{W},\mathcal{B},\mathcal{E}_O} \\ \delta_{\mathcal{B},\mathcal{E}_O}(z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}) &= a_{\mathcal{W},\mathcal{B}}.\end{aligned}$$

Let

$$a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \equiv \frac{1+\alpha\beta}{(1+\beta)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)} w.$$

Notice that

$$\begin{aligned}\delta_{\mathcal{W},\varepsilon_O}(a_{\mathcal{W},\varepsilon_O,\varepsilon_L}) &= z_{\mathcal{W},\varepsilon_O,\varepsilon_L} \\ \delta_{\varepsilon_O,\varepsilon_L}(z_{\mathcal{W},\varepsilon_O,\varepsilon_L}) &= a_{\mathcal{W},\varepsilon_O,\varepsilon_L} \\ \delta_{\mathcal{W},\varepsilon_L}(z_{\mathcal{W},\varepsilon_O,\varepsilon_L}) &= a_{\mathcal{W},\varepsilon_O,\varepsilon_L},\end{aligned}$$

so as long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\varepsilon_O,\varepsilon_L}$ there will exist entrepreneurs that neither borrow nor save.

$a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\varepsilon_O,\varepsilon_L}$ and $\delta_{\mathcal{B},\varepsilon_O}(z) \geq \delta_{\varepsilon_O,\varepsilon_L}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^L}$.

Finally notice that

$$\begin{aligned}\delta_{\mathcal{W},\varepsilon_L}(z_{\mathcal{W},\mathcal{B},\varepsilon_L}) &= a_{\mathcal{W},\mathcal{B}} \\ \delta_{\mathcal{B},\varepsilon_L}(z_{\mathcal{W},\mathcal{B},\varepsilon_L}) &= a_{\mathcal{W},\mathcal{B}}.\end{aligned}$$

If $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^L}$ there will only be entrepreneurs that borrow in equilibrium. $\square$

Lemma 16 determines the skill level $z_{\mathcal{W},\varepsilon_L}^0$ above which there will be no workers.

Lemma 16. *Let*

$$z_{\mathcal{W},\varepsilon_L}^a = \left(\frac{w - \underline{a} \left(\left(\frac{1+r^L}{1+r^D} \right)^{\frac{\beta}{1+\beta}} - 1 \right)}{1 - \alpha} \right)^{1-\alpha} \left(\frac{w}{\alpha} \right)^\alpha (1 + r^L) \left(\frac{1 + r^D}{1 + r^L} \right)^{\frac{\beta(1-\alpha)}{1+\beta}}$$

Then

$$\delta_{\mathcal{W},\varepsilon_L}(z_{\mathcal{W},\varepsilon_L}^a) = \underline{a}.$$

Appendix B Characterizing the boundaries that determine the occupational choice as $\lambda \rightarrow \infty$

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w and r . Consumers will choose their occupation depending on the utility they can derive from it (see (9)). Lemma 17 characterizes the utility derived from each occupation.

Lemma 17. *The utility of consumer (z, a) is*

$$u(z, a) = \begin{cases} (1 + \beta) \ln \frac{a+w}{1+\beta} + \beta \ln \beta(1 + r) & \text{if } (z, a) \in \mathcal{W} \\ (1 + \beta) \ln \left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta} \left(\frac{z}{1+r} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \right) + \beta \ln \beta(1 + r) & \text{if } (z, a) \in \mathcal{E}. \end{cases}$$

Proof. In this model $c_2(z, a) = \beta(1 + r)c_1(z, a)$ for all consumers (z, a) . Therefore

$$u(z, a) = (1 + \beta) \ln c_1 + \beta \ln \beta(1 + r).$$

The proof of the lemma follows from Lemmas 4 and 5. $\square$

We now compare the utility derived from the two occupations. Lemma 18 shows the boundary that arises from this comparison. The proof of this lemma follows from Lemma 17. It is worth noticing that this boundary only depends on the skill level.

