FINANCIAL OPTIONS FOR PRICE-QUANTITY HEDGING IN COMPETITIVES POWER MARKETS. IDEAS FOR THE COLOMBIAN CASE.

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Abstract

Starting from the conceptual framework defined by Oum, Oren and Deng in [1], this paper seeks to design financial instruments to offer simultaneous price-quantity hedging in wholesale electricity markets. Instruments designed minimize replicating errors for both power producers and retailers from a market maker's perspective. An infinite collection of derivatives (*"exotic option"*) emerges as the solution of this price-quantity hedging. This exotic option is then replicated with a portfolio composed by risk free bonds, forward/futures contracts, and standard options.

A dynamic hedging strategy to rebalance agents' position through time is also proposed. This strategy is found by maximizing a static expected utility problem at options' maturity subject to constraints.

The proposed approaches are proven within the Colombian wholesale electricity market framework. Results allow to experimentally validate some theoretical results presented in [1] and evidence improvements for power producers and retailers' profits by following hedging strategies proposed. Intuition suggests financial instruments proposed would help to address major problems in the Colombian power market such as lack of liquidity and anonymity present in the current trading scheme.

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Introduction

Deregulation in early nineties has driven deep changes in the architecture of the power industry around the world. Centrally planned monopolies are getting extinguished and replaced by profit oriented and competitive electricity industries [2]. The Nordic market is one of the most outstanding examples to mention, next to experiences in the United Kingdom, Switzerland, Australia, among others [3],[4],[5],[6].

Most literature in the subject suggest deregulation in the power sector as a good medicine to improve efficiency in electricity generation, a mechanism to reduce electricity prices, improving transparency for better consumer choices and allowing the emergence of efficient financial trade in electricity [7],[6],[5]. Even so, authors like Tishler et al. in [8] found that the net benefit from deregulating the electricity sector in Israel will most likely be negative. Woo and Zarnikau state that a reduction in prices due to the electricity market reform is unlikely to occur without the assumption that the post-reform market has marginal costs below average costs [9].

After all, liberalization of the electricity markets necessarily imply price risks due to price volatilities in wholesale markets mainly driven by steeply power generation supply curves, and quantity risks due to uncertain and inelastic demands of a non-storable commodity (paid at fixed rates), among others. Thus, liberalized markets would also bring plenty of possibilities and one of the keys to success is the ability to manage these new risks [10].

To overcome these situations and manage risks, policy makers have adopted experiences from financial markets and tend to create power derivatives. Deng and Oren studied some electricity derivatives in [11]. While Willems and Morbee studied in [12] how options affect hedging and investments in the electricity sector. Other relevant references are [13],[14],[15]. While at present the liquidity of power derivatives is limited, it is expected that better understanding of how such instruments can be used will increase their utilization and liquidity [11]. Literature have focused in two major issues: pricing of derivatives and portfolio optimization for risk management purposes.

When it comes to the pricing of electricity derivatives there are lots of references. The fact that electrical power is not physically storable eliminates the direct application of the no-arbitrage methodology from financial mathematics [16], therefore, different techniques must be applied. Just to mention a few references in this direction, in [15] it is discussed the difficulty of pricing options. Vehviläinen, Lucia and Schwartz in [17] and [6] study properties of the instruments available at NordPool, while Wilhelm in [16] converts the electricity market into a virtual base market consisting of zero bonds and an additional risky asset to elaborate a risk neutral price dynamic. In turn, Fusai et al. in [18] derive a closed-form formula for the fair value of a discretely monitored Asian-style, by employing the Fourier transform pricing method proposed by Carr and Madan in [19].

On the other hand, portfolio optimization has appeared since lessons learned from the financial markets suggest that financial derivatives, when well understood and properly utilized, are beneficial to the sharing and controlling of undesired risks through properly structured hedging strategies [11],[13]. Prior literature on the subject relies on the original Markowitz mean-variance portfolio optimization theory [20] while most recent literature does not maximize agents' expected utility anymore, but minimize its risk exposition [21]. Three major paths could be identified in portfolio optimization: price hedging, quantity (volumetric) hedging, and simultaneous price-quantity hedging.

Price risk hedging strategies exploit electricity derivative properties in order to do achieve the goal: Ahn et al minimize the Value at Risk (VaR) using options in [22], while Kleindorfer and Li consider the portfolio optimization including derivative instruments, subject to a Value-at-Risk (VaR) constraint [23]. Näsäkkälä and Keppo found an analytical solution for a portfolio value distribution [4], and Huisman et al. carried out a mean-variance framework to address the concept of structuring the portfolio and focuses on how to optimally allocate positions in peak and off-peak forward contracts [24].

Quantity risk hedging has not been done supported on electricity derivative features since it is not hedgeable with standard contracts, but there are some works. Näsäkkälä and Keppo in [25] describe the difficulties to achieve so given load uncertainty and the eventual correlation between spot prices and water inflows to reservoirs. These kinds of applications exploit the advantages of flexible power generation technologies (mainly hydropower generation plants) to manage quantity risk. Just to mention, Doege et al. show how the volume risk can be managed through an intelligent dispatch strategy [26].

Literature about simultaneous price-quantity hedging strategies with derivatives in electricity markets is scarce. References in the subject face simultaneous price and quantity risk taking advantage of the production flexibility of some power generation technologies and the available contracts in the market. Fleten et al. discuss a risk management model (stochastic programming) for a hydropower producer operating in a competitive electricity market. The portfolio at risk includes own production and a set of power contracts for delivery or purchase, including contracts of financial nature [3]. Unger studies risk management in the electricity market in general and the interaction between physical production and electricity contracts in particular. As he states in his work, the flexibility in some production plants, such as the hydro storage plant, even makes them suitable to hedge not only price risk, but also volume risk, which currently is not possible in the standardized market [10].

Oum, Oren and Deng proposed in [11] a price-quantity hedging approach involving the use of standard forward electricity contracts and price-based power derivatives. The approach proposed by authors addresses the problem of developing an optimal hedging portfolio for a risk-averse LSE when price and volumetric risks are present and correlated. The idea constitutes an alternative to weather derivatives given the speculative image they have on markets.

We seek to apply and extend findings in [1] to design suitable instruments for an electricity derivative market which would allow agents to build up optimal zero-cost hedging strategies. This is done by exploiting the correlation between load and prices. Oum et al. demonstrate how such a static hedging strategy can be implemented through a portfolio of standard forwards and a spectrum of call and put options with various strike prices which we propose to be optimized in order to satisfy agents' hedging requirements. Regarding the stochastic feature of both spot price and load, we also study and analyze the dynamics of this sort of portfolios through time in order to rebalance them at each decision time.

The remaining of this paper is organized as follows: section 1.1 briefly describes the price-quantity hedging approach in [1]. Section 1.2 formulates an optimization framework to find optimum strike prices for put and call options suitable for LSEs and GENCOs interacting in a wholesale market. In Section 1.3 price and quantity dynamics are studied to come up with a rebalancing strategy for price-quantity hedging portfolios. An application to the Colombian power market is presented in Section 2.1. Finally, conclusions and discussions about the Colombian market are presented in Section 3.1. We leave Section 3.2 for further work.

Chapter 1

Theoretical framework

1.1 Price-quantity risk hedging

Weather derivatives may be an alternative in extra tropical power markets due to the strong correlation between power demand and weather but most of the power markets do not include these instruments or find themselves, like Colombia, within the tropic. As it was mentioned above, in [1] authors propose a different approach for managing volumetric risks by exploiting the correlation between load and spot prices.

Power load is strongly correlated to spot prices due to the non-storability feature of electricity as a commodity and to a steeply rising supply function. In Colombia it is about 0.62 according to recent estimations [27], which is close estimations in California (0.54), Spain (0.70), Britain (0.58) and Scandinavia (0.53) [1].

This paper relies on the original formulation in [1] which is a model with one single setting period to offer static price-quantity hedging to a LSE. Hedging instruments are purchased at time 0 and all payoffs are received at time 1. It is assumed that the hedging portfolio has an overall payoff structure $x(p)$, depending on the realization of the spot price when serving a load q_R at a fixed tariff r. The total profit of a LSE after receiving payoffs from the contracts in the hedging portfolio can be defined as:

$$Y_R(p, q_R, x_R(p)) = (r - p) \cdot q_R + x_R(p) \tag{1.1}$$

To develop a competitive power derivative market bidders and askers must be considered. Hence, the analysis in [1] is extended to a GENCO which is assumed to be continuously dispatched with marginal production cost c and power generation q_G. Thus,

$$Y_G(p, q_G, x_G(p)) = (p - c) \cdot q_G + x_G(p) \tag{1.2}$$

Agents' risk preferences are characterized with a concave utility function U_R/U_G defined over the corresponding total profit Y_R/Y_G at time 1. The realization of spot price p and load q is characterized by a joint probability function $f(p; q)$ which is defined on the probability measure P. A risk-neutral probability measure Q is defined to price hedging instruments. Let $g(p)$ be the probability density function of p under Q. Oum et al. formulate agent's problem as follows:

$$\begin{aligned} &\max_{x(p)} E[U(Y(p, q))] \\ &\text{s.t. } E^Q[x(p)] = 0 \end{aligned} \tag{1.3}$$

where $E[\cdot]$ and $E^Q[\cdot]$ denote expectations under the probability measure P and Q, respectively. Constraint in Eq. (1.3) implies that purchasing derivative contracts may be financed from selling other derivative contracts or from the money market accounts. Further details about the settings of the model on which this paper relies can be found in [1].

