Incentives in Demand Response Programs

by

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Abstract

In this work we prove the inefficiency (in the sense of Pareto) of the electricity system, as well as its resemblance with the tragedy of the commons. Furthermore, a non-flat aggregated demand of the population is evidenced in the optimal consumption profile of the population. We propose two incentives, namely economic and social, in order to achieve an efficient flat consumption. On the one hand, efficiency in the electricity consumption is achieved by means of economic incentives. The economic incentive might be seen as an indirect revelation mechanism. These incentives (monetary rewards or punishments) are calculated for each user, based solely on the consumption of the population. On the other hand, a flat demand is achieved by means of social incentives. In this case, social rewards are provided by a coordination game in which the members of a networked take part. We consider that the economic and social incentives are associated with Smith dynamics and opinion dynamics, respectively. Some simulation results show the ideas presented in this paper.

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Chapter 1

Introduction

The *smart grid* (SG) concept has entailed profound changes in the conception of electricity systems. These changes are motivated by a set of improvements that are possible thanks to the development of new technologies in some fields, such as control, communications, and power generation, among others. Particularly, efficiency in both electricity generation and consumption is one of the main goals of the SG [1]. On this respect, since consumers become active agents in the SG, the technological development might be insufficient by itself to achieve the desired goals [2]. Efficiency is expected to be achieved by means of an active cooperation of consumers in the electricity systems. Here, *demand response* (DR) programs arise as a tool intended to promote cooperation of consumers with the electricity system. Hence, *demand response* (DR) programs are considered essential tools for the success of the SG.

DR programs deal with the problem of providing incentives to the consumers in order to modify their electricity consumption behavior. DR has been considered from different perspectives, mainly based on economic incentives. The incentives used in the literature include pricing mechanism, by which either higher or lower consumption is encouraged through changes in the electricity prices along the day. The design of incentives is not a trivial task, since the pricing scheme might impact on the volatility or robustness of the system [3].

Also, incentives must be designed having into account the characteristics of the consumers. Particularly, consumers might be either *price takers* or *price anticipators*.

On the one hand, a price taker agent makes decisions without considering future implications of its decisions. On the other hand, a price anticipator (or strategic agent in the context of *game theory*) make decisions having into account the consequences of its actions on the system. Hence, the behavior of consumers might leads to different outcomes. Particularly, Johari and Tsitsiklis show that a strategic behavior lead to inefficient outcomes with respect to the optimal outcome, in the context of network resource allocation [4].

The optimal electricity consumption in a population of strategic agents might be achieved by means of *mechanism design* (see [5,6]). Mechanism design addresses the problem of designing the rules of a game, in order to achieve the desired outcome. Some mechanisms require consumers to report their preferences on electricity consumption to design prices that lead to an optimal outcome. However, agents might be unwilling to report their true preferences due to strategic or privacy issues. For example, telling lies might be the best action for a consumer in order to maximize its welfare or protect its interests. Therefore, incentives must be designed guaranteeing efficiency requirements as well as truth revealing properties [7]. A class of mechanism that meets these requirements is known as *direct revelation mechanisms* [7]. Direct revelation mechanisms require a central agent that gathers information and computes the optimal resource allocation. Some disadvantages of this class of mechanism are its information and computational burden [8]. Therefore, we propose a scheme of economic incentives that can be seen as an *indirect revelation mechanism* that is based on the Clarke pivot mechanism [7]. Unlike direct revelation mechanisms, our mechanism does not requires information related to the consumption preferences of each user. Instead, our mechanism only uses consumption signals from consumers, which are assumed to be known by the utility. Since we do not ask users preferences, we are using less information and computations. In this case, the resource allocation process is decentralized, because each agent allocates energy consumption along the day according to the incentive signals provided by a central agent.

The aforementioned economical incentives are characterized for achieving a social optimal in terms of a perfect competition market. However, unlike what is sought in the SG, we show that the optimal equilibrium is not completely flat when the preferences vary along the day. Hence, a second type of incentives, namely social incentives, is proposed in order to promote a flat demand profile in the population [9]. Here, social incentives are used to modify the preferences of the users. Therefore, an optimal and flat demand might be achieved by the combination of social and economical incentives. These incentives might be provided by an environment in which users interact. Particularly, Milinski et al. have shown that by means of social incentives it is possible to promote cooperation in populations [10].

The main contributions of this paper are as follows: 1) we formulate the demand response problem as a tragedy of the commons dilemma, highlighting the efficiency loss in the electricity system when price signals are not controlled; 2) we propose a novel scheme of economic incentives for achieving optimal demand profiles in a population of strategic agents; 3) we prove that the optimal demand profile of a population might not be completely flat if the consumption preferences (or energy necessities) are time varying; and 4) we propose a novel approach of social incentives, that are intended to promote changes in the preferences of the consumers to achieve flat demand profiles. In particular, the influence over the preferences of a population have been considered in [11–13], and the references therein. On the one hand, [11,12] consider that the preferences are related to the proclivities of each agent toward buying certain product. In these cases, the preferences might be influenced by means of advertising or marketing strategies. On the other hand, [13] assumes that the process of opinion formation is influenced by external factors, such as media influence. In this case, we do not consider the problem of adoption of a technology. Instead, we consider a scenario in which the SG technology (such as smart meters) has already been installed and the incentives objectives are to promote an efficient consumption. Specifically, we model social interactions by means of *opinion dynamics* together with *environmental incentives* [14]. In this case, the environmental incentives are described by a coordination game, in which we consider the existence of *prominent agents* (agents with major influence over other agents). In this model, the preferences of each user are the parameters that determine the valuation of the electricity in each time interval.

In this case, we assume that a change in the preferences of an agent are translated in a modification in the consumption through a learning process. Particularly, we assume that the learning process is carried out using the *Smith dynamics* [15]. However, these learning dynamics are based on some rationality assumptions: 1) agents try to maximize their profit; 2) agents are provided with sufficient information about the electricity prices; 3) users are able to calculate the consumption level that leads to maximum profit; and 4) users have the ability to adjust their consumption as required. Since these assumptions are not valid in some real scenarios, one might consider the existence of an automation tool in charge of *load management*, i.e., one tool that decides the best energy consumption action of some appliances in each time instant. Some load management approaches, such as the proposed in [16] [17], require estimation or information about the scheduled energy consumption of all users. In our approach, such tools would schedule the energy consumption of each consumer based only on a price signal and the information about the utility function of each consumer.

This work is outlined as follows. In Chapter 2 we introduce an electricity market model, some efficiency results, and a model of opinion dynamics with environmental incentives. Chapter 3 presents an outline of the economical incentives that lead to an efficient outcome. Here we show the properties and limitations of this mechanism of incentives. The modeling of the social influence in the formation of preferences, as well as a consumption adjustment processes are introduced in Chapter 4. Chapter 5 presents some simulation results, while some concluding remarks are made in Chapter 6.

Chapter 2

Literature Review

In this section, we introduce a market model of the electricity system, as well as the model that describes the social dynamics of a population. Our social dynamics admits clustering or consensus, among other properties. We aim to model the possible behaviors in a real population.

2.1 Electricity Market Model

Here we assume that the electricity system is composed by three elements namely consumers, producers, and an independent system operator (ISO). In this case, we consider a set $\mathcal{V} = \{1, \ldots, N\}$ of N consumers and a unique producer, who are restricted to consume and sell electricity, respectively. On the other hand, the ISO is in charge of regulate the market by equating the generation to the demand of energy [3]. The ISO is just a coordinator, and do not influence the market.

In particular, we consider that their energy consumption along a day is determined by time varying preferences. These preferences reflect the changing necessities of energy along the day. Hence, we partition the day in T time intervals, in which the energy necessities are roughly the same. In this way, we take into account the time varying preferences of each user.

Along these lines, we consider that the i^{th} user might decide the amount of energy q_i^t that consumes at the time interval $t \in \{1, \ldots, T\}$. The daily consumption profile of a user might be represented by the vector $\boldsymbol{q}_i = [q_i^1, \ldots, q_i^T]^\top \in \mathbb{R}^T_{\geq 0}$. Without loss of generality, we restrict the electricity consumption to null or positive values, i.e., $q_i^t \geq 0$ for all agents $i \in \mathcal{V}$ and any time interval $t \in \{1, \ldots, T\}$. Likewise, the electricity consumption of the population is denoted by the vector $\boldsymbol{q} = [\boldsymbol{q}_1^\top, \ldots, \boldsymbol{q}_N^\top]^\top \in \mathbb{R}^{T \cdot N}_{\geq 0}$.