Lemma 18. *Let*

$$z_{\mathcal{W},\mathcal{E}} \equiv \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1+r).$$

If $z < z_{\mathcal{W},\mathcal{E}}$ then $(z, a) \in \mathcal{W}$ and if $z \geq z_{\mathcal{W},\mathcal{E}}$ then $(z, a) \in \mathcal{E}$.

Proposition 4 characterizes the sets of consumers.

Proposition 4. *Consumer occupations are characterized by*

$$\begin{aligned} \mathcal{W} &= \{(z, a) : z < z_{\mathcal{W},\mathcal{E}}\} \\ \mathcal{E} &= \{(z, a) : z \geq z_{\mathcal{W},\mathcal{E}}\}. \end{aligned}$$

Appendix B.1 Solving the model

Let

$$A \equiv \int a dG(z \times a)$$

Since for each individual, consumption in the two periods is related by

$$c_2(z, a) = \beta(1+r)c_1(z, a).$$

This must also hold in summation, which implies:

$$\int c_2(z, a) dG(z \times a) = \int \beta(1+r)c_1(z, a) dG(z \times a). \quad (\text{B1})$$

We use Lemmas 5 and 18 and the market clearing condition for labor to solve for $C_{w,r} \equiv w(1+r)$. This allows us to have an expression for total production. Then we use A and total production to solve for r using (B1). Finally we solve for w .

Appendix C Social Planner Problem

The Social Planner Problem is

$$\begin{aligned} & \max_{l(z), c_1(z, a), c_2(z, a), o} \int (v(c_1(z, a) + \beta v(c_2(z, a))) dG(z \times a) \\ & \int c_1(z, a) dG(z \times a) = \int a dG(z \times a) \\ & \int c_2(z, a) dG(z \times a) = \int_{\mathcal{E}} z l(z)^\alpha dG(z \times a) \\ & \int_{\mathcal{E}} l(z) dG(z \times a) = \int_{\mathcal{W}} dG(z \times a). \end{aligned}$$

Notice that aggregate consumption in period 1 is exogenous, so maximizing social welfare implies maximizing production in the second period. Now, the marginal productivity of labor is constant across all firms, but adding an extra entrepreneur implies increasing the marginal productivity of labor, since the average size of a firm decreases. So solving the Social Planner Problem reduces to finding a boundary above which consumers will become entrepreneurs. We know that

$$l(z) = \left(\frac{\alpha z}{MPL} \right)^{\frac{1}{1-\alpha}}.$$

From the labor market clearing condition we have

$$\left(\frac{\alpha}{MPL} \right)^{\frac{1}{1-\alpha}} \int_{z \geq z_{\mathcal{W}, \mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z \times a) = \hat{G}(z_{\mathcal{W}, \mathcal{E}}),$$

where

$$\hat{G}(z') \equiv \int_{z < z'} dG(z \times a).$$

Then

$$MPL = \alpha \left(\frac{(1 - \hat{G}(z_{\mathcal{W}, \mathcal{E}})) \mathbf{E} \left[z^{\frac{1}{1-\alpha}} | z \geq z_{\mathcal{W}, \mathcal{E}} \right]}{\hat{G}(z_{\mathcal{W}, \mathcal{E}})} \right)^{1-\alpha}.$$

The Social Planner Problem is equivalent to

$$\begin{aligned} \max_{z_{\mathcal{W}, \mathcal{E}}} H(z_{\mathcal{W}, \mathcal{E}}) & \equiv \max_{z_{\mathcal{W}, \mathcal{E}}} \int_{\mathcal{E}} z f(l(z)) dG(z \times a) \\ & = \max_{z_{\mathcal{W}, \mathcal{E}}} \left(\frac{\alpha}{MPL(z_{\mathcal{W}, \mathcal{E}})} \right)^{\frac{\alpha}{1-\alpha}} \int_{z \geq z_{\mathcal{W}, \mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z \times a) \\ & = \max_{z_{\mathcal{W}, \mathcal{E}}} \left(1 - \hat{G}(z_{\mathcal{W}, \mathcal{E}}) \right)^{1-\alpha} \mathbf{E} \left[z^{\frac{1}{1-\alpha}} | z \geq z_{\mathcal{W}, \mathcal{E}} \right]^{1-\alpha} \hat{G}(z_{\mathcal{W}, \mathcal{E}})^\alpha. \end{aligned}$$