Oum et al. derive optimality conditions and find LSE's optimal payoff function $x^*(p)$ for CARA and mean-variance utility functions assuming a probability distribution function for spot prices and loads served. Then, they use Carr and Madan argument exposed in [28] to express the optimal payoff function $x^*(p)$ as a continuum collection of risk free bonds, forward/futures contracts, put and call options:

$$x(p) = x(F) + x'(F)(p - F) + \int_0^F x''(K)(K - p)^+ dK + \int_F^\infty x''(K)(p - K)^+ dK \tag{1.4}$$

where $(\cdot)^+ \equiv max(\cdot, 0)$. These last options have as underlying asset the spot price p, fact that could represent a difficulty since electricity itself is not a regular commodity. In this sense it is not possible to carry forward power generated in one time period to the next, making the pricing of instruments underlying on it a tough task. Additionally, power markets usually are incomplete and hence no unique risk-neutral probability measure can be found [26]. Even so, these difficulties and the pricing of such instruments can be achieved following recent literature [29],[18].

A constraining fact in Eq. (1.4) is that it requires the electricity derivative markets to offer a continuous spectrum of options' strike prices, in order to perfectly replicate the payoff profile of $x^*(p)$. In practice, just a few number of strike prices are offered, so in [1] authors describe one strategy to define the amount of bonds, futures/forwards, and options to purchase so that the total payoff from those options get close enough to the payoff provided by the optimal payoff function $x^*(p)$. We present an enhanced replicating strategy to obtain smaller replicating errors, by applying the linear regression approach of Generalized Least Squares (GLS). The methodology will be further explained in Section 1.2.

1.2 Optimal instruments for the market

From a market maker's perspective, developments of [1] could be properly used to design the instruments to be traded in a power derivative market composed by forward/futures contracts and standard options with the spot price as underlying asset (risk free bonds will also be necessary). Assuming that spot price p and power demand/generation $q_{R/G}$ are static random variables in a single time period (maturity), the optimal payoff function for each LSE and each GENCO participating in the market is found by optimizing as shown in Eq. (1.3). Seeking to aggregate agents' hedging profiles, two unique optimal payoff functions are defined independently; each one represents the collective intention of the retail market and the power generation market, separately. This is done by weighting up each agents profile in the corresponding market: retailers or power producers. Hereafter identified as retail market payoff curve ($\bar{x}_R(p)$) and generation market payoff curve ($\bar{x}_G(p)$).

At this stage, it is proposed an optimization routine to define optimum strike prices for a discrete number of put and call options assumed to be offered in the electricity derivatives market, next to forward/futures contracts and to risk free bonds. These instruments could then be used either by retailers or power producers to build up a price-quantity hedging portfolio close to their own optimal payoff function from Eq. (1.3).

Optimal strikes' are obtained by simultaneously minimizing the replicating error of both retail and generation market payoff curves using a discrete number of options, jointly with the already mentioned instruments. This procedure also enhances complementary features of the power market since options required by LSEs can be offered by GENCOs and vice versa. In the following, a detailed description of the procedure above is presented.

1.2.1 Market payoff functions

The weighting procedure to determine market payoff functions $\bar{x}_\kappa(p)$, $\kappa = \{R, G\}$ is presented in Eq. (1.5).

$$\bar{x}_\kappa(p) = \sum_{i=1}^{N_\kappa} w_i x_i^*(p) \qquad w_i = \frac{\int_0^S x_i^*(p)dp}{\sum_{i=1}^{N_\kappa} \int_0^S x_i^*(p)dp} \tag{1.5}$$

Parameter S corresponds to a price-cap value defined under the market maker or clearing house criteria and N to the number of market participants either LSEs or GENCOs. Under this setting, $\bar{x}_R(p)$ and $\bar{x}_G(p)$ will represent the joint needs of all the agents and could be used to determine general hedging instruments (suitable for all), by no favoring any agent and giving them equal hedging opportunities.

1.2.2 Replicating strategy

Assuming there will be offered in the market i put and j call options, it is possible to rewrite Eq. (1.4) as follows:

$$x(p) = \beta_0+\beta_1(p-F)+\beta_2(k_1-p)^+...+\beta_{i+2}(k_i-p)^++\beta_{i+3}(p-k'_1)^+...+\beta_{i+j+2}(p-k'_j)^++\epsilon \tag{1.6}$$

where β_x is the coefficient of each instrument and ϵ denotes the error, such that $\epsilon \rightarrow 0$ as $(i,j) \rightarrow (\infty,\infty)$. As real markets will always have a finite number of options, the aim is to find the coefficients that minimize $x(p) - \hat{x}(p) = \epsilon$. This can be done by applying GLS to Eq. (1.6), so the coefficients can be estimated from:

$$\hat{\beta} = (Z'Z)^{-1}Z'x(p) \tag{1.7}$$

where

$$Z = \begin{bmatrix} 1 & p_0-F & (k_1-p_0)^+-V_{k_1} & ... & (k_i-p_0)^+-V_{k_i} & (p_0-k'_1)^+-V_{k'_1} & ... & (p_0-k'_j)^+-V_{k'_j} \\ 1 & p_1-F & (k_1-p_1)^+-V_{k_1} & ... & (k_i-p_1)^+-V_{k_i} & (p_1-k'_1)^+-V_{k'_1} & ... & (p_1-k'_j)^+-V_{k'_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & p_n-F & (k_1-p_n)^+-V_{k_1} & ... & (k_i-p_n)^+-V_{k_i} & (p_n-k'_1)^+-V_{k'_1} & ... & (p_n-k'_j)^+-V_{k'_j} \end{bmatrix}$$

In Eq. (1.7), Z corresponds to the independent variables matrix with n rows defined by the set of prices from 0 to S and m columns, each corresponding to a financial instrument including the risk free bonds (unitary column) and the futures/forward contracts. In matrix Z, V_{k_i} and $V_{k'_j}$ denote the value of the put and call option, respectively.

1.2.3 Optimizing strike prices

The optimization procedure aims to minimize the square difference between $\bar{x}_R(p)$ and $\bar{x}_G(p)$ and their corresponding replicating strategies (see (1.8)). Three constraints rule the procedure: (i) express the agents' market payoff function as a linear combination of the available financial instruments plus an error (to be minimized), and (ii) estimates the coefficients related to each instrument by using Eq. (1.7). Finally, (iii) corresponds to a compatibility constraint. Hence,

$$\min_{\bar{K},\bar{K}'} \sum_{p=0}^{S} [(\bar{x}_R(p) - \hat{x}_R(p))^2 + (\bar{x}_G(p) - \hat{x}_G(p))^2] \tag{1.8}$$

$$\text{s.t.} \begin{cases} \cdot(i_R) \quad \hat{x}_R(p) = \beta_0 + \beta_1(p-F) + \beta_2(k_1 - p)^+ + ...+ \\ \quad \beta_{i+2}(k_i - p)^+ + \beta_{i+3}(p - k'_1)^+ + ... + \beta_{i+j+2}(p - k'_j)^+ \\ \cdot(i_G) \quad \hat{x}_G(p) = \alpha_0 + \alpha_1(p-F) + \alpha_2(k_1 - p)^+ + ...+ \\ \quad \alpha_{i+2}(k_i - p)^+ + \alpha_{i+3}(p - k'_1)^+ + ... + \alpha_{i+j+2}(p - k'_j)^+ \\ \cdot(ii_R) \quad \hat{\beta} = (Z'Z)^{-1}Z'x_R(p) \\ \cdot(ii_G) \quad \hat{\alpha} = (Z'Z)^{-1}Z'x_G(p) \\ \cdot(iii_{R/G}) \quad 0 < k_1 < ... < k_i < F < k'_1 < ... < k'_j < S \end{cases}$$

This formulation depends on the price thick resolution given for its solution (e.g. definition of parameter n). Under these settings, $\bar{K}$ and $\bar{K}'$ are finite vectors containing i and j strike prices for put and call options, respectively, which replicate the retail and generation payoff curves with minimum error. F is the futures electricity price and β/α are the linear regression coefficients for LSEs/GENCOs.

Including the GLS approach represents an improvement to the original model in [1] since the optimization not only minimizes the replicating errors by means of the strike prices but also by means of the coefficients. Then, in the financial sense, GLS provides the number of instruments of each type (i.e. put or call options) to be purchased/sold for the portfolio, which will minimize the replicating error ϵ.