The preferences of each user, together with the electricity consumption define a degree of welfare that each customer receives. This welfare might be represented by the benefit earned, minus the cost of the energy consumed. On the one hand, the benefit might be represented by means of a *valuation function* $v_i^t : \mathbb{R} \to \mathbb{R}$, where $v_i^t(q_i^t)$ represents the welfare experienced by the i^{th} user when consuming q_i^t electricity units in the t^{th} time interval. An agent daily utility $v_i : \mathbb{R}^T_{\geq 0} \to \mathbb{R}$ is the aggregation of the welfare experimented in each time interval t, i.e., the aggregate welfare of the i^{th} agent is defined as $v_i(\boldsymbol{q}_i) = \sum_{t=1}^T v_i^T(q_i^t)$, where $t \in \{1, \ldots, T\}$. On the other hand, the costs of the energy in a time interval t is denoted by λ^t. Without loss of generality, we consider null and positive prices, i.e., $\lambda^t \in \mathbb{R}_{\geq 0}$. The daily energy costs might be represented by the vector $\boldsymbol{\lambda} = [\boldsymbol{\lambda}^1, \ldots, \boldsymbol{\lambda}^T]^\top$. Consequently, the profit of the i^{th} consumer might be represented by

$$U_i(\boldsymbol{q}) = v_i(\boldsymbol{q}_i) - \boldsymbol{q}_i^\top \boldsymbol{\lambda}. \tag{2.1}$$

Prices are determined by the generation capabilities of the producer. Particularly, we assume that the generation g^t at each time interval t is coordinated by the ISO in order to guarantee that the generation equals the total demand, i.e., $g^t = \sum_{i=1}^N q_i^t$. Also, we assume that the energy price is enough to cover the production costs. That is

$$\sum_{t=1}^{T} g^t \lambda^t = \sum_{t=1}^{T} C(g^t), \tag{2.2}$$

where $C(g^t)$ is the production cost associated with a generation of g^t energy units at the time interval t. The selection of prices might be made using either marginal or average price, among others. On the one hand, marginal price requires an infinite population. On the other hand, average price does not have restrictions on the number of users. Particularly, average prices are used commonly in the electricity system. The average price function $p : \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$p(\hat{q}^t) = \frac{C(\hat{q}^t)}{\hat{q}^t} = \lambda^t, \tag{2.3}$$

where $\hat{q}^t \in \mathbb{R}_{\geq 0}$ is the total demand of energy at the t^{th} time interval, i.e., the electricity price at time t is calculated having into account the aggregate consumption of the whole population defined as $\hat{q}^t = \sum_{j=1}^{N} q_j^t$. We consider that the generation cost is the same at every time interval. The profit of the i^{th} agent might be rewritten as

$$U_i(\boldsymbol{q}) = v_i(\boldsymbol{q}_i) - \sum_{t=1}^{T} q_i^t p\Big(\sum_{j=1}^{N} q_j^t\Big) \tag{2.4}$$

Now, let us define the optimal consumption profile, in the sense of Pareto, as the equilibrium of the population when agents are price-takers. Accordingly, the profile μ that maximizes the social welfare is a solution to

$$\begin{aligned} \underset{\boldsymbol{q}}{\text{maximize}} \quad & \sum_{i=1}^{N} U_i(\boldsymbol{q}) = \sum_{i=1}^{N} \left(v_i(\boldsymbol{q}_i) - \sum_{t=1}^{T} q_i^t p\Big(\sum_{j=1}^{N} q_j^t\Big) \right) \\ \text{subject to} \quad & q_i^t \geq 0, i \in \mathcal{V}, \, t = \{1, \ldots, T\}. \end{aligned} \tag{2.5}$$

Here, if the following first order condition (FOC) is satisfied for all user $i \in \mathcal{V}$

$$\begin{aligned} \frac{\partial}{\partial q_i^t} \sum_{i=1}^{N} U_i(\boldsymbol{\mu}) = {} & \frac{\partial}{\partial q_i^t} v_i^t(\mu_i^t) \\ & - p\Big(\sum_{j=1}^{N} \mu_j^t\Big) - \sum_{h=1}^{N} \mu_h^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \mu_j^t\Big) = 0, \end{aligned} \tag{2.6}$$

then the equilibrium $\boldsymbol{\mu}$ is efficient in the sense of Pareto.

On the other hand, the scenario in which agents are price anticipators is characterized by strategic interactions among agents. Consequently, the electricity system might be seen as a game where each agent endeavor to maximize independently its own welfare. The equilibrium concept used in game theory is the the Nash equilibrium. In this sense, the Nash equilibrium ξ is a solution to the following maximization problem for all $i \in \mathcal{V}$

$$\begin{aligned} \underset{\boldsymbol{q}_i}{\text{maximize}} \quad & U_i(\boldsymbol{q}_i, \boldsymbol{q}_{-i}) = v_i(\boldsymbol{q}_i) - \sum_{t=1}^{T} q_i^t p\Big(\sum_{j=1}^{N} q_j^t\Big) \\ \text{subject to} \quad & q_i^t \geq 0, i = \{1, \ldots, N\}, t = \{1, \ldots, T\}. \end{aligned} \tag{2.7}$$

Accordingly, when Assumption 1 is satisfied, it is known that the optimization problem described in Eq. (2.7) has a unique solution inside the feasible set. Now, let the vector $\boldsymbol{\xi}$ in $\mathbb{R}^{NT}$ be the solution to the the first order conditions described by

$$\begin{aligned} \frac{\partial}{\partial q_i^t} W_i(\boldsymbol{\xi}_i, \boldsymbol{\xi}_{-i}) = & \frac{\partial}{\partial q_i^t} v_i^t(\xi_i^t) \\ & - p\Big(\sum_{j=1}^{N} \xi_j^t\Big) - \xi_i^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \xi_j^t\Big) = 0. \end{aligned} \tag{2.8}$$

where the vector $\boldsymbol{\xi}_i$ in $\mathbb{R}^T$ is the demand profile of the i^{th} agent, for all $i \in \mathcal{V}$. If condition 2.8 is satisfied for all agent, i.e., for all $i \in \mathcal{V}$, we say that $\boldsymbol{\xi}$ is the Nash equilibrium of the game described by Eq. (2.4).

Now, the existence of a market equilibrium inside of the feasible area is ensured if the following assumptions are satisfied.

Assumption 1.

i. The valuation function $v_i^t(\cdot)$ is differentiable, concave, and non-decreasing.

ii. The generation cost $C(\cdot)$ is a differentiable, convex, and non-decreasing function.

iii. $\frac{\partial}{\partial q_i^t} U_i(\boldsymbol{q})\Big|_{\boldsymbol{q}=\boldsymbol{0}} > 0$.

These assumptions are reasonable in the context of economic theory. Some typical functions that are used in the literature [16, 18] are

$$v_i^t(q_i^t) = \alpha_i^t \log(1 + q_i^t),\ \alpha_i^t > 0, \tag{2.9}$$

$$C(\|\boldsymbol{q}\|_1) = \beta(\|\boldsymbol{q}\|_1)^2,\ \beta > 0, \tag{2.10}$$

$$p(\|\boldsymbol{q}\|_1) = \beta\|\boldsymbol{q}\|_1,\ \beta > 0, \tag{2.11}$$

for any agent $i \in \mathcal{V}$ and any time instant $t = \{1, \ldots, T\}$. Here α_i^t represents the preferences that each agent have with respect to the energy consumption in the t^{th} time interval. On the other hand, β represents the generation characteristics of the producer.

In [19], it is shown that the equilibrium of the market might be inefficient when agents are price anticipating. That is, the equilibrium is characterized by a higher consumption of resources and a lower social welfare of the whole population. This result can be stated as follows:

Lemma 1. (C. Barreto et al., 2013) *Suppose that Assumption 1 is satisfied. Then, a game of the form stated in Eq. (2.7) has a unique Nash equilibrium* $\boldsymbol{\xi}$ *such that it satisfies the following conditions.*

i. $\|\boldsymbol{\xi}\|_1 > \|\boldsymbol{\mu}\|_1$,

ii. $\boldsymbol{\xi}$ *is not efficient in the sense of Pareto.*

$\boldsymbol{\mu}$ *is the the Pareto efficient equilibrium of the system described by Eq. (2.5) and* $\|\cdot\|_1$ *is the* L^1*-norm.*

Proof. Since Assumption 1 is satisfied, we know that the optimization problems in Eq. (2.5) and (2.7) have an unique solution that we refer as $\boldsymbol{\xi}$ and $\boldsymbol{\mu}$. Now, the proof of numeral i. is made by contradiction. Let us suppose that $\|\boldsymbol{\xi}\|_1 \leq \|\boldsymbol{\mu}\|_1$. Since $v_i(\cdot)$ is a non-decreasing concave function we have

$$\sum_{i=1}^{N} \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\xi}_i) \geq \sum_{i=1}^{N} \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\mu}_i) \tag{2.12}$$

Operating over Eq. (2.6) and (2.8) and replacing in (2.12), we have

$$\sum_{i=1}^{N} \xi_i^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \xi_j^t\Big) \geq \sum_{i=1}^{N} \sum_{h=1}^{N} \mu_h^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \mu_j^t\Big) \tag{2.13}$$

Since $p(\cdot)$ is a convex increasing function, we have

$$\frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \xi_j^t\Big) \leq \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \mu_j^t\Big).$$

Therefore,

$$\sum_{i=1}^{N} \xi_i^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \xi_j^t\Big) < \sum_{h=1}^{N} \mu_h^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \mu_j^t\Big)$$

The former equation might be rewritten as

$$\sum_{i=1}^{N} \xi_i^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \xi_j^t\Big) < \sum_{i=1}^{N} \sum_{h=1}^{N} \mu_h^t \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \mu_j^t\Big) \tag{2.14}$$

Since Eq. (2.13) and (2.14) are in contradiction, we conclude that $\|\boldsymbol{\xi}\|_1 > \|\boldsymbol{\mu}\|_1$.