Appendix D Select Proofs

Appendix D.1 Proof of Lemma 6

To prove this Lemma it is sufficient to consider aggregate wealth across the different occupational choices. That is, let $A_{\mathcal{O}}$ denote the aggregate wealth endowed to consumers who choose occupation $\mathcal{O}$, where $\mathcal{O} \in \{\mathcal{W}, \mathcal{B}, \mathcal{E}_L, \mathcal{E}_O, \mathcal{E}_D\}$. Additionally, let

$$\begin{aligned} Z_{\mathcal{E}_D} &\equiv \left[\int_{\mathcal{E}_D} z^{\frac{1}{1-\alpha}} dG(z \times a) \right]^{1-\alpha} \\ Z_{\mathcal{E}_L} &\equiv \left[\int_{\mathcal{E}_L} z^{\frac{1}{1-\alpha}} dG(z \times a) \right]^{1-\alpha}. \end{aligned}$$

That is, $Z_{\mathcal{E}_D}^{\frac{1}{1-\alpha}}$ ($Z_{\mathcal{E}_L}^{\frac{1}{1-\alpha}}$) is the $\frac{1}{1-\alpha}$ -th moment of the skill endowed to the entrepreneurs that are depositors (borrowers). Finally let $M_{\mathcal{W}}$ be the mass of workers:

$$M_{\mathcal{W}} \equiv \int_{\mathcal{W}} dG(z \times a).$$

Then the market clearing condition for deposits can be written as

$$\begin{aligned} &\frac{\beta}{1+\beta} (wM_{\mathcal{W}} + A_{\mathcal{W}}) + \frac{\beta}{1+\beta} A_{\mathcal{E}_D} - \frac{1+\alpha\beta}{1+\beta} \left(\frac{Z_{\mathcal{E}_D}}{1+r^D} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \\ &= \lambda \frac{\beta}{1+\beta} A_{\mathcal{B}}; \end{aligned} \tag{D2}$$

for loans as

$$\frac{\beta}{1+\beta} A_{\mathcal{E}_L} - \frac{1+\alpha\beta}{1+\beta} \left(\frac{Z_{\mathcal{E}_L}}{1+r^L} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} + (1+\lambda) \frac{\beta}{1+\beta} A_{\mathcal{B}} = 0; \tag{D3}$$

and for labor as

$$M_{\mathcal{W}} = \left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w} \right)^{\frac{1}{1-\alpha}} + \frac{\alpha\beta}{1+\alpha\beta} \frac{A_{\mathcal{E}_O}}{w} + \left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w} \right)^{\frac{1}{1-\alpha}}. \tag{D4}$$

(D2) and (D3) can be rewritten as (D5) and (D6), respectively.

$$\left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w} \right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta} (A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}) \frac{\alpha}{w} \tag{D5}$$

$$\left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w} \right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta} (wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}) \frac{\alpha}{w}. \tag{D6}$$

Plugging (D5) and (D6) into the labor market clearing condition we get

$$M_W = \frac{\beta}{1 + \alpha\beta} (A_{\mathcal{E}_L} + (1 + \lambda)A_B) \frac{\alpha}{w} + \frac{\alpha\beta}{1 + \alpha\beta} \frac{A_{\mathcal{E}_O}}{w} + \frac{\beta}{1 + \alpha\beta} (wM_W + A_W + A_{\mathcal{E}_D} - \lambda A_B) \frac{\alpha}{w}. \quad (\text{D7})$$

The result follows from reorganizing (D7).