1.3 Hedging portfolio dynamics

Now, it is illustrated how a LSE/GENCO may use the options designed in Section 1.2 to mitigate its risk exposure at each decision time before maturity. It is done considering that spot electricity price and power demand/generation are no longer static random variables but stochastic processes.

Given that the goal is to minimize agents' profit risk exposition (individually) at maturity, it is used the mean-variance utility function presented in Eq. (1.9) for all market participants.

$$E[U(Y_{R/G})] = E[Y_{R/G}] - \frac{1}{2}a \cdot Var(Y_{R/G}) \tag{1.9}$$

Parameter a corresponds to a risk aversion coefficient. With that in mind, the analytical outcome from the optimization problem in Eq. (1.3) will be:

$$x^*(p) = \frac{1}{a}\left(1 - \frac{g(p)/f_p(p)}{E^Q[g(p)/f_p(p)]}\right) - E[y(p,q)|p] + E^Q[E[y(p,q)|p]]\frac{g(p)/f_p(p)}{E^Q[g(p)/f_p(p)]} \tag{1.10}$$

where $f_p(p)$ is the marginal density function of p under the probability measure P. The arithmetic procedure to find this expression is clearly developed in [1]. As a first approach the assumption that $P \equiv Q$ is held[1]. Hence, Eq. (1.10) becomes:

$$x^*(p) = E[y(p,q)] - E[y(p,q)|p] \tag{1.11}$$

Maximizing the mean-variance utility function (1.9) with the zero-cost constraint in Eq. (1.3) is equivalent to just minimizing the variance of profit after hedging.

[1] This assumption has been held in other works like [30] and [4]

Then, the optimal portfolio is the one making the expected total profit for any given price equal to the expected profit before hedging.

Before exploring the dynamics of the optimal portfolio (1.11), it is necessary to study price and quantity statistical properties through time.

1.3.1 Modeling power demand and generation

We rely on time series analysis to forecast power generation and load based on historic data. Particularly, *Autoregressive Integrated Moving Average* (ARIMA) models will be employed given the realized non-stationary behavior of this kind of data after performing stationarity tests.

Let q_t represent the power demand/generation at time t of a certain agent either LSE or GENCO. According to the notation expressed in [31], an ARIMA(p,d,q) model can be written as:

$$\phi_p(B)\nabla^d q_t = \theta_q(B)a_t$$

where B is the backward shift operator, $\phi_p(B)$ is the stationary autoregressive operator of order p, $\theta_q(B)$ is the moving average operator of order q. Note that these variables (p and q) are different from the spot price p and power demand/generation $q_{R/G}$ defined earlier. ∇^d corresponds to the *d-th* difference operator and a_t is the series of independent "shocks". Thus, the value of power demand/generation at any lead time l can be expressed in terms of a weighted sum of previous known values q_{t-j} and the shock a_{t+l}, as follows:

$$q_{t+l} = \sum_{j=1}^{\infty} \pi_j^{(l)} q_{t-j+1} + a_{t+l} \tag{1.12}$$

where

$$\pi_j^{(l)} = \sum_{i=1}^{l} \psi_{l-i}\pi_{i+j-1}$$

$$\psi_i = \varphi_1\psi_{i-1} + \cdots + \varphi_{p+d}\psi_{i-p-d} - \theta_i \qquad \psi_0 = 1$$

$$\pi_i = \psi_i - \psi_{i-1}\pi_1 - \cdots - \psi_1\pi_{i-1} \qquad i = 2, 3, 4, ...$$

Coefficients φ_i constitute the non-stationary autoregressive operator $\varphi_p(B) = \phi_p(B)\nabla^d$, such that d roots of $\varphi_p(B) = 0$ are unitary and the remainder lie outside the unit circle. Eq. (1.12) is useful to determine the expected forward value of power consumption or generation given the information up to time t, with variance expressed as,

$$Var(q_{t+l}) = \left(1 + \sum_{j=1}^{l-1} \psi_j^2\right) \sigma_a^2$$

where σ_a corresponds to the standard deviation of independent shocks a_t [31]. Both values (mean and variance), jointly with the first moment and second central moment of spot price p_t, will be the inputs of Eq. (1.11) to determine agents' optimal payoff function.

1.3.2 Electricity price

To build up a hedging strategy it is required an estimation of electricity price and its volatility at maturity. In a mature competitive market of derivatives, this information is contained in the forward electricity price curve[2]. Forward curve is not strictly a forecast of future spot prices. It is instead the current price of the commodity to be delivered at that future time, considering the *"cost of carry"* of the commodity from one period to the next.

Although, due to non-storability feature of electricity, future electricity prices are not directly related to current spot prices, so one could think the electricity delivered

[2] Set of prices as of today for the delivery of electricity at different points of time in the future.

today is a quite different commodity than the electricity delivered tomorrow. It is a deeply studied problem. See [32],[33],[34],[35].

A thorough and precise estimation of the forward electricity price curve is beyond the scope of this work, thus, it is used the forward curve to forecast the spot price at maturities [3].

Once again ARIMA methods are used as a first approach to fit the logarithm of spot price data to a stochastic process.

$$lp_{t+l} = \sum_{j=1}^{\infty} \pi_j^{(l)} Lp_{t-j+1} + e_{t+l} \tag{1.13}$$

$$Var(lp_{t+l}) = \left(1 + \sum_{j=1}^{l-1} \psi_j^2\right) \sigma_e^2 \tag{1.14}$$

where $lp_t = log(p_t)$, σ_e^2 is the variance of independent shocks e_t.

Hereafter, since spot price p and electricity demand/generation $q_{R/G}$ are no longer being considered as static random variables, they will be re-denoted as p_t and q_t, respectively. Each referring to their value at time t.

1.3.3 Mathematical formulation to rebalance portfolios

Given the information up to time t, assume $\{q_{t+l}\}_{t+l\in[0,T)}$ and $\{lp_{t+l}\}_{t+l\in[0,T)}$ follow the time series equations stated in (1.12) and (1.13), respectively. Also assume p_t and q_t are jointly represented by a bivariate lognormal-normal probability density function at time T. Assuming $P \equiv Q$, the optimal payoff function $x_t^*(p_T)$ maximizing each agent's mean-variance utility function (1.9) at maturity T, is obtained as presented in 1.15 for retailers and in 1.16 for generators (details about the proof can be found

[3] This approach is similar to the one developed in [33] for the Spanish market.

on appendix).

$$x_t^*(p_T) = rm_q - (\rho s_q s_p + m_q)e^{m_p+\frac{1}{2}s_p^2} - (r - p_T)\left(m_q + \frac{\rho s_q}{s_p}(log(p_T) - m_p)\right) \quad (1.15)$$

$$x_t^*(p_T) = (\rho s_q s_p + m_q)e^{m_p+\frac{1}{2}s_p^2} - cm_q - (p_T - c)\left(m_q + \frac{\rho s_q}{s_p}(log(p_T) - m_p)\right) \quad (1.16)$$

where

$$m_q = E_t[q_T] = \sum_{j=1}^{\infty} \pi_{q,j}^{(l)} q_{t-j+1}$$

$$m_p = E_t[log(p_T)] = \sum_{j=1}^{\infty} \pi_{lp,j}^{(l)} q_{t-j+1}$$

$$s_q = \sqrt{Var_t(q_T)} = \left(1 + \sum_{j=1}^{l-1} \psi_{q,j}^2\right)\sigma_a^2$$

$$s_p = \sqrt{Var_t(log(p_T))} = \left(1 + \sum_{j=1}^{l-1} \psi_{lp,j}^2\right)\sigma_e^2$$

$$l = T - t$$

$$\rho = Corr(p_t, q_t)$$

As soon as new market information becomes available about what is expected to happen at maturity, the portfolio built at any time t should be rebalanced in order to incorporate this new data into it and meet the original target once the portfolio expires. This means that it is required to calculate the change of the optimal payoff function given the changes of its underlying parameters.

For any agent who has built a price-quantity hedging portfolio once, when information is revealed by the market, the new optimal payoff function is obtained as follows:

$$x_t = x_{t-1} + \Delta x_t \quad (1.17)$$

where the term Δx_t represents the change of x_{t-1} with respect to x_t, and it can be expressed in the following form,

$$\Delta x_t = \frac{\partial x}{\partial m_q}\Delta m_q + \frac{\partial x}{\partial m_p}\Delta m_p + \frac{\partial x}{\partial s_q}\Delta s_q + \frac{\partial x}{\partial s_p}\Delta s_p + \epsilon_{m_p} + \epsilon_{s_p} \tag{1.18}$$

The above partial derivatives must be calculated from Eq. (1.15) for retailers and for generators from Eq. (1.16). Find details on the appendix.

Terms ϵ_{m_p} and ϵ_{s_p} in Eq. (1.18) represent linearization errors, because this equation comes from a linear approximation of x_{t-1} using Taylor series. These errors tend to zero as changes in parameters become infinitesimal.