Now, proof of numeral ii. is made by direct proof. From assumption 1 we know that the competitive equilibrium is unique, and corresponds to the best possible outcome for the population. Hence, the competitive equilibrium is efficient in the sense of Pareto. On the other hand, from numeral i. we conclude that $\|\boldsymbol{\xi}\|_1 > \|\boldsymbol{\mu}\|_1$. Therefore, $\boldsymbol{\xi} \neq \boldsymbol{\mu}$, which implies that the Nash equilibrium $\boldsymbol{\xi}$ is not equal to the competitive equilibrium. Hence, the the Nash equilibrium of the game defined by Eq. (2.5) is not efficient in the sense of Pareto. □

Lemma 1 reveals that the proposed electricity system conforms a social dilemma similar to the 'tragedy of the commons' [10]. This situation arises when a shared resource is overused by the people, even if they know the consequences of overusing it. In the case of the electricity system, the Pareto efficient outcome $\boldsymbol{\mu}$ requires the consumption of less electricity resources. However, at least one agent has incentives to deviate from the Pareto efficient outcome. Therefore, the social welfare maximizer is not a stable point. This fact illustrates the requirements for implementing incentives. In this approach we consider economical incentives for achieving demand response objectives.

The inefficiency of the market equilibrium motivates the design of economic incentives in order to force the population to an Pareto optimal outcome. The proposed incentives are social and economical.

Remark 1. *The discrimination of time periods is made in order to model different preferences of users along the day. However, the consumption made in each time period is considered independent with respect to the past consumption. That is, the optimization problems described in Eq. (2.5) and (2.7) can be separated in T independent optimization problems. Therefore, without loss of generality, we can analyze the case of $T = 1$ and the results made can be extended to cases with $T > 1$. Accordingly, the notation used hereafter do not have into account the time period.*

2.2 Opinion Dynamics with Environmental Incentives

The main concern of classical opinion dynamics theory is to model the decision processes between agents of a population and characterize the possible equilibrium that might arise, such as consensus or polarization of the population beliefs [20]. In such models, information about a particular subject might be transmitted and evaluated by each member of the population. Also, each agent adopts certain opinion according to the available information. However, classical models ignore the implications of a belief in a social environment. For example, if the actions or decisions of an agent are shaped according to its beliefs, then the welfare of an agent might depend indirectly on its beliefs. Therefore, the adoption of certain belief might be conditioned to the impact on the agent's welfare.

In this case, we assume that the consumers that are part of the electricity system conform a social network described by the connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of consumers and $\mathcal{E}$ is the set of social links between them. Here, the opinion of the i^{th} consumer is denoted by $0 \leq x_i \leq 1$. In this case, the opinion of each agent is related with its willingness to cooperate with the electricity system (in terms of efficient consumption). Here, $x_i = 0$ indicates null interest in cooperation, conversely, $x_i = 1$ demonstrates a disposition in cooperation.

In this model, we consider that the social environment of an agent is composed by its neighborhood. Hence, the social welfare of the i^{th} agent, denoted by $F_i(x) \in \mathbb{R}_+$, is considered as a function that depends on the opinion of the agent with respect to the opinion of its neighbors, where $\boldsymbol{x} = [x_1, \ldots, x_N]^\top$ is the opinion profile of the population. Particularly, we assume that the social welfare function $F_i(x)$ might be represented by a coordination game [11]. Hence, social incentives are represented by [9]

$$F_i(x) = \sum_{j \in \mathcal{N}_i} a_{i,j}\big(1 - \|x_i - x_j\|_\infty^2\big), \tag{2.15}$$

where $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$ is the set of neighbors of the i^{th} agent, i.e., nodes connected to the i^{th} agent by a link defined in $\mathcal{E}$. $a_{i,j}$ is a weight that represents the importance that the i^{th} agent gives to its j^{th} neighbor, such that $\sum_{j \in \mathcal{N}_i} a_{i,j} = 1$. Hence, an agent will have higher payoffs if its opinion is closer to the opinion of its neighborhood. Particularly, $F_i(x)$ is maximum when $x_i = \sum_{j \in \mathcal{N}_i} a_{i,j} x_j$, i.e., when the opinion of the i^{th} agent is equal to the weighted average of its neighbors' opinions. On the other hand, the maximum social utility, defined as $\max_x \sum_{i=1}^N F_i(x)$, corresponds to the case of consensus in the population, i.e., $x_i = x_j$ for all $i, j \in \mathcal{V}$. In this case, we assume that opinion modifications are made having into account only social incentives. Now, let us introduce the dynamics involved in the opinion updating process carried out by each agent. In this case, we consider that each agent updates its opinion according to a R_i rate Poisson alarm clock. Each time an agent is selected for updating its opinion, it observes the opinion of one of its neighbors and might update its opinion. The probability that the i^{th} agent observes the j^{th} is defined as p_{ij}, where $\sum_{j \in \mathcal{N}_i} p_{ij} = 1$ for all $i \in \mathcal{V}$. Now, we consider that the actualization is carried out by means of

$$x_i(k+1) = x_i(k)(1 - \lambda_{ij}) + x_j(k)\lambda_{ij}, \tag{2.16}$$

where λ_{ij} is the degree of confidence that the ith agent has on the j^{th} opinion, where j belongs to the neighborhood of i. In order to make λ_{ij} dependent on both the welfare improvement and the opinions distance let us define

$$\lambda_{ij} = \delta_{ij}\sigma_{ij}, \tag{2.17}$$

where $0 \leq \delta_{ij} \leq 1$, and $0 \leq \sigma_{ij} \leq 1$. δ_{ij} is a factor that represents the convenience of adopting the j^{th} agent's opinion from the point of view of the i^{th} agent. The convenience is estimated by comparing the welfare of both i^{th} and j^{th} agent's opinions, namely $F_i(x)$ and $F_j(x)$ respectively. In this case we use

$$\delta_{ij} = \frac{[F_j(x) - F_i(x)]_+}{F_j(x)}. \tag{2.18}$$

On the other hand, σ_{ij} represents a cost for the i^{th} agent due to changing opinion. This cost is in concordance with the inertia assumption. In the remainder of the document we use

$$\sigma_{ij} = 1 - (\|x_i(k) - x_j(k)\|_\infty)^q. \tag{2.19}$$

Note that q can be used for changing the disposition of a given agent to change its opinions regardless the payoff incentives. For example, using $q > 1$ may represent that the i^{th} agent is not too restricted by the distance of opinions. On the other hand, when $q < 1$ the distance in opinion is important. These characteristics may model different types of agents.

Remark 2. *σ_{ij} may be related with the bounded confidence parameter of opinion dynamics models because it limits the actualization processes as a function of the distance between the i^{th} and j^{th} agent's opinions.*

According to the previous definitions, we consider that $0 \leq \lambda_{ij} \leq 1$ to guarantee that the opinion $x_i(k)$ satisfies the constraint $0 \leq x_i^d \leq 1, \forall i \in \mathcal{V}, d \in \{1, \ldots, n\}$. Moreover, agents' opinions evolve in a subspace Ω determined by the convex hull of the initial population opinions $x(0)$, i.e., $\Omega = \text{co}(x(0))$.

Remark 3. *If $\lambda_{ij} = \frac{1}{2}$ for all $i \in \mathcal{V}$ and $j \in N_i$, this dynamic is equivalent to the opinion dynamics Deffuant-Weisbuch model [21] with parameter $\varepsilon_i = 1, \forall i \in \mathcal{V}$ and a fixed network topology.*

In order to analyze the convergence of the model, we are interested in the value at which the opinion of the i^{th} agent converges in time. We assume that k is large enough for the system to reach its steady state. This can be guaranteed when the future state is equal to the present state. We define the expected value of the opinion $x_i(k+1)$ as follows:

$$\mathbb{E}[x_i(k)(1-\lambda_{ij}) + x_j(k)\lambda_{ij}] = \sum_{j \in N_i} (x_i(k)(1-\lambda_{ij}) + x_j(k)\lambda_{ij}) p_{ij}, \quad (2.20)$$

where p_{ij} is the probability that the i^{th} agent selects the j^{th} agent of its neighbors to update its opinion, where $j \in N_i$, $\sum_{j \in N_i} p_{ij} = 1$. Equation (2.20) can be rewritten as

$$\mathbb{E}[x_i(k)(1-\lambda_{ij}) + x_j(k)\lambda_{ij}] = x_i(k) + \sum_{j \in N_i} (x_j(k) - x_i(k))\lambda_{ij} p_{ij}. \quad (2.21)$$

A discrete time system has reached an equilibrium, when the future state is equal to the present state, i.e., $x_i(k+1) = x_i(k)$. Thus, we obtain that

$$x_i(k) = x_i(k) + \sum_{j \in N_i} (x_j(k) - x_i(k))\lambda_{ij} p_{ij}, \quad (2.22)$$

which is only valid when $\sum_{j \in N_i} (x_j(k) - x_i(k))\lambda_{ij} p_{ij} = 0$. This condition holds when any of the following conditions holds for a given i and for all $j \in N_i$:

(i) $x_i(k) = x_j(k) \; \forall i \in \mathcal{V}, \forall j \in N_i$.

(ii) $F_i(x) \geq F_j(x)$.

(iii) $1 = \|x_i(k) - x_j(k)\|_\infty$.

Condition (i) can be interpreted as if the i^{th} agent and its neighbors have reached consensus. On the other hand, according to Equations (2.17), (2.18), and (2.19), condition (ii) and (iii) holds if $[F_j - F_i]_+ = 0$ or $\|x_i(k) - x_j(k)\|_\infty = 1$ respectively.

Based on the definition of $[\cdot]_+$, $[F_j(x) - F_i(x)]_+$ is equal to zero when $F_j(x) \leq F_i(x)$. This can be understood as the i^{th} agent has no incentive to change its opinion toward the j^{th} agent opinion. If for all $i \in \mathcal{V}$, conditions (i), (iii) or (ii) are satisfied, then the population state has reached an equilibrium.