Appendix D.2 Assumptions on parameters in Section 5.2

The following assumptions on parameters guarantee that the prices stated in Proposition 1 are an equilibrium:

$$z_2 > \left(\frac{1}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1 + r^L) > z_1.$$

This assumption guarantees that consumers with $z = z_2$ choose to become entrepreneurs are entrepreneurs while the other consumers don't.

$$a_2 > \frac{w}{\left(\frac{1 + r^B}{1 + r^D} \right)^{\frac{\beta}{1 + \beta}} - 1} > a_1.$$

This assumption guarantees that consumers with $a = a_2$ choose to become bankers and consumers with $a = a_1$ choose to become workers.

$$\bar{a} > \frac{1 + \alpha\beta}{\beta} \left(\frac{z_2}{1 + r^D} \right)^{\frac{1}{1 - \alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1 - \alpha}}.$$

This assumption guarantees that there are entrepreneurs that deposit.

$$0 < (\lambda - \alpha\beta)(1 - \delta_a)\delta_z a_2 - \alpha\beta(1 - \delta_z)\frac{\bar{a}}{2} - (1 + \alpha\beta)\delta_a\delta_z a_1 < (1 - \delta_z)\frac{\bar{a}}{2}.$$

The first inequality is consistent with the existence of consumers other than workers that save; namely, entrepreneurs that deposit. The second inequality is consistent with the existence of consumers that do not save; namely, entrepreneurs that borrow and entrepreneurs that neither borrow nor deposit.

$$(1 + \lambda)(1 - \delta_a)\delta_z a_2 < \left(\left(\frac{1 - \delta_z}{2\bar{a}} \right)^{\frac{1}{2}} - \left((\lambda - \alpha\beta)(1 - \delta_a)\delta_z a_2 - \alpha\beta(1 - \delta_z)\frac{\bar{a}}{2} - (1 + \alpha\beta)\delta_a\delta_z a_1 \right)^{\frac{1}{2}} \right)^2.$$

This final assumption guarantees that $r^L > r^D$, which is necessary to have an equilibrium.

Appendix D.3 Proof of Proposition 1

The strategy to prove this proposition will be to assume that parameter values are such that an equilibrium exist. Then we show that the conditions on this parameters satisfy what is stated in Appendix D.2. We will also build on the proof of Lemma 6. Given the definitions in Appendix D.1, under $\tilde{G}(\cdot)$ we have

$$\begin{aligned}
M_{\mathcal{W}} &= \delta_a \delta_z \\
A_{\mathcal{W}} &= a_1 \delta_a \delta_z \\
A_{\mathcal{B}} &= a_2 (1 - \delta_a) \delta_z \\
A_{\mathcal{E}_D} &= \frac{1 - \delta_z}{\bar{a}} \int_{\delta_{\mathcal{E}_D, \mathcal{E}_O}(z)}^{\bar{a}} ada \\
A_{\mathcal{E}_L} &= \frac{1 - \delta_z}{\bar{a}} \int_0^{\delta_{\mathcal{E}_O, \mathcal{E}_L}(z)} ada \\
Z_{\mathcal{E}_D} &= z_2 \left(\frac{1 - \lambda_z}{\bar{a}} \right)^{1-\alpha} (\bar{a} - \delta_{\mathcal{E}_D, \mathcal{E}_O}(z))^{1-\alpha} \\
Z_{\mathcal{E}_L} &= z_2 \left(\frac{1 - \lambda_z}{\bar{a}} \right)^{1-\alpha} (\delta_{\mathcal{E}_O, \mathcal{E}_L}(z))^{1-\alpha}.
\end{aligned} \tag{D8}$$