However, Eq. (1.17) just provides the change of the optimal payoff function in response to changes of its underlying parameters, but it does not state how the current portfolio should be rebalanced. That is, what is the amount of financial instruments the agent must sell or buy to update its portfolio according to the market state. To do so, the replicating strategy proposed in Section 1.2.2 is employed as follows:

$$\Delta\beta_t = (Z'Z)^{-1}Z'\Delta x_t \tag{1.19}$$

where Z corresponds to the independent variables matrix defined earlier. Hence, the new portfolio coefficients are,

$$\beta_t = \beta_{t-1} + \Delta\beta_t \tag{1.20}$$

An agent at time $t = 0$ (or at any other future time) could find its optimal payoff function $x_0(p_T)$ to maximize the expected value of its utility function (1.9) at maturity T. Same agent could then build up a replicating portfolio by holding positions in bonds, futures/forward contracts, and options in the market. See Eqs. (1.5) and (1.7). Thereafter, at any decision time, agent could use Eq. (1.12) and (1.13) to update its expectations at maturity, Eq. (1.14) to calculate Δx_t, and finally Eq. (1.19) to know how many instruments of each type to sell/buy, in order to rebalance its prior position (portfolio). That way, any agent may achieve a dynamic price-quantity hedging with the static criteria of Eq. (1.3).

The following section presents the results obtained by applying the theoretical developments of the sections 1.2 and 1.3 to the Colombian wholesale electricity market.

Chapter 2

Case study

2.1 The Colombian power market

Colombia started its power industry deregulation in 1994 and created a wholesale electricity market since 1995. At that time power generation, transmission, distribution and retailing activities were unbundled by law. Further details about Colombian power market deregulation can be found in [36].

Today, Colombian power market is structured in the following way: (i) a physical delivery day-ahead spot market (hourly basis), (ii) a firm energy market for capacity adequacy (see [37],[38] for details), (iii) a non-standardized bilateral contract market (taylor made cash-settled contracts), and (iv) a secondary market for ancillary services (e.g. AGC). Colombia also has an international electricity trading agreement with Ecuador. Colombia exported (imported) 510 (37) GWh in 2008 and 1,077 (21) GWh in 2009.

According to recent findings in the Colombian power market, the bilateral contracting system faces lack of liquidity (non-standardized contracts) and anonymity, not to mention the absence of an adequate and reliable signal for future electricity prices(see [39],[40],[41]).

In this Section, we develop an application of the methodology proposed in prior sections with information available for the Colombian power market from January 2001 to February 2009. This application is undertaken exploiting the aforementioned hourly correlation between spot price and regulated demand which is deeply studied in [27].

It was found in [27] that monthly and quarterly loads served during on-peak periods by each of five randomly selected LSEs actively participating in the market[1], as well as monthly on-peak power generated by each of nine different GENCOs[2] fit to Normal probability density functions[3] (pdf). It was also found that monthly and quarterly on-peak spot prices fit to a Log-Normal pdf.

These pdfs are used to determine agents' optimal payoff functions with Eq. (1.10) assuming a utility function as in Eq. (1.9). The parameters of each pdf come from a calibrated forecasting model based on historical data of each agent's demand/generation and spot price. We divide this application in two parts. The first, aimed to find optimal strike prices from a market maker perspective. It is assumed that two (monthly) put options and three (monthly) call options expiring at February 2010 will be offered in the market, next to forward contracts and risk free bonds. Second part illustrates how a randomly chosen LSE/GENCO might build up its hedging portfolio at a particular month to later rebalance it in a monthly manner to achieve a price-quantity hedging at maturity.

[1]LSEs selected aggregate 72% of the national regulated demand.

[2]Five hydropower generation plants were selected, next to two coal and two natural gas power generation plants. All of them contribute (on average) with 52% of the national energy production.

[3]Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests were conducted in order to conclude so.

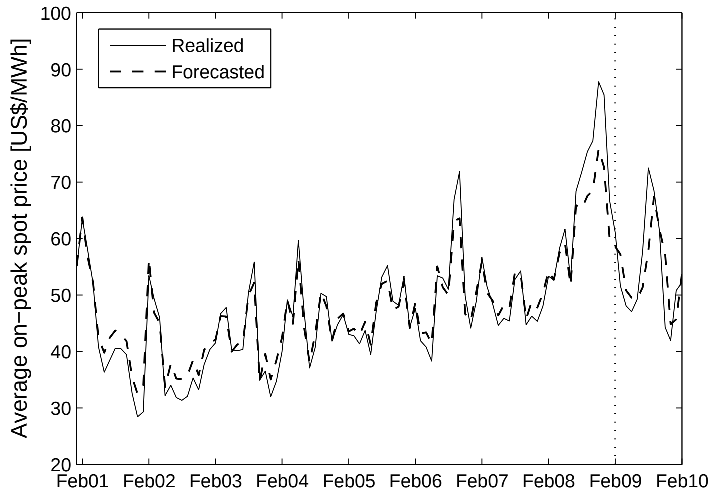

Figure 2.1: Realized and forecasted monthly average of the on-peak spot price.

2.1.1 Optimal strike prices at maturity

As higher prices and loads, as well as volatilities take place during on-peak hours in the Colombian electricity system, this application focuses on designing financial options to offer hedging solely within those hours. It is considered that during off-peak periods, agents can fully hedge themselves with forward/futures contracts (see [27] for details).

The parameters of the stochastic processes $ARIMA(p,d,q)(P,D,Q)_{12}$ for each agent are listed in Table 3.1 at the appendix. Values of the remaining parameters required to perform the procedure described throughout Section 1.2, such as risk aversion coefficient a, correlation coefficient ρ, regulated tariff r and marginal production cost c, are displayed in Tables 3.2 and 3.3 at the appendix.

On the other hand, the underlying asset, that is, the monthly on-peak average time series of spot price logarithm lp_t, was fitted to an $ARIMA(3,1,1)$ process as Figure 2.1 illustrates. Note that in this fashion, the options proposed would be actually

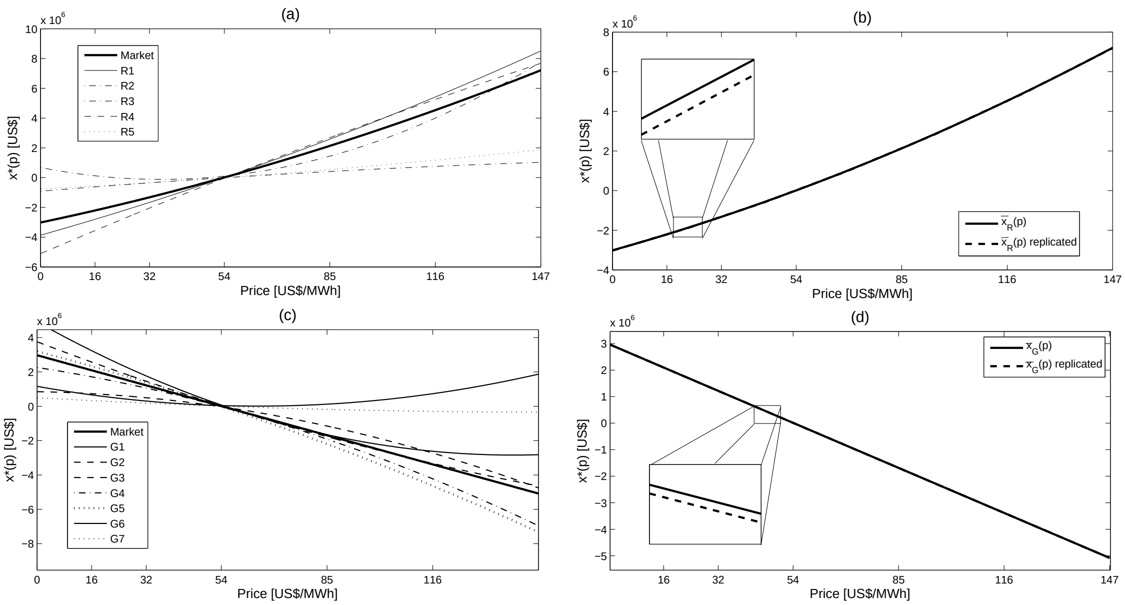

Figure 2.2: Application case results for February 2010.

Asian-style options.

According to historic information, it is expected that on February 2010 $F = 54.21\ US\$/MWh$ and that the spot price follows a Log-Normal pdf $log(p_T) = lp_T \sim N(3.97, 0.19^2)$. The price cap S, is set to be 147 $US\$/MWh$, according to the way scarcity price for the firm energy market is calculated in Colombia.

Once all parameters are clearly defined, the options' optimal strike prices can be found following the process of section 1.2. General results are presented in Figure 2.2. The upper row is related to LSEs and the lower to GENCOs[4]. On the left column, Figures 2.2a and 2.2c illustrate $x^*_R(p)/x^*_G(p)$ for each LSE/GENCO considered, respectively. They were calculated as proposed in [1] (See Eq. (1.3)). Right over these curves there are also depicted in darker lines the market payoff functions $\bar{x}_R(p)/\bar{x}_G(p)$

[4]According to the analysis, plants G8 and G9 are not supposed to operate in this month so they were exclude from the analysis.

calculated as proposed in Eq. (1.5).