We are also interested in including prominent agents in the model. A prominent agent is an agent with higher probabilities of being chosen by its neighborhood to update its beliefs, and it is also less influenced by other agents. Two options can be found depending on the probability of being chosen which are defined next.

Definition 1. *An agent j is a prominent agent if for all $i \in \mathcal{V}$ the condition $p_{ij} \geq p_{ik}$ is true for all $k, j \in N_i$, where $p_{ij} \neq 0$ and there is at least one l for which $p_{ij} > p_{il}$.*

According to the previous definition we proceed now to define the desirable states of the society. These are states in which the agents obtain maximum fitness or they are in an equilibrium.

Definition 2. *Let $\Theta_i = \arg\max_x F_i(x)$ be the set of states x that maximizes the function $F_i(x)$. These states are the desirable states of the i^{th} agent.*

However, it is possible that the desirable states are not reachable in all cases. Therefore, we define the best state that the population might achieve under the dynamics defined previously.

Definition 3. *Let $\Theta_i^\Omega = \arg\max_{x\in\Omega} F_i(x)$ be the set of states $x \in \Omega$ that maximizes the function $F_i(x)$. These states are the best possible states of the i^{th} agent.*

We assume that the the welfare of the i^{th} agent is the same for all the population's agents and its value depends only on its own opinion, i.e., $F_i(x) = F_i(x_i) = F(x_i), \forall i \in \mathcal{V}$. Thus, the welfare function The possibility of reaching the best possible states is defined by the following proposition.

Proposition 1. *Suppose a social network described by the connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a concave and Lipschitz continuous welfare function $F(x_i(k)) \in \mathbb{R}$, with Lipschitz constant L, and a set $\Gamma \subseteq \mathcal{V}$, such that for all $\gamma \in \Gamma$, $F(x_\gamma(0)) \geq F(x_i(0)) \geq L\frac{q^q}{(1+q)^{q+1}}$, with $q > 0$. Then, $F(x_i(k)) \to F(x_\gamma(0))$ as $k \to \infty$, where $x_i(k) \in \mathbb{R}^n$ is the opinion of the i^{th} agent that satisfies that $0 \leq x_i^d \leq 1, d \in \{1, \ldots, n\}$, $\forall i \in \mathcal{V}$.*

Proof. According to Eq. (2.20), if $\gamma \in N_i$, $F(x_\gamma(k)) \geq F(x_i(k))$, and $(\|x_\gamma(k) - x_i(k)\|_\infty)^q < 1$, the i^{th} agent updates its opinion $x_i(k)$ with respect to the γ^{th} agent's opinion according to

$$x_i(k+1) = \lambda_{i\gamma} x_\gamma(k) + (1 - \lambda_{i\gamma}) x_i(k), \tag{2.23}$$

where $0 < \lambda_{i\gamma} \leq 1$. Since $F(x_i(k))$ is concave, we know that $F(x_i(k+1)) \geq \lambda_{i\gamma} F(x_\gamma(k)) + (1 - \lambda_{i\gamma}) F(x_i(k))$. This can be rewritten as

$$\begin{aligned} &F(x_i(k+1)) \geq \\ &\qquad F(x_i(k)) + \lambda_{i\gamma}(F(x_\gamma(k)) - F(x_i(k))) > F(x_i(k)), \end{aligned} \tag{2.24}$$

Thus, the i^{th} agent obtains an increment in its welfare each time that it updates its opinion. Now, consider the Lipschitz continuity condition

$$\|F(x_i(k+1)) - F(x_i(k))\|_\infty \leq L\|x_i(k+1) - x_i(k)\|_\infty. \tag{2.25}$$

From Eq. (2.24) we know that $F(x_i(k+1)) - F(x_i(k)) > 0$. Therefore, $\|F(x_i(k+1)) - F(x_i(k))\|_\infty = |F(x_i(k+1)) - F(x_i(k))| = F(x_i(k+1)) - F(x_i(k))$. Thus, we rewrite the left side of Eq. (2.25) as

$$F(x_i(k+1)) - F(x_i(k)) \leq L\|x_i(k+1) - x_i(k)\|_\infty.$$

Replacing Eq. (2.23) on the right side we obtain

$$F(x_i(k+1)) \leq F(x_i(k)) + L\lambda_{i\gamma}\|x_\gamma(k) - x_i(k)\|_\infty.$$

Replacing (2.17), (2.18), and (2.19) results

$$\begin{aligned} F(x_i(k+1)) \leq F(x_i(k)) + L\frac{[F(x_\gamma) - F(x_i)]_+}{F(x_\gamma)} \ldots \\ \ldots \Big(1 - (\|x_\gamma(k) - x_i(k)\|_\infty)^q\Big)\|x_\gamma(k) - x_i(k)\|_\infty. \end{aligned}$$

Here, the product $\big(1-(\|x_\gamma(k)-x_i(k)\|_\infty)^q\big)\|x_\gamma(k)-x_i(k)\|_\infty$ has maximum value when $\|x_\gamma(k)-x_i(k)\|_\infty=\frac{1}{1+q}$. On the other hand, since $[F(x_\gamma(k))-F(x_i(k))]_+>0$, we rewrite $[F(x_\gamma(k))-F(x_i(k))]_+=F(x_\gamma(k))-F(x_i(k))$. Hence, replacing $\|x_\gamma(k)-x_i(k)\|_\infty=\frac{1}{1+q}$ and substracting $F(x_\gamma(k))$ on both sides of the inequality we obtain

$$F(x_i(k+1))-F(x_\gamma(k))\leq \Big(F(x_\gamma(k))-F(x_i(k))\Big)\Bigg(-1+L\frac{q^q}{(1+q)^{q+1}}\frac{1}{F(x_\gamma)}\Bigg),$$

where $\big(F(x_\gamma(k))-F(x_i(k))\big)>0$. Now, if $F(x_\gamma(k))\geq L\frac{q^q}{(1+q)^{q+1}}$ we can guarantee that

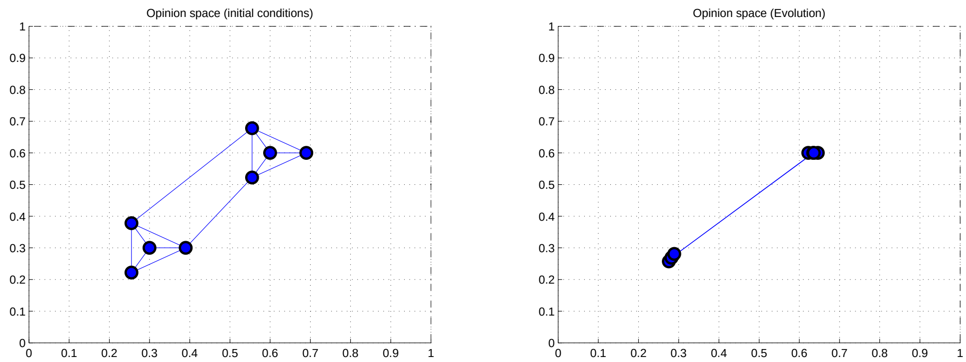
$$F(x_i(k+1))\leq F(x_\gamma(k)). \tag{2.26}$$

Along these lines, Eq. (2.24) shows that each actualization of the i^{th} agent opinions entails an increment in its own welfare. However, Eq. (2.26) states that the i^{th} agent's welfare is bounded by the γ^{th} agent welfare, i.e., $F(x_i(k+1))\leq F(x_\gamma(k))$ for all $k\in\{0,1,\ldots\}$. Therefore, the i^{th} agent welfare get closer to the γ^{th} agent welfare as the updating process takes place. Thus, $F(x_i(k))\to F(x_\gamma(k))$ as $k\to\infty$. Furthermore, as the i^{th} agent welfare increases, its neighbors are motivated to imitate its beliefs. Then, $F(x_j(k))\to F(x_i(k))\to F(x_\gamma(k))$ as $k\to\infty$, where $j\in N_i, j\notin N_\gamma$ and $i\in N_\gamma$. On the other hand, the γ^{th} agent's opinion remains unmodified because $\lambda_{\gamma i}=0,\forall i\in\mathcal{V}$, i.e., $x_\gamma(0)=x_\gamma(k),\forall k\in\{0,1,\ldots\}$. Consequently, since the graph $\mathcal{G}$ is connected, we have that $F(x_i(k))\to F(x_\gamma(0))$ as $k\to\infty,\forall i\in\mathcal{V}$.

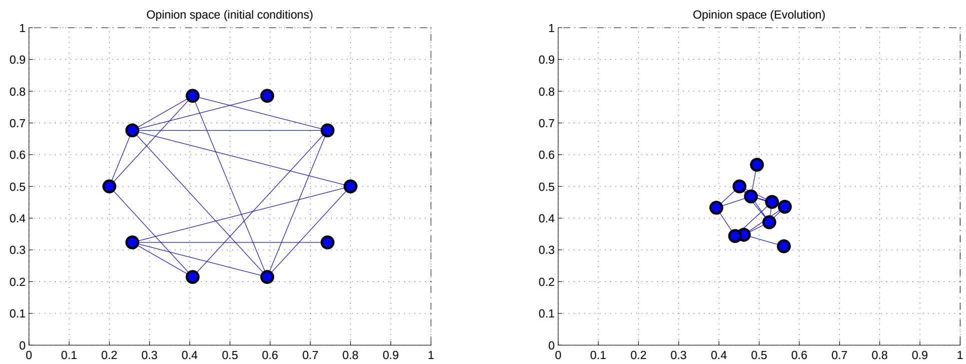
□

The model proposed in [14] also includes prominent agents, i.e., agents that might have a major influence in the society. A prominent agent is an agent that has higher probabilities of being chosen by its neighbors to update its belief, and is less influenced by other agents.