Reorganizing (D5) and (D6) we get

$$\begin{aligned}
1 + r^L &= Z_{\mathcal{E}_L} \left(\frac{\alpha}{w} \right)^\alpha \left(\frac{1 + \alpha\beta}{\beta (A_{\mathcal{E}_L} + (1 + \lambda)A_{\mathcal{B}})} \right)^{1-\alpha} \\
1 + r^D &= Z_{\mathcal{E}_D} \left(\frac{\alpha}{w} \right)^\alpha \left(\frac{1 + \alpha\beta}{\beta (wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}})} \right)^{1-\alpha}.
\end{aligned} \tag{D9}$$

The result follows from plugging (D8) into (D9), and using the result from Lemma 6. The proof that the assumptions stated in Appendix D.2 are sufficient restrictions on the parameters arises from guaranteeing that there are no negative roots in the resulting r^L and r^D and by using the characterization of the boundaries that determine the occupational choices in the model (See Appendix A).

Appendix D.4 Proof of Proposition 2

The proof of this proposition relies on the result stated in Proposition 1. Notice that $M_{\mathcal{W}}$ and A are constant in the model. Then we can write r^L and r^D as

$$\begin{aligned}
1 + r^L &= \frac{z_2}{\beta} (1 + \alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A} \right)^\alpha \mathcal{C}_L^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta} (1 + \alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A} \right)^\alpha \mathcal{C}_D^{1-\alpha}.
\end{aligned}$$

Notice that $r^L - r^D$ is a function of $\mathcal{C}_L - \mathcal{C}_D$. Furthermore, from the corollary to Proposition 1 this difference is decreasing in λ .

Now, total production in this economy is given by the sum of what is produced by each type of entrepreneur. Let $Y_{\mathcal{T}}$ be total production by entrepreneur of type $\mathcal{T}$, where $\mathcal{T} \in \{L, O, D\}$. Taking into account the results in Proposition 1 and Lemma 3 we have

$$\begin{aligned} Y_L &= z_2 \frac{1 - \delta_z}{\bar{a}} \left(\frac{1}{1 + \alpha\beta} \right)^\alpha \left(\frac{M_{\mathcal{W}}}{A} \right)^\alpha \left(\frac{1}{\mathcal{C}_L} \right)^{\frac{1+\alpha}{2}} \\ Y_O &= z_2 \frac{1 - \delta_z}{\bar{a}} \left(\frac{1}{1 + \alpha\beta} \right)^\alpha \left(\frac{M_{\mathcal{W}}}{A} \right)^\alpha \frac{1}{1 + \alpha} \left[\left(\frac{1}{\mathcal{C}_D} \right)^{1+\alpha} - \left(\frac{1}{\mathcal{C}_L} \right)^{\frac{1+\alpha}{2}} \right] \\ Y_D &= z_2 \frac{1 - \delta_z}{\bar{a}} \left(\frac{1}{1 + \alpha\beta} \right)^\alpha \left(\frac{M_{\mathcal{W}}}{A} \right)^\alpha \left(\bar{a} - \frac{1}{\mathcal{C}_D} \right) \left(\frac{1}{\mathcal{C}_D} \right)^\alpha. \end{aligned}$$

The result follows from adding $Y_{\mathcal{T}}$ for $\mathcal{T} \in \{L, O, D\}$.

Appendix D.5 Proof of Lemma 7

The proof of this lemma relies on what is proven in Appendix A. First, from Proposition 3 we see that a consumer with skill z_1 and wealth a_1 choose to be an entrepreneur that is able to use its wealth to pay for workers. Additionally, if $\underline{a} = 0$ a consumer with skill z_2 and wealth $a = 0$ chooses to become an entrepreneur that needs to borrow to pay for workers according to Lemma 10 and Proposition 3. Finally,

$$z_2 - z_1 = \left(\frac{1}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha (1 + r^D)^{\frac{\beta(1-\alpha)}{1+\beta}} \left[(1 + r^L)^{\frac{1+\alpha\beta}{1+\beta}} - (1 + r^D)^{\frac{1+\alpha\beta}{1+\beta}} \right],$$

which is increasing in $r^L - r^D$.