Figures 2.2b and 2.2d show $\bar{x}_R(p)/\bar{x}_G(p)$ next to the optimal replicating strategy employing matrix Z to determine GLS coefficients (see Eq. (1.7)). It can be seen that the piecewise linear replicating strategies closely follow the original payoff function. A zoom on a small segment of the curves on each figure was made in order to amplify the differences between the lines. The grid on the x-axis is defined according to the optimal strike prices found through the optimization procedure (1.8) and showed in Table 2.1. The value of each option V_{k_i} and $V_{k'_j}$ to calculate matrix Z was neglected,

Table 2.1: Options' optimal strike prices for February 2010.

	k_1^*	k_2^*	$k_1'^*$	$k_2'^*$	$k_3'^*$
$[US\$/MWh]$	16.33	32.26	54.28	85.50	116.50

given that pricing these derivatives is beyond the scope of this work. Considerations about how the analysis should be modified once a proper valuation methodology is developed, are discussed in section 3.1.

2.1.2 Building and rebalancing portfolios

Once the market maker has launched these five options, it is worthwhile to inquiry how the agents can use of these instruments. Here, an illustration of how two agents, R_4 and G_2, could hedge their own price-quantity risk exposition from time $t = 0$ (Feb. 2009) to time $t = T$ (Feb. 2010).

Given their own information about expected power demand/generation at $t = T$ (see Tables 3.2 and 3.3 at the appendix), as well as the market information about spot price up to time $t = 0$, R_4 and G_2 create a former portfolio by replicating the their respective optimal payoff function showed in Figures 2.2a and 2.2c, using Eq.

1.7. This portfolio corresponds to the first row of Tables 2.2 and 2.3, respectively. Hereafter, random paths of average on-peak spot price and power demand/generation were generated for 12 months ahead from February 2009 (see Figure 2.1), then, as soon as updated information becomes available, R_4 and G_2 rebalance their own portfolio with Eqs. (1.17) and (1.14)[5] until expiration on Feb. 2010[6]. Tables 2.2 and 2.3 summarize the portfolios held by the agents at each time t, with the average percentage replicating error shown in the last column.

Table 2.2: Monthly replicating portfolio of R_4 from 0 to T.

Time	*Futures*	Put_1	Put_2	$Call_1$	$Call_2$	$Call_3$	*Error*
0	93,410	-1,123	-6,497	-7	-7,433	-4,166	3.30%
1	95,747	-921	-6,449	409	-7,196	-3,849	3.24%
2	96,472	-816	-6,395	615	-7,044	-3,673	3.22%
3	96,550	-809	-6,390	630	-7,034	-3,660	3.19%
4	96,358	-807	-6,381	628	-7,022	-3,654	3.23%
5	94,634	-722	-6,201	745	-6,765	-3,459	3.21%
6	94,626	-716	-6,209	750	-6,766	-3,458	3.39%
7	94,820	-655	-6,255	826	-6,746	-3,419	8.58%
8	96,099	-649	-6,023	1,000	-6,539	-3,205	2.41%
9	95,972	-667	-5,997	977	-6,534	-3,210	2.39%
10	94,699	366	-7,255	1,683	-6,584	-3,100	6.31%
11	97,155	416	-7,310	1,757	-6,585	-3,073	3.63%
12	98,336	-614	-7,281	370	-7,616	-4,190	2.44%

It is noteworthy that in Tables 2.2 and 2.3, no risk-free bonds are issued or lent in the portfolios, in order to be consequent with the zero-cost portfolio constraint of Eq. (1.3), since options' premium were neglected from the beginning.

[5]Recall that the terms of Eq. (1.14) for GENCOs are presented in the appendix.

[6]Here, the rebalancing has been carried out in a monthly basis, however, it can be done more often depending on the market transaction costs.

Table 2.3: Monthly replicating portfolio of G_2 from 0 to T.

Time	*Futures*	Put_1	Put_2	$Call_1$	$Call_2$	$Call_3$	*Error*
0	-63,430	3,116	6,726	4,210	9,535	7,206	7.19%
1	-63,373	3,117	6,723	4,213	9,534	7,206	6.97%
2	-63,184	3,119	6,715	4,222	9,528	7,207	6.68%
3	-63,004	3,123	6,704	4,232	9,521	7,207	7.21%
4	-63,186	3,117	6,714	4,217	9,525	7,203	6.56%
5	-62,775	3,121	6,693	4,234	9,508	7,201	6.28%
6	-62,191	3,118	6,675	4,241	9,487	7,193	4.19%
7	-62,243	3,062	6,747	4,138	9,500	7,159	2.39%
8	-61,869	3,183	6,434	4,371	9,322	7,149	1.90%
9	-63,095	3,130	6,409	4,250	9,250	7,054	1.83%
10	-61,957	1,756	8,635	2,153	9,959	6,671	5.25%
11	-58,518	1,819	8,344	2,337	9,740	6,624	4.00%
12	-58,974	2,782	7,211	3,951	9,629	7,200	2.88%

Figure 2.3a compares, after 10,000 scenario Monte-Carlo simulations, the distribution of R_4' profits at delivery time, after price hedge (holding positions only on futures/forward contracts) and after price-quantity hedge (holding the last portfolio of Table 2.2). However, with a price hedge the mean profit was US\$-703,000 with a 29% of standard deviation, while with a price-quantity hedge the mean profit was US\$-705,095 with a 6.8% of standard deviation, which shows meaningful improvements in reducing profit variance during on-peak periods on Feb. 2010 with a low profit mean reduction ($\approx 0.3\%$) , when price-quantity hedging strategy is undertaken. The expected value of this LSE's profit is negative because the expected spot price for on-peak hours F is greater than its regulated tariff r.

Equivalent results for G_2 are showed in Figure 2.3b, where the mean profit with price hedge was US\$151,310 with a 15.5% of standard deviation, while with a price-quantity hedge the mean profit was US\$152,870 with a 13.6% of standard deviation. Figures 2.3c and 2.3d depict how the dynamic hedge works onto the agents' profit

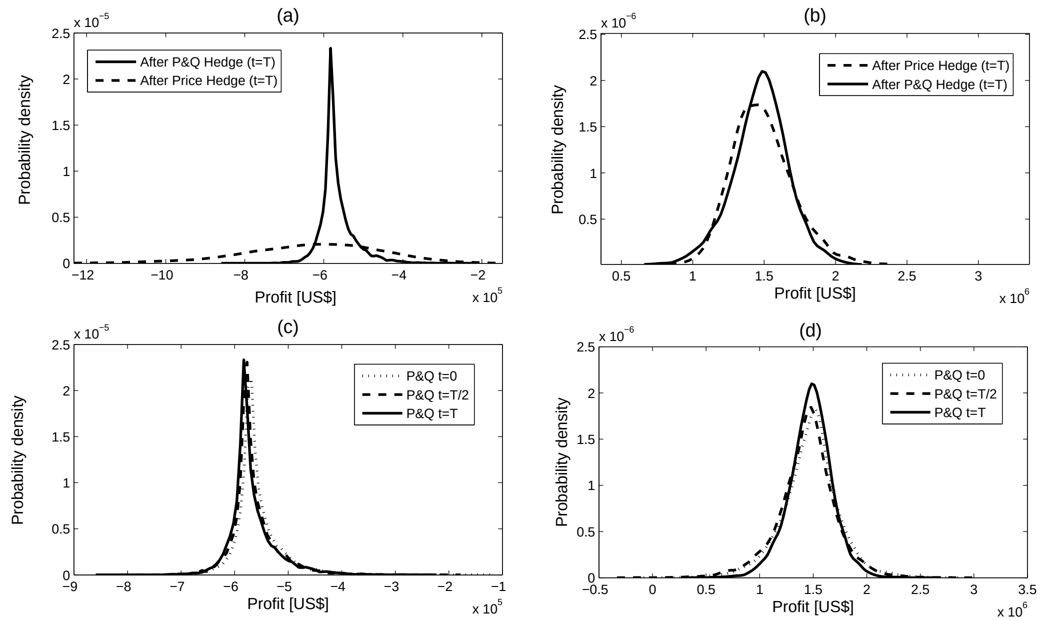

Figure 2.3: Profits' distribution comparison under different scenarios.

at maturity, for R_4 and G_2 respectively. This picture highlights the fact that the monthly rebalancing performed allows both agents to take advantage of current market conditions to maximize their own utility function at maturity. This strategy is always more suitable, in terms of profit's mean-variance, than an strategy using futures/forwards contracts, exclusively. Promising results were obtained for remaining agents as well.