Figs. 2-1 and 2-2 show some examples of the opinion dynamics of a population of agents (considering a two dimensional opinion space). On the one hand, Fig. 2-1 shows cases of polarization and fragmentation in the population opinions. On the



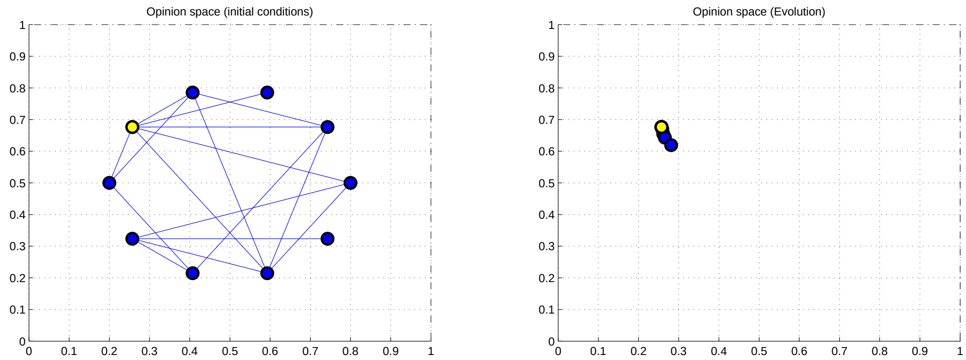
(a) Occurrence of polarization in opinions



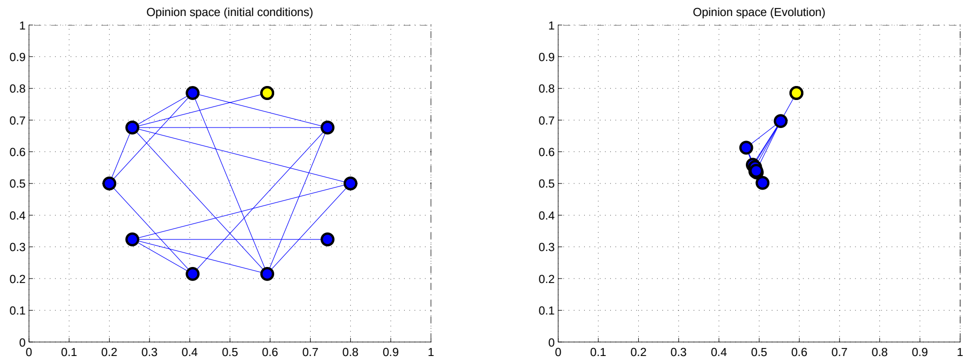
(b) Occurrence of fragmentation of opinions

Figure 2-1: Initial state and evolution of the opinions of two populations without prominent agents

other hand, Fig. 2-2 shows the social dynamics presence of prominent agents. We can see that the final state of the population is dependent on both the graph structure and the initial opinion profile. Also, the influence of the prominent agents is determined by its social links. Particularly, Fig. 2-2a and 2-2b show two scenarios in which the prominent agents are able or unable to influence the population, respectively. Here, the influence is the capacity to attract others opinions toward its own.



(a) Occurrence of consensus in the population due to the action of a prominent agent



(b) Case in which a prominent agent is not able to attract the population

Figure 2-2: Initial state and evolution of the opinions of two populations with prominent agents

Chapter 3

Economic Incentives

In the previous Chapter, we have observed that the society is not able to achieve a social optimal because the Nash equilibrium is not a Pareto optimal solution, i.e., when the users are price anticipating, the stable outcome is inefficient with respect to profits and resources usage. Accordingly, the design of economic incentives is required in order to guarantee that the competitive equilibrium is a stable outcome. In this section, we design and analyze an *indirect-revelation mechanism* for the DR problem.

The design of incentives is strongly related to the mechanism design. Mechanism design arises in the context of strategic situations that conform a game among some agents. The main objective in mechanism design is to design the rules that guarantee the realization of a particular objective function, rather than predict the outcome of a game [7, 22]. Mechanism design is applied mainly in scenarios with hidden information, i.e., situations such as auctions and resource allocation problems. In these situations, the players of the game have private information that is required to find the optimal outcome.

One of the mot commonly used mechanisms are the *direct-revelation mechanism.* A direct-revelation mechanisms is a game in which players are asked explicitly to reveal the information required to calculate the ideal outcome, i.e., the set of strategies of each player is a set of private information (see Vickrey second price auction) [7]. On the other hand, an indirect-revelation mechanism is a mechanism in which the outcome is calculated through information about the agents that implicitly contains the preferences of each agent. The direct-revelation mechanisms are characterized by being inefficient with respect to both communication and computation resources [8].

Specifically, the characteristics of the direct revelation mechanisms are unsuitable for the DR problem. The demand profile of each user contains information about the user preferences. Although the preferences of each user remains unrevealed, the demand profile information can be considered as a truthful mechanism for assigning incentives. Since the prices are set *ex post*, i.e., prices are set once the consumption has been made, intentionally false consumptions lead to looses rather than gains. We assume that the utility has information about the consumption profile of each user. This is reasonable because that information is basic when calculating the total price of the consumption.

An initial intuition about the mechanism is that incentives should reflect the contribution that an agent makes to the population surplus. The main idea is to redistribute the social gains among agents according to their impact on the population welfare. Thus, agents that cooperate are provided with higher incentives than agents that do not cooperate. From Lemma 1 we know that the welfare of the population increases as the agents approach to the Pareto optimal outcome. The profit change that the i^{th} user experiences when it unilaterally deviates from the Nash equilibrium ξ_i to $\hat{q}_i = \xi_i - \delta$, with $\delta > 0$, is represented by $\Delta W_i = W_i(\hat{\boldsymbol{q}}_i, \boldsymbol{\xi}_{-i}) - W_i(\xi_i, \boldsymbol{\xi}_{-i})$. On the other hand, the profit change in the population is represented by $\Delta \sum_{i=1}^N W_i = \sum_{i=1}^N \big(W_i(\hat{q}_i, \boldsymbol{\xi}_{-i}) - W_i(\xi_i, \boldsymbol{\xi}_{-i}) \big)$. Since the Nash equilibrium is not efficient in the sense of Pareto, we have

$$\Delta \sum_{i=1}^N W_i > \Delta W_i$$

for all any i in of the population. In order to align the users welfare function with the population objective function, we consider incentives of the form:

$$I_i(\boldsymbol{q}) = \sum_{h \neq i} q_h \left(h_i(\boldsymbol{q}_{-i}) - p\Big(\sum_{j=1}^N q_j \Big) \right), \tag{3.1}$$

where $h_i(\boldsymbol{q}_{-i})$ is a design term that do not have strategic implications in the mechanism, but has some relevance with respect to the properties of the mechanism. The incentives modify the marginal price is charged to each user. These incentives are calculated in function of the relative consumption of each user with respect to the population consumption profile. The form of this incentive is related to the price used in the Vickrey-Clarke-Groves mechanism [7] and some potential games utility functions [23]. These incentives lead to welfare functions of the form

$$\hat{W}_i(q_i, \boldsymbol{q}_{-i}) = v_i(q_i) - \left(q_i^t p\Big(\sum_{j=1}^{N} q_j^t \Big) - I_i(\boldsymbol{q}) \right). \tag{3.2}$$

The Nash equilibrium of this system is the vector $\boldsymbol{\omega}$ that satisfies the first order condition

$$\frac{\partial}{\partial q_i^t} \hat{W}_i(\boldsymbol{\omega}) = \frac{\partial}{\partial q_i^t} v_i(\omega_i) - p\Big(\sum_{j=1}^{N} \omega_j^t \Big) - \sum_{h=1}^{N} \omega_h \frac{\partial}{\partial q_i^t} p\Big(\sum_{j=1}^{N} \omega_j^t \Big) = 0. \tag{3.3}$$

Since Eq. (2.6) and (2.8) are equivalent, the Nash equilibrium $\boldsymbol{\omega}$ of the game with incentives specified in Eq. (3.2) is equal to the competitive equilibrium $\boldsymbol{\mu}$. Therefore, the Nash equilibrium $\boldsymbol{\omega}$ is efficient in the sense of Pareto. The equilibrium of the game with incentives is independent of the term $h_i(\boldsymbol{q}_{-i})$, since $h_i(\boldsymbol{q}_{-i})$ is independent of q_i. However, form of $h_i(\boldsymbol{q}_{-i})$ influences the welfare of the population. For example, $h_i(\boldsymbol{q}_{-i}) = 0$ lead to a unrealistic pricing mechanism in which each user is charged with the total generation cost $C(\sum_{j=1}^{N} q_j)$. A more reasonable mechanism is provided by the Clarke pivot rule [7], that calculate incentives with respect to the contribution made by an agent to the society. Based on the Clarke pivot rule we propose $h_i(\boldsymbol{q}_{-i}) = p\big(\sum_{j \neq i}^{N} q_j + f(\boldsymbol{q}_{-i}) \big)$, where $f(\boldsymbol{q}_{-i})$ is a function that represents the alternative behavior of the i^{th} agent. Although the Clarke pivot rule defines $f(\boldsymbol{q}_{-i}) = 0$, we find that the cases of $f(\boldsymbol{q}_{-i}) \neq 0$ provide some interesting properties. In this case, we consider that $f(\boldsymbol{q}_{-i})$ is a linear combination of the vector $\boldsymbol{q}_{-i}$ of the form

$$f(\boldsymbol{q}_{-i}) = \sum_{j \neq i} \alpha_j q_j, \tag{3.4}$$

where $\alpha_i \in \mathbb{R}$ for all $i \in \mathcal{V}$. The next result shows that when we consider an affine marginal price function, there is no function $f(\cdot)$ of the form in Eq. (3.4) such that the mechanism of incentives is budget balanced, that is, the sum of the incentives on the population is equal to zero. Therefore, the mechanism requires subsidies from an external agent.

Theorem 1. *Suppose Assumption 1 is satisfied, also that* $p(z) = \beta z + b$, *where* $z \in \mathbb{R}$, *with* $\beta > 0$ *and* $b > 0$, *and a population of* $N \geq 2$ *agents. Then there not exists a function* $f(\cdot)$ *such that the mechanism satisfies the* budget balance *property.*

Proof. This proof is made by contradiction. First, we assume that there exists a function $f(\cdot)$ such that the mechanism is budget balanced, i.e., $\sum_{i=1}^{N} I_i(\boldsymbol{q}) = 0$. Now we express the incentives in matrix form. On that purpose, we first we define

$$[f(\boldsymbol{q}_{-1}), \dots, f(\boldsymbol{q}_{-N})]^\top = F\boldsymbol{q},$$

as a vector with the i^{th} element equal to $f(\boldsymbol{q}_{-i})$ from Eq. (3.4), where $F = (\boldsymbol{e}\alpha^\top - \mathrm{diag}(\alpha_1, \dots, \alpha_N))$, $\alpha = [\alpha_1, \dots, \alpha_N]^\top$, $\mathrm{diag}(\alpha_1, \dots, \alpha_N)$ is a diagonal matrix, and is $\boldsymbol{e}$ a vector in $\mathbb{R}^N$ with all its components equal to 1. Since $p(\cdot)$ is an affine function, Eq. (3.1) can be expressed as

$$\sum_{i=1}^{N} I_i(\boldsymbol{q}) = \beta \sum_{i=1}^{N} \Big(\sum_{j \neq i}^{N} q_j \Big) \Big(f(\boldsymbol{q}_{-i}) - q_i \Big). \tag{3.5}$$

This can be rewritten in matrix form as

$$\sum_{i=1}^{N} I_i(\boldsymbol{q}) = \beta \boldsymbol{q}^\top \Phi (F\boldsymbol{q} - \boldsymbol{q}), \tag{3.6}$$

where $\Phi = (\boldsymbol{e}\boldsymbol{e}^\top - I)$ is a square matrix of ones with a zero diagonal, and I is the identity matrix in $\mathbb{R}^{N\times N}$.

Now, considering the budget balance condition we have

$$\boldsymbol{q}^\top \Phi F \boldsymbol{q} = \boldsymbol{q}^\top \Phi \boldsymbol{q}. \tag{3.7}$$

This equation is satisfied if either $q_i = 0$ for all $i \in \mathcal{V}$, or if $F = I$. Since none of these conditions is satisfied for all $q_i \in \mathbb{R}$, we conclude that budget balance is impossible for the mechanisms, for all vector $\boldsymbol{q} \in \mathbb{R}^N_+$.

□

Theorem 1 states the impossibility of finding a mechanism in the form of Eq. (3.1) and (3.4) that do not require external influence in form of subsidies or taxes. This result is not unexpected, since the the Myerson-Satterthwaite Theorem states the impossibility of mechanisms with ex post efficiency and without external subsidies [24]. Now, let us state some desirable conditions for the mechanism. First, consider that all users are equivalent for the utility. Therefore, we make the following assumption.

Assumption 2. Incentives for the i^{th} and j^{th} agents are equivalent whether their consumption is the same, i.e., $I_i(\boldsymbol{q}) = I_j(\boldsymbol{q})$ if and only if $q_i = q_j$. Particularly, if $q_i = q_j$ for all $i, j \in \mathcal{V}$, then $I_i(\boldsymbol{q}) = I_j(\boldsymbol{q}) = 0$. On the other hand, a higher power consumption deserves a lower incentive, i.e., $I_i(\boldsymbol{q}) > I_j(\boldsymbol{q})$ if and only if $q_i < q_j$.

We find that the unique function $f(\boldsymbol{q}_{-i})$ that satisfies Assumption 2 corresponds to an average of $\boldsymbol{q}_{-i}$. This result is summarized in th following proposition.

Proposition 2. *Assume a population of $N \geq 2$ agents, incentives of the form in Eq. (2) and (3.4), and that Assumption 2 is satisfied. Then the function $f(\cdot)$ has the form*

$$f(\boldsymbol{q}_{-i}) = \frac{1}{N-1} \sum_{j \neq i} q_j, \tag{3.8}$$

for all $q_j \geq 0$ with $j, i \in \mathcal{V}$.

Proof. Since $p(z) = \beta z + b$, incentives from Eq. (3.1) might be rewritten as

$$I_i(\boldsymbol{q}) = \Big(\sum_{j\neq i}^{N} q_j\Big)\beta\big(f(\boldsymbol{q}_{-i}) - q_i\big) \tag{3.9}$$

Consider a population consumption profile $\boldsymbol{q}$ such that $q_j = q_i = \sigma$, for all $i, j \in \mathcal{V}$. Now we define two population profiles $\hat{\boldsymbol{q}} = q + \delta\boldsymbol{e}_i$ and $\tilde{\boldsymbol{q}} = q + \delta\boldsymbol{e}_j$, where $\boldsymbol{e}_i$ is an N-dimensional vector with the i^{th} entry equal to 1 and 0 otherwise, for all $j, i \in \mathcal{V}$. According to Assumption 2, $I_j(\hat{\boldsymbol{q}}) = I_i(\tilde{\boldsymbol{q}})$. Therefore,

$$\begin{aligned} I_i(\hat{\boldsymbol{q}}) &= \beta\Big((N-1)\sigma\Big)\big(f(\hat{\boldsymbol{q}}_{-i}) - \sigma - \delta\Big) \\ &= I_j(\tilde{\boldsymbol{q}}) = \beta\Big((N-1)\sigma\Big)\big(f(\tilde{\boldsymbol{q}}_{-j}) - \sigma - \delta\Big) \end{aligned} \tag{3.10}$$

Therefore, $f(\hat{\boldsymbol{q}}_{-i}) = f(\tilde{\boldsymbol{q}}_{-j})$. This is equivalent to

$$\sigma\sum_{h\neq i}\alpha_h = \sigma\sum_{h\neq j}\alpha_h.$$

The last expression is satisfied only if

$$\alpha_i = \alpha_j = \alpha,$$

for all $j, i \in \mathcal{V}$.

Now, suppose that all agents have the same consumption profile, i.e., the population consumption profile is $\boldsymbol{q}$, such that $q_j = q_i = \sigma$ for all $j, i \in \mathcal{V}$. According to the Assumption 2, $I_j(\boldsymbol{q}) = I_i(\boldsymbol{q}) = 0$. Therefore $I_i(\boldsymbol{q}) = \Big(\sum_{j\neq i}^{N} q_j\Big)\beta\big(f(\boldsymbol{q}_{-i}) - q_i\Big) = 0$. Since $q_j = q_i = \sigma$, we have

$$\beta\big((N-1)\sigma\big)\big((N-1)\alpha\sigma - \sigma\Big) = 0$$

Thus we have that

$$\alpha = \frac{1}{N-1}$$

$\square$

According to Proposition 2, the unique mechanism that satisfies Assumption 2 is given by

$$I_i(\boldsymbol{q}) = \beta\Big(\sum_{j\neq i} q_j\Big)\Big(\frac{N}{N-1}\sum_{j\neq i} q_j - q_i\Big) \tag{3.11}$$

for all $q_i \geq 0$, $i, j \in \mathcal{V}$. The population incentives can be expressed as

$$\sum_{i=1}^{N} I_i(\boldsymbol{q}) = q^\top A q \tag{3.12}$$

where $A = \beta\Big(\frac{-1}{N-1}ee^\top + \frac{N}{N-1}I\Big)$. From Theorem 1 we conclude that a budget balance mechanism is not possible. Based on the mechanism incentives defined in Eq. (3.11), we know that the sum of incentives of the population is greater than zero. Therefore, the mechanism requires subsidies. The following proposition describe it.