Figures 2.4 illustrates R_4's expected profit and variation coefficient trade-off for different mean spot prices (Fig. 2.4a) and for different standard deviation of spot price as well (Fig. 2.4b). This is done by considering no hedge at all, price hedge and price-quantity hedge. It can be inferred from these pictures that, for a given expected profit level, the price-quantity hedging implies less risk on this profit than the other strategies do, and vice versa. That is, whatever the mean and standard

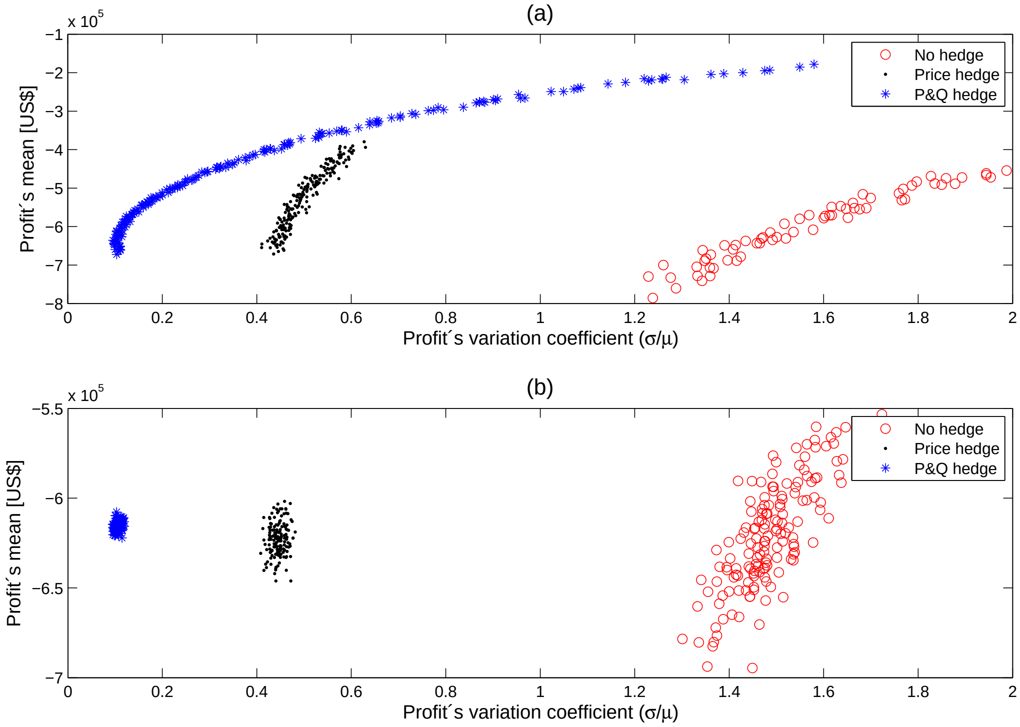

Figure 2.4: Profit's mean/risk trade-off for R_4 with different hedging strategies.

deviation of spot price are realized, it is always more financially efficient to undertake a price-quantity strategy compared to no hedge and even price hedge. Nevertheless, at a certain spot price mean level, the outcome obtained with both strategies, price and price-quantity, are very close (See Fig. 2.4a), so the agent could chose either strategy to hedge its profit.

On the other hand, Figures 2.5a and 2.5b show the number of instruments that the retailing market would demand and the power production market would sell, respectively, in order to build their hedging strategies month by month before maturity. This figure suggests that it would be possible to develop a market to trade these

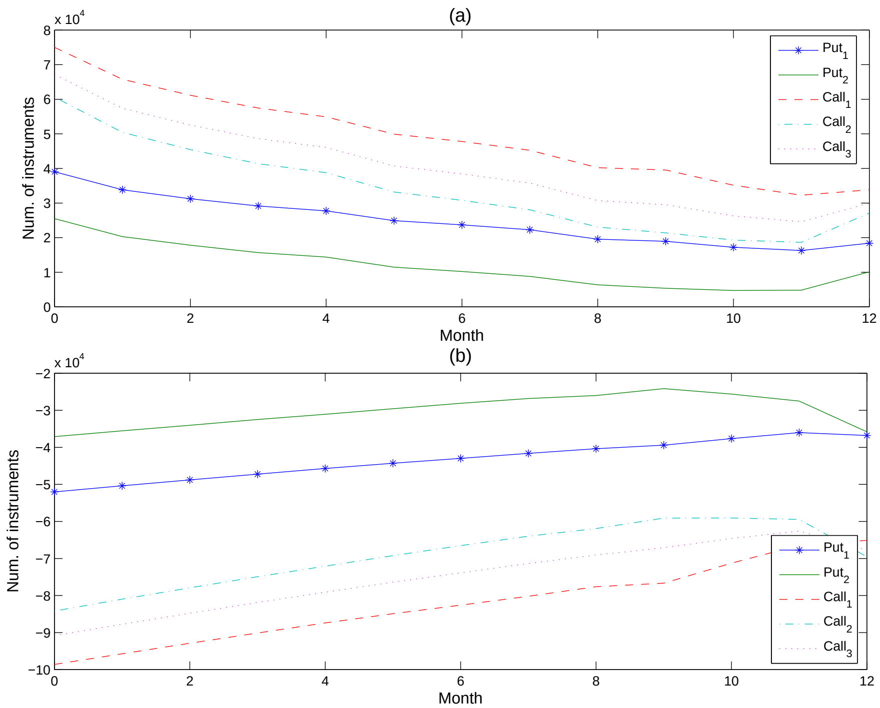

Figure 2.5: Number of instruments required for each market throughout the year.

options, which is promising result. Five major results should be highlighted from the exercise: (i) the solution of the global optimization process in section 5.1 does not favor any agent at all, by giving the same hedging opportunities to each of them. This is because the optimization problem is formulated in order to minimize the replicating error of both, retail and generation market payoff curves, and also because of the way these curves are calculated. (ii) Regarding the LEGO© approach theory presented in [42] and the Eq. (1.4), demonstrated in [28] and financially interpreted in [1], it is possible to replicate any payoff profile through a linear combination of the four basic financial instruments: bonds, futures/forward, swaps and options. Nevertheless, the markets are incomplete and it is not likely to have an infinite spectrum of options' strike prices as it is required for Eq. (1.4) to be held, so once a finite number of options is defined, it is necessary to deal with a certain level of replicating error, which could be mitigated with the replicating strategy proposed in section 3.2. GLS model's suitability and goodness is that it finds the coefficients' estimators that minimize this residual error produced because of the lack of infinite options' strike prices, then, it is likely to achieve a good fit by employing this methodology. The last column of Tables 2.2 and 2.3 shows replicating error at each rebalancing time, which could be considered low compared with other replicating strategies. (iii) It is important to note that for the agents whose load/generation is highly correlated with spot price and their risk aversion is high, this price-quantity hedging strategy gives better results in terms of expected value and dispersion of profits than just price hedging. However, those agents with low price-quantity correlation and low risk aversion, could achieve similar benefits by implementing either strategy. (iv) The dynamic hedge strategy with a static criteria presented in section 4, provides the agents an opportunity to build a portfolio and rebalance it during several months in advanced of delivery time, in order to offset probable profit's losses at maturity coming from spot price and power demand/generation uncertainty. (v) Finally, the results obtained suggest that there would be enough supply and demand for these options, so a derivatives market to trade them may be set up.

Chapter 3

Conclusions and future work

3.1 Conclusions

This work has developed a practical application framework to implement the static price-quantity hedging proposed in [1]. It was tested with real information from the Colombian power market. The analysis suggests that by offering two put and three call monthly options with optimal strike prices, an enhancement in agents' risk hedging positions might be achieved.

Prior to the creation and implementation of standard options, standard forward/futures contracts with the same time basis or even longer should be launched, in order to be able to build up these portfolios. Standardized contracts could solve the lack of liquidity on the current bilateral contracting system in Colombia and simultaneously, by creating a clearing house to manage such contractual relationships, the lack of anonymity can be solved. Recently, the Colombian power regulator (CREG[1]) as well as the Colombian power market administrator (XM[2]) have been working on the development of an standardized futures contract but nothing has been settled

[1] www.creg.gov.co
[2] www.xm.com.co

definitively, and any intention to offer electricity derivatives such as options or more complex instruments is foreseeable in the near future.

It can be said, that derivative instruments are suitable and viable to overcome some of the current Colombian power market problems, to improve and encourage competition among agents, and as Chao and Wilson say in [43], could help to suppress the spot price volatility and mitigate market power situations. Moreover, two noteworthy facts: (i) derivatives suggested are widely used internationally (Asian-style options), which create an added value market by offering different hedging strategies with simple and understandable financial instruments. (ii) if Colombia effectively gets interconnected with Central-America and other South-American countries, the Colombian derivatives market could be established as a strong electricity pool considering its experience on the subject in the region.

Another advantage derived from this proposal is that by using the spot price as underlying asset it would be encouraged a signal clean up beneficial for end users, avoiding external interventions, taxes and the impact of foreigner loads, making of the spot price a reliable and strong index to back up transactions in the market. Not to mention that in the meanwhile, next to the standardized forward/futures contracts to be offered, it could also be strengthen up a futures electricity price signal which after a while, if considered necessary, could shift the spot price signal.

An additional remark is that even along this paper power producers and retailers were treated as so, eventually, a non-dispatched power generator could be considered as a retailer and a retailer with excess of electricity, could be treated as a power producer. Instruments here proposed are still suitable for any agent's scenario.