Proposition 3. *Suppose that Assumptions 1 and 2 are satisfied. Given a mechanism of the form in Eq. (3.11), then the incentives required by the population are positive,*

$$\sum_{i=1}^{N} I_i(\boldsymbol{q}) \geq 0$$

for all $q \in \mathbb{R}^N$*,*

Proof. First, consider $q_i^2 + q_j^2 + 2(q_i q_j) = (q_i + q_j)^2 \geq 0$ for all $q_i \in \mathbb{R}_+^T$. Hence, we have that

$$q_i^2 + q_j^2 \geq -2q_i q_j.$$

Now, summing in both sides of the previous equation we obtain

$$\sum_{j}^{N}\sum_{j\neq i}(q_i^2 + q_j^2) \geq \sum_{j}^{N}\sum_{j\neq i} -2q_i q_j,$$

which is equivalent to

$$2(N-1)\sum_{i=1}^{N} q_i^2 \geq \sum_{j}^{N}\sum_{j\neq i} -2q_iq_j.$$

Dividing both sides of the inequality results

$$\sum_{i=1}^{N} q_i^2 \geq \frac{-1}{N-1}\sum_{j}^{N}\sum_{j\neq i} q_iq_j. \tag{3.13}$$

Now, let $A_{i,j} = \frac{-1}{N-1}$ if $i \neq j$ and $A_{i,i} = 1$ for all $i, j \in \mathcal{V}$. Therefore, Eq. (3.12) can be expressed as

$$q^\top Aq = \sum_{j}^{N} q_j \Big(\sum_{i}^{N} q_i A_{i,j}\Big).$$

This can be rewritten as

$$\boldsymbol{q}^\top A\boldsymbol{q} = \frac{-1}{N-1}\sum_{j}^{N} q_j \Big(\sum_{h\neq i} q_h\Big) + \sum_{j}^{N} q_j^2.$$

From Eq. (3.13), it can be seen that $\boldsymbol{q}^\top A\boldsymbol{q} \geq 0$, for all $\boldsymbol{q} \in \mathbb{R}_+^N$. □

This proposition establishes that the mechanism requires the subsides from an external institution. Furthermore, as the population size increases, the total incentives of the population increases, i.e., the subsidies that the mechanism requires increases with the population size. Particularly, we have

$$\lim_{N\to\infty} \boldsymbol{q}^\top A\boldsymbol{q} = \boldsymbol{q}^\top\boldsymbol{q} > \boldsymbol{q}^\top A\boldsymbol{q}$$

.

Chapter 4

Social Incentives

Unlike economic theory predictions, experimental results have shown that the behavior of decision making agents do not follow rationality assumption. This assumption implies that each agent behaves in a selfish way in order to maximize its own revenue. However, cooperation or altruism might arise in presence of *social incentives*, such as rewards and punishments.

On the one hand, Milinski et at. show that social rewards might help to solve social dilemmas such as the tragedy of the commons [10]. In this case, agents play two alternating games, namely a public goods game and an indirect reciprocity game. On the one hand, public goods games are known to be cases of the tragedy of the commons, in which agents tend to be selfish. On the other hand, the reciprocity game offer rewards to the agents with good reputation, that is build by cooperating in the public goods game. Experiments show that the introduction of a reciprocity game encourages people to cooperate in the public goods game.

On the other hand, Fehr and Rockenbach show that the use of punishments with moral legitimacy tend to promote altruistic acts [25]. Here, cooperation is adopted in order to avoid the punishments. Particularly, Krupka and Weber find that agents are concern about the utility reported by their actions as well as the social appropriateness of their actions [26]. However, if the cooperation is not reciprocal, the cooperation decreases over the time [27].

Based in previous results we aim to achieve the demand response objectives by providing social incentives. On this respect, we model decision making processes of populations by means of an opinion dynamics model that has into account environmental incentives. In this case, the environmental incentives are social rewards modeled according to Eq. (2.15). In this Chapter, we show the influence that opinions might have on the consumption preferences of each consumer, as well as some learning dynamics that can be used by each agent to adjust its consumption.

4.1 Opinions Influence on Electricity Consumption

The literature about the implications of an opinion over the consumption preferences of a good is not extensively treated in the literature. An approach that connects the electricity consumption of an agent with its opinion is presented in [9]. This approach assumes that agents tend to modify its valuation function $v_i^t(\cdot)$ (see Eq. (2.9)) according to their opinions. In this case, we assume that the utility encourages flatten demand profiles. Hence, agents who believe in cooperation will attempt to flatten their demand profiles. This behavior is modeled as a tendency to equate the consumption preferences along the day as x_i tends to 1. On the other hand, agents who are reluctant to cooperating with the utility, might hold their preferences unchanged. The aforementioned adjustment of preferences is captured by means of the following valuation function

$$\hat{v}_i^t(q_i^t, x_i) = \left(\alpha_i^t(1 - x_i) + \bar{\alpha}_i x_i\right) \log(1 + q_i^t), \tag{4.1}$$

where q_i^t is the consumption of the i^{th} agent at the t^{th} time interval and $\bar{\alpha}_i = \sum_{\tau=1}^{T} \alpha_i^\tau$. Hence, if $x_i = 0$, then

$$\hat{v}_i^t(q_i^t, x_i = 0) = \alpha_i^t \log(1 + q_i^t) = v_i^t(q_i^t).$$

This reflects the unwillingness of the i^{th} agent to cooperate. On the other hand, if $x_i = 1$, the disposition for cooperating of the i^{th} agent is reflected in a change in the preferences

$$\hat{v}_i^t(q_i^t, x_i = 1) = \bar{\alpha}_i \log(1 + q_i^t).$$

Remark 4. *Although there is a change in the preferences, that change do not implies a social optimal equilibrium. That is, the proposed social incentives are not able to encourage an optimal consumption by themselves.*

However, changes in the valuation of the electricity do not imply instantaneous changes in the electricity consumption. It is assumed that each agent carries out a learning process by which a consumption adjustment is made as preferences changes. This learning process is introduced before.

4.2 Consumption Learning Dynamics

The learning process carried out by each consumer is viewed as a method to solve a resource allocation problem. From the perspective of population games [15, 28], the resource to be allocated is the daily power consumption, the strategy set is composed by T time intervals, and fitness functions are defined as the derivative of the profit function defined in Eq. (2.4), i.e., the fitness of a population without incentives is defined as

$$f_i^t(\boldsymbol{x}, \boldsymbol{q}^t) = \frac{\partial U_i^t(\boldsymbol{q}^t)}{\partial q_i^t} = \frac{\Big(\alpha_i^t(1 - x_i) + \bar{\alpha}_i x_i\Big)}{1 + q_i^t} - \beta\Big(\sum_{j=1}^{N} q_j^t + q_i^t\Big). \tag{4.2}$$

On the other hand, when incentives have been introduced, the fitness function takes the following form

$$f_i^t(\boldsymbol{x}, \boldsymbol{q}^t) = \frac{\partial (U_i^t(\boldsymbol{q}^t) + I_i^t(\boldsymbol{q}^t))}{\partial q_i^t} = \frac{\Big(\alpha_i^t(1 - x_i) + \bar{\alpha}_i x_i\Big)}{1 + q_i^t} - 2\beta\Big(\sum_{j=1}^{N} q_j^t\Big). \tag{4.3}$$

Previous considerations imply that the learning process is made faster than the updates in the opinion of an agent. Here, the learning process is approximated by means of Smith dynamics, applied to resource allocation problem. Particularly, Smith dynamics satisfies the conditions of positive correlation (PC) and noncomplacency (NC), that are desirable for stability and convergence purposes [15]. Smith dynamics are described by the differential equation

$$\dot{q}_i^t = \sum_{\tau=1}^{T} q_i^{\tau} [f_i^t(\boldsymbol{x}, \boldsymbol{q}^t) - f_i^{\tau}(\boldsymbol{x}, \boldsymbol{q}^{\tau})]_+ \\ - q_i^t \sum_{\tau=1}^{T} [f_i^{\tau}(\boldsymbol{x}, \boldsymbol{q}^{\tau}) - f_i^t(\boldsymbol{x}, \boldsymbol{q}^t)]_+. \quad (4.4)$$

The resource allocation achieved by means of the Smith dynamics is the solution to the optimization problem described in Eq. (2.7).

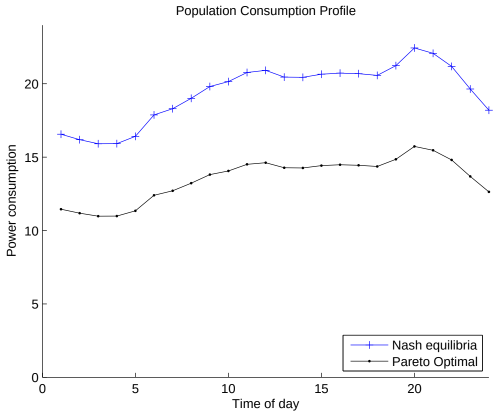
Chapter 5

Simulations

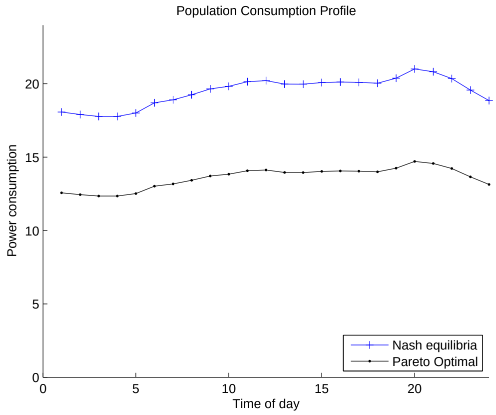
In this Chapter, we illustrate the aforementioned ideas concerning economical and social incentives. First, let us exemplify the content of Chapter 4 by considering an homogeneous population of agents, i.e., a population of agents that satisfies $\alpha_i^t = \alpha_j^t$ for all $i, j \in \mathcal{V}$ and $t \in \{1, \ldots, T\}$ (see Eq. (2.9)). An homogeneous population is characterized because all its members have the same demand profile at the equilibrium. Now, consider that all the agents have the same opinion, i.e., $x_i = x_j$ for all $i, j \in \mathcal{V}$. The demand profile at the equilibrium of our population is shown in Fig. 5-1. In this case, we consider the equilibrium with different opinions of the population. It can be seen that as the opinion of the agents approaches 1, the demand profile tends to be flatten. However, a flatten demand profile do not implies efficiency.

Note that the Nash equilibrium depicted in Fig. 5-1 is obtained by means of the Smith dynamics described in Chapter 4.2 together with the fitness function defined in Eq. (4.2), i.e., a case in which we do not consider incentives. On the other hand, the Pareto optimal outcomes shown in Fig. 5-1 are obtained when the Smith Dynamics are combined with the defined in Eq. (4.3), i.e., a scenario in which we introduce incentives.

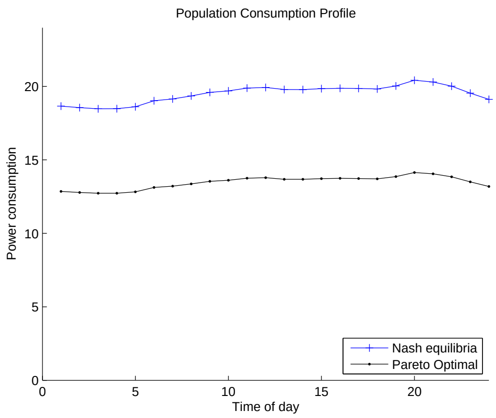
Now, let us analyze the ideas introduced in Chapter 3. Particularly, we are interested in the dynamical behavior of the economic incentives. In this case we consider both homogeneous and heterogeneous populations. In order to the analyze the response of the population to the economic incentives, we introduce incentives in the



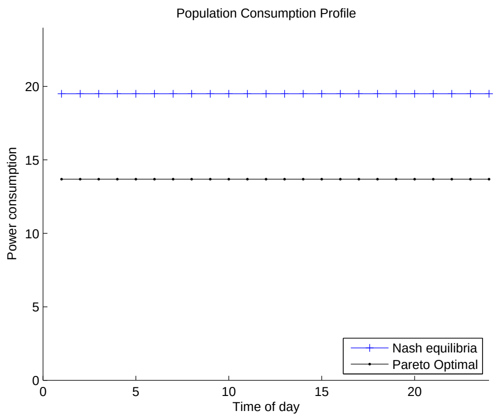
(a) $x_i = 0$.



(b) $x_i = 0.5$.



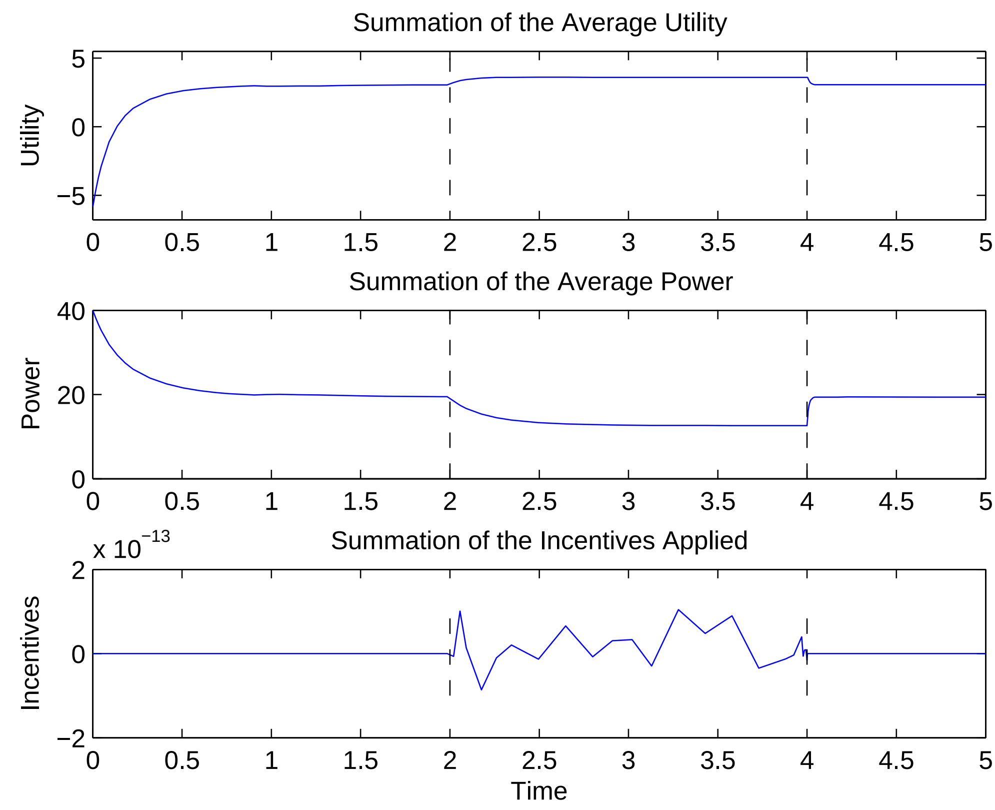
(c) $x_i = 0.7$.



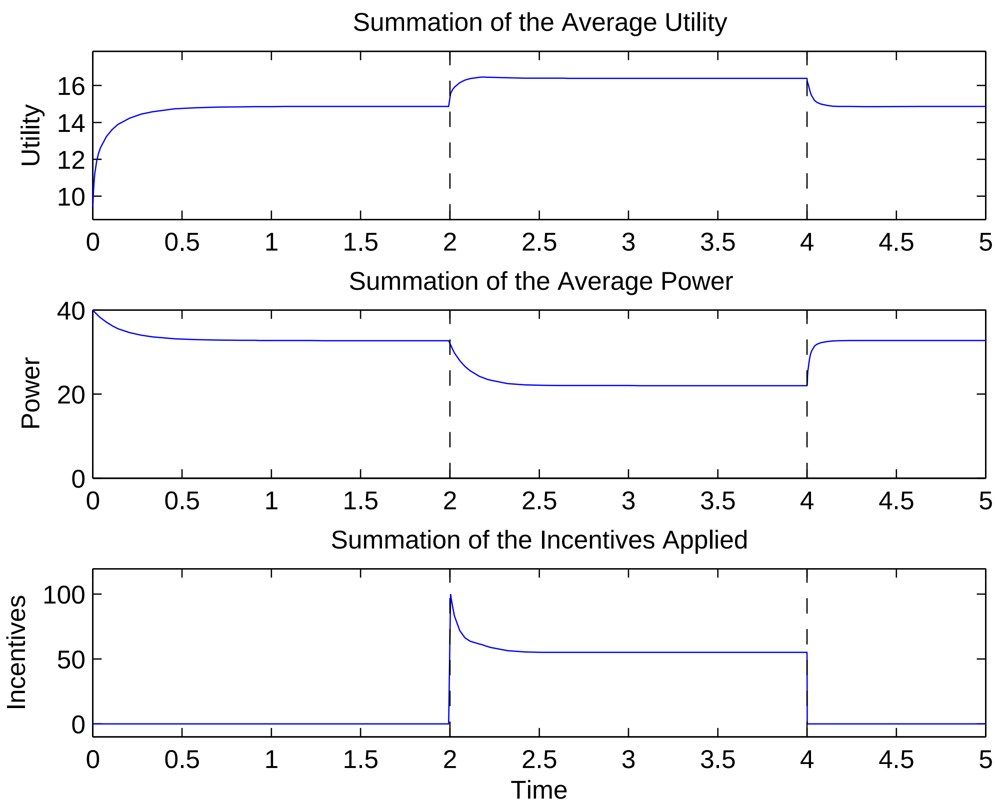
(d) $x_i = 1$.

Figure 5-1: Demand profile of an agent with different opinions.

time period comprehended between 2 and 4 second (see Fig. 5-2). We can observe that the introduction of the incentives produces an increment in population average utility, as well as a reduction in the average consumption. Also, note that the total incentives delivered to the homogeneous population is close to zero. Specifically, this happens because the following conditions are satisfied: 1) the population is homogeneous; 2) agents' initial conditions are identical; and 3) their consumption updates are synchronized. These conditions imply that the consumption adjustment of the agents are identical at each time instant. Hence, incentives property **ii.** guarantee that the total amount of incentives is null. In contrast, the heterogeneous population requires incentives different from zero, even in the equilibrium.



(a) Dynamics of a homogeneous population.



(b) Dynamics of a heterogeneous population.

Figure 5-2: Dynamics of two populations in the presence of incentives.

Chapter 6

Conclusions and Future Directions

In this paper, we prove the inefficiency (in the sense of Pareto) in the electricity system. We also demonstrate that the electricity system resemblances with the tragedy of the commons. Furthermore, we show that unlike the SG goals, the efficient consumption of a population (in the sense of Pareto) is not characterized by a flat demand. In order to achieve a efficient flat demand, we propose the use of two incentives, namely economic and social incentives. The economic incentives lead to an efficient outcome when each agent optimizes its own electricity consumption in a distributed fashion. Particularly, each agent optimizes its consumption based on information about the population consumption. Nevertheless, security failures related with the information that the smart meters report might lead to failures of the mechanism. The analysis of security problems is out of the scope of the present work, although would be an interesting research direction. A drawback of this mechanism is the requirement of external subsidies for its implementation. However, the model used does not consider the cost associated to the required infrastructure to handle peaks. The inclusion of such costs in the market model would justify the inversion required by the incentives mechanism. Future work will be focused on the analysis of the transition dynamics between inefficient and efficient outcomes.

On the other hand, a flat demand is promoted through social incentives. We consider that social interactions might influence the formation of preferences. Simulation results show that the preferences of a population might be influenced by means of prominent agents. Particularly, the proposed model of opinion dynamics allows the emergence of consensus, polarization or fragmentation in the population opinions when the social incentives are considered as a coordination game. The theoretical conclusions extracted from the proposed model are worth to be analyzed in practical experimentation in future works.

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