Taking the spot price as an underlying asset for the proposed options could be regarded as a difficulty or a shortcoming for this application since most if not all the electricity derivative markets around the world underlie on forward contracts considering that forward contracts can be traded while spot electricity does not [44].

Even though it implies difficulties at the time of pricing the instruments, because the methodology of Black-Scholes-Merton cannot be directly applied, it can still be done. Doege et al. highlight the importance of a risk neutral spot price model and that these models can only be estimated by incorporating futures contracts [26]. Some references like [14],[15],[6],[45],[46] and [29], handle this issue as well. In fact, the methodology undertaken in [18] would be suitable as a first approach to tackle the pricing problem, since the authors applied their framework to find the fair value of an Asian-style options with an underlying asset that follows a mean reversion process, which is actually a feature of the Colombian spot electricity price.

Despite the options' value were neglected in the case study, this issue should be discussed. Once a proper valuation method is developed, some features of the hedging strategy must be modified. In first place, the intercept in Eq. (1.5) has to be enabled[3], since it represents the number of bonds in the portfolio to offset the losses or gains from positions in the options. Secondly, GLS approach no longer works to replicate the agent's optimal payoff function dynamicly, since changes through time in both instruments, bonds and futures/forward contracts, are going to be mainly determined by the greeks *theta* ($\partial V_k/\partial t$) and *delta* ($\partial V_k/\partial p$) respectively, in order to hedge the options sold or wrote as well as the agent's profit. Thus, *Conditional Least Squares* (CLS) method is proposed, rather than GLS, to estimate the coefficient of each instrument in the replicating portfolio, given that this method allows to impose restrictions on the coefficients of a linear regression.

Finally, one limitation of the approach here proposed, is that it does not consider the portfolio of technologies available for each power producer, who could have in his assets not only hydro power generation plants, but also natural gas and diesel or nuclear power generation plants. This fact would light up its portfolio of investment on electricity derivatives due to its available technological flexibility.

[3]Recall that the application case was carried out regardless this coefficient (β_0).

3.2 Further work

Different research lines can be oriented from this work:

- Estimating the forward electricity price curve properly in an incomplete and low liquid market such as the Colombian.
- Pricing these type of options with the electricity spot price as underlying asset.
- Modifying the replicating strategy into the dynamic hedging to take into account the changes induced by the value of options.
- To define what kind of options should be implemented in Colombia: European, American or Asiatic kind, since the approach developed does not impose any constraint in this sense. Deep research should be done on this direction to make clear that decision.
- To discuss and analyze the practical aspects of implementing these financial options and create a strong institution for trading them, as it is mentioned in [43].

Bibliography

[1] Y. Oum, S. Oren, and S. Deng, "Hedging quantity risk with standard power options in a competitive wholesale electricity market," *Naval Research Logistics*, vol. Vol. 53, pp. pp. 697–712, 2006.

[2] R. Wilson, "Architecture of power markets," *Econometrica*, vol. Vol. 70, No. 4, pp. pp. 1299–1340, 2002.

[3] S.-E. Fleten, S.-W. Wallace, and W. Ziemba, "Hedging electricity portfolios via stochastic programming," *Decision making under uncertainty: energy and power*, pp. pp. 71–93, 2002.

[4] E. Näsäkkälä and J. Keppo, "Electricity load pattern hedging with static forward strategies," *Managerial Science, special issue energy pricing and risk management*, vol. Vol. 31, No. 6, pp. pp. 116–137, 2005.

[5] E. Mork, "Emergence of financial markets for electricity: a european perspective," *Energy Policy*, vol. Vol. 29, pp. pp. 7–15, 2001.

[6] J. Lucia and E. Schwartz, "Electricity prices and power derivatives: evidence from the nordic power exchange," *Review of Derivatives Research*, vol. Vol. 5, pp. pp. 5–50, 2002.

[7] N. Von der Fehr and D. Harbord, "Competition in electricity spot markets: Economic theory and international experience," Memorandum, University of Oslo, Tech. Rep., 1997.

[8] A. Tishler, J. Newman, I. Spekterman, and C. Woo, "Assessing the options for a competitive electricity market in israel," *Utilities Policy*, vol. Vol. 16, pp. pp. 21–29, 2008.

[9] C. Woo and J. Zarnikau, "Will electricity market reform likely reduce retail rates?" *Article in press. Electricity Journal*, vol. Vol. 22, Issue 2, 2009.

[10] G. Unger, "Hedging strategy and electricity contract engineering," Ph.D. dissertation, No. 14727. ETHZ., 2002.

[11] S. Deng and S. Oren, "Electricity derivatives and risk management." *Energy*, vol. Vol. 31, pp. pp. 940–953, 2006.

[12] B. Willems and J. Morbee, "Market completeness: how options affect hedging and investments in the electricity sector," *Energy Economics*, vol. doi:10.1016/j.eneco.2009.10.019, 2009.

[13] A. Eydeland and K. Wolyniec, *Energy and Power Risk Management. New developments in modeling, pricing and hedging.* John Wiley and Sons, Inc., Hoboken, New Jersey., 2003.

[14] D. Pilipovic, *Energy risk: valuing and managing energy derivatives.* McGraw-Hill, New York., 1997.

[15] A. Eydeland and H. Geman, *Fundamentals of electricity derivatives in energy modeling and the management of uncertainty.* Risks Books, New York., 2000.

[16] M. Wilhelm, "Modeling, pricing and risk management of power derivatives," Ph.D. dissertation, dissertation No. 17062. ETHZ., 2007.

[17] I. Vehviläinen, "Basics of electricity derivative pricing in competitive markets," *Applied Mathematical Finance*, vol. Vol. 9, pp. pp. 45–60, 2002.

[18] G. Fusai, M. Marena, and A. Roncoroni, "Analytical pricing of discretely monitored asian-style options: Theory and application to commodity markets," *Journal of Banking & Finance*, vol. Vol. 32, p. pp. 20332045, 2008.

[19] P. Carr and D. Madan, "Option valuation using the fast fourier transform," *Journal of Computational Finance*, vol. Vol. 2 (4), pp. pp. 61–73, 1999.

[20] H. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. Vol. VII, No. 1, 1952.

[21] M. Xu, "Risk measure pricing and hedging in incomplete markets," Finance 0406004, EconWPA, Tech. Rep., 2004.

[22] D. Ahn, J. Boudoukh, M. Richardson, and R. Whitelaw, "Optimal risk management using options," NBER series No. 6158., Tech. Rep., 1997.

[23] P. Kleindorfer and L. Li, "Multi-period var-constrained portfolio optimization with applications to the electric power sector," *Energy Journal*, vol. Vol. 26 (1), pp. pp. 1–26, 2005.

[24] R. Huisman, R. Mahieu, and F. Schlichter, "Electricity portfolio management: optimal peak/off-peak allocations," *Energy Economics*, vol. Vol. 31, pp. pp. 169–174, 2009.

[25] E. Näsäkkälä and J. Keppo, "Hydropower production planning and hedging under inflow and forward uncertainty." *Systems analysis research reports E15*, 2005.

[26] J. Doege, P. Schiltknecht, and H. Lüthi, "Risk management of power portfolios and valuation of flexibility," *OR spectrum*, vol. Vol. 28, pp. pp. 267–287, 2006.

[27] G. Vizcaíno, J. Alzate, A. Cadena, and J. Benavides, "Plain vanilla options for price-quantity hedging in the Colombian wholesale electricity market," in *32nd IAEE International Conference. Energy-Economy-Environment. San Franciso, CA, USA.*, 2009.

[28] P. Carr and D. Madan, "Optimal positioning in derivative securities," *Quantitative Finance*, vol. Vol. 1, p. pp. 1937, 2000.

[29] P. Boyle, M. Broadie, and P. Glasserman, "Monte carlo methods for security pricing," *Journal of Economic Dynamics and Control*, vol. Vol. 21, pp. pp. 1267–1321, 1997.

[30] N. Audet, P. Heiskanen, J. Keppo, and I. Vehvilainen, "Modeling electricity forward curve dynamics in the nordic market," *Modelling Prices in Competitive Electricity Markets, D.W. Bunn (ed.), Wiley*, pp. pp. 251–265, 2004.

[31] G. Box and G. Jenkins, *Time Series Analisys: forecasting and control*, E. Robinson, Ed. Holden-Day, 1994.

[32] S.-E. Fleten and J. Lemming, "Constructing forward price curves in electricity markets," *Energy Economics*, vol. Vol. 25, pp. pp. 409–424, 2003.

[33] M. Vázquez, E. F. Sánchez-Úbeda, A. Berzosa, and J. Barquín, "Short-term evolution of forward curves and volatility in illiquid power markets," University Library of Munich, Germany, MPRA Paper 8932, Feb. 2008. [Online]. Available: http://ideas.repec.org/p/pra/mprapa/8932.html

[34] T. Kristiansen, "Pricing of monthly forward contracts in the nord pool market," *Energy Policy*, vol. Vol. 35, pp. pp. 307–316, 2007.

[35] H. Bessembinder and M. L. Lemmon, "Equilibrium pricing and optimal hedging in electricity forward markets," *the Journal of Finance*, vol. Vol. 57 (3), 2002.

[36] J. Moreno, "Trading strategies modeling in colombian power market using artificial intelligence techniques," *Energy Policy*, vol. Vol. 37, pp. pp. 836–843, 2009.

[37] P. Cramton and S. Stoft, "Colombia firm energy market," in *Proceedings of the Hawaii International Conference on System Sciences*, 2007.

[38] J. Alzate, A. Cadena, and J. Benavides, "Analysis of the firm energy market mechanism for the colombian electricity market." in *IEEE PES Transmission and Distribution Conference and Exposition. Colombia.*, 2008.

[39] CREG, "Documento CREG 065 de 2006 - Definición de la componente de generacióndе la fórmula tarifaria de energía eléctrica," 2006.

[40] A. Castro, "Análisis de la implementación de un sistema de derivados financieros enel sector eléctrico colombiano," Master's thesis, Universidad de los Andes, 2005.

[41] A. Ceballos, "Impacto del sistema eléctronico de contratos sec en las empresas del sectoreléctrico colombiano," Master's thesis, Universidad de los Andes, 2006.

[42] C. Smithson, "A LEGO approach to financial engineering: An introduction to forwards, futures, swaps and options," *Midland Corporate Finance Journal*, 1987.

[43] H. Chao and R. Wilson, "Resource adequacy and market power mitigation via option contracts." in *POWER Ninth Annual Research Conference*, 2004.

[44] F. Kjaerland, "A real option analysis of investments in hydropower - the case of norway," *Energy Policy*, vol. Vol. 371, pp. pp. 543–551, 2006.

[45] V. Kholodnyi, "Valuation and hedging of european contingent claims on power with spikes: a non-markovian approach," *Journal of Engineering Mathematics*, vol. Vol. 49, p. pp. 233252, 2004.

[46] M. Burger, B. Klar, A. Müller, and G. Schindlmayr, "A spot market model for pricing derivatives in electricity markets," *Quantitative Finance*, vol. Vol. 4, pp. pp. 109–122, 2004.

Appendix

Proof

Here, we present the deduction for Eq. (1.16) considering that Eq. (1.15) is clearly deduced in [1]. Taking into account the mean-variance utility function (1.9) and the assumption that $P \equiv Q$ Eq. (1.10) reduces itself to Eq. (1.11):

$$x^*(p_T) = E[y(p_T, q_T)] - E[y(p_T, q_T)|p_T]$$

By replacing Eq. (1.2) in the above equation we obtain:

$$x^*(p) = E[p_T q_T] - cE[q_T] - (p_T - c)E[q_T|p_T] \tag{3.1}$$

Now, suppose the marginal distributions of p_T and q_T as follows under P and Q: $log(p_T) \sim N(m_p, s_p^2)$, $q \sim N(m_q, s_q^2)$ and $\rho = Corr(p_t, q_t)$, then $q_T|p_T \sim N(m_q + \rho\frac{s_q}{s_p}(log(p_T) - m_p), s_q^2(1-\rho^2))$. Hence, we can write the following equations:

$$\begin{aligned}
E[p_T q_T] &= \rho\sqrt{Var(lp_T)Var(q_T)}E[p_T] + E[q_T]E[p_T] \\
E[p_T] &= e^{m_p + \frac{1}{2}s_p^2} \\
E[q_T] &= m_q \\
Var(lp_T) &= s_p^2 \\
Var(q_T) &= s_q^2
\end{aligned}$$

Finally, Eq. (1.16) is obtained by replacing these results in Eq. (3.1).

Partial derivatives calculation

Calculating partial derivatives of Eq. (1.15) for a retailer agent we obtain,

$$\begin{aligned}
\frac{\partial x}{\partial m_q} &= p_T - e^{m_p+\frac{1}{2}s_p^2}\\
\frac{\partial x}{\partial m_p} &= \frac{\rho s_q}{s_p}(r-p_T) - (\rho s_p s_q + m_q)e^{m_p+\frac{1}{2}s_p^2}\\
\frac{\partial x}{\partial s_p} &= \frac{\rho s_q}{s_p^2}(r-p_T)(log(p_T)-m_p) - (\rho s_q + \rho s_q s_p^2 + m_q)e^{m_p+\frac{1}{2}s_p^2}\\
\frac{\partial x}{\partial s_q} &= -\rho s_p e^{m_p+\frac{1}{2}s_p^2} - \frac{\rho}{s_p}(r-p_T)(log(p_T)-m_p)\\
\epsilon_{m_p} &= (\rho s_q s_p + m_q)(\Delta m_p + 1 - e^{\Delta m_p})e^{m_p+\frac{1}{2}s_p^2}\\
\epsilon_{s_p} &= \left[\rho s_q \Delta s_p\left(1 - e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p}\right) + m_p\left(\Delta s_p - e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p}+1\right)\right]e^{m_p+\frac{1}{2}s_p^2}\\
&\quad + \rho s_p s_q\left(s_p\Delta s_p - e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p}+1\right)e^{m_p+\frac{1}{2}s_p^2} + \rho s_q(r-p_T)(log(p_T)-m_p)\left(\frac{1}{s_p} - \frac{\Delta s_p}{s_p^2} - \frac{1}{s_p+\Delta s_p}\right)
\end{aligned}$$

Likewise, Eq. (1.16) must be used to calculate its partial derivatives for generators. In this way:

$$\begin{aligned}
\frac{\partial x}{\partial m_q} &= e^{m_p+\frac{1}{2}s_p^2} - p_T\\
\frac{\partial x}{\partial m_p} &= (\rho s_p s_q + m_q)e^{m_p+\frac{1}{2}s_p^2} + \frac{\rho s_q}{s_p}(p_T - c)\\
\frac{\partial x}{\partial s_p} &= \frac{\rho s_q}{s_p^2}(p_T-c)(log(p_T)-m_p) + (\rho s_q + \rho s_q s_p^2 + m_q)e^{m_p+\frac{1}{2}s_p^2}\\
\frac{\partial x}{\partial s_q} &= \rho s_p e^{m_p+\frac{1}{2}s_p^2} - \frac{\rho}{s_p}(r-p_T)(log(p_T)-m_p)\\
\epsilon_{m_p} &= (\rho s_q s_p + m_q)(e^{\Delta m_p} - \Delta m_p - 1)e^{m_p+\frac{1}{2}s_p^2}\\
\epsilon_{s_p} &= \left[\rho s_q \Delta s_p\left(e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p} - 1\right) + m_p\left(e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p} - \Delta s_p - 1\right)\right]e^{m_p+\frac{1}{2}s_p^2}\\
&\quad + \rho s_p s_q\left(e^{\frac{\Delta s_p^2}{2}+s_p\Delta s_p} - s_p\Delta s_p - 1\right)e^{m_p+\frac{1}{2}s_p^2} + \rho s_q(r-p_T)(log(p_T)-m_p)\left(\frac{1}{s_p} - \frac{\Delta s_p}{s_p^2} - \frac{1}{s_p+\Delta s_p}\right)
\end{aligned}$$

Parameters of application case

Table 3.1: ARIMA processes' parameters.

Agent/ARIMA	p	d	q	P	D	Q
R_1	2	1	3	0	0	1
R_2	3	2	3	0	0	1
R_3	5	2	0	0	0	1
R_4	4	1	3	1	0	1
R_5	5	2	4	0	0	1
G_1	1	2	2	1	0	1
G_2	1	1	1	-	-	-
G_3	1	1	1	-	-	-
G_4	0	1	3	-	-	-
G_5	4	1	4	-	-	-
G_6	3	2	2	-	-	-
G_7	5	1	2	-	-	-
G_8	1	2	4	-	-	-
G_9	2	2	1	-	-	-

Table 3.2: LSEs' parameters.

Retailer	μ_R[MWh]	σ_R [MWh]	r *[US$/MWh]*	a_R	ρ_R
R_1	83,656.38	4,417.17	53.70	0.92	0.28
R_2	13,709.17	2,875.50	45.48	1.00	-0.14
R_3	47,399.28	11,409.27	56.23	0.96	0.50
R_4	87,812.84	5,066.46	47.02	0.92	-0.15
R_5	17,927.15	2,551.20	57.38	0.91	0.14

Table 3.3: GENCOs' parameters.

Generator	μ_G[MWh]	σ_G [MWh]	*c [US$/MWh]*	a_G	ρ_G
G_1	5,867.10	16,709.49	28.77	0.784	-0.21
G_2	61,693.97	11,361.47	30.33	0.257	-0.14
G_3	20,369.69	6,449.32	14.17	0.670	0.60
G_4	59,974.77	8,215.22	46.00	0.784	0.27
G_5	67,431.71	11,276.94	32.55	0.776	0.14
G_6	63,485.78	8,100.54	35.30	0.948	-0.57
G_7	6,144.02	2,284.42	47.21	0.909	-0.16