A location-allocation model under uncertainty applied to disasters

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Abstract
Preparedness can reduce the impact of a disaster by strategically locating facilities and quantities of various types of emergency based on emergency risk maps. We model a general location-allocation problem in disaster management as a mixed-integer problem that considers uncertainty in demand. To solve the model, we use two-stage stochastic programming with risk constraints to minimize the total expected cost and limit the downside risks. We illustrate the performance of our methodology and apply it to a case study in the city of Bogotá (Colombia).

Keywords: Disaster management, Stochastic program, Expected downside risk, Facility location, Inventory planning, Resource allocation

1. Introduction

The Center for Research on the Epidemiology of Disasters (CRED), defines a disaster as an event which overwhelms local capacity, requiring external help at the national or international level; an unforeseen and often sudden event that causes great damage, destruction, and human suffering [4]. These disasters can be categorized in two groups: natural and technological.

Natural disasters have traditionally been regarded as unavoidable and to a large extent beyond human control. Historical data found in [4] shows that in 2011 natural disasters worldwide killed a total of 30,773 people, caused 244.7 million victims, and the financial losses were the highest ever registered with an estimated US$ 366.1 billion. Some of the most recent natural disasters include the hurricane Katrina on August 23, 2005; one of the deadliest hurricanes in the United States, in which at least 1100 people died [13] and its monetary loss was estimated over 100 billion USD [15]. Another example is the Haiti earthquake on January 12, 2010 which is the second deadliest earthquake in the history with over 222,570 deceased [6].

Technological disasters are the result of man-made products failure and are typically accidental. These can be categorized as transport system accidents (large-scale road, air and maritime accidents), collapse of constructions, large fires, and technological and toxic accidents (nuclear power plant accidents, leakage of hazardous substances) [22]. In many urban areas of the world the limited space to build causes residential zones to be in proximity of industrial facilities, and hence being exposed to high risk of technological disasters. Examples around the world are extensive, one of the most remembered is the Chernobyl nuclear disaster on April 26, 1986 [23].

To reduce the severity of damage caused by all types of disasters, communities get prepared by stockpiling inventory and building adequate structures. Each of these decisions and actions could make a community more resilient to natural disasters, if done properly [19, 10]. Nevertheless, the decision-making process to support emergency response is not an easy process and differs greatly from business logistics, because it involves a high level in uncertainty of the number of people affected [2, 21].

Our study focuses on the location of capacitated response facilities and the allocation of the relief supplies in order to improve the effectiveness of the post-disaster relief operations in response to sudden disasters with stochastic demand. To this end, we model the problem as a mixed-integer programming problem and use Stochastic Programming (SP) to model the uncertainty in the demand via a finite set of scenarios. In order to limit the risk of having high allocation costs we incorporate a downside risk constraint to this model.

The rest of this paper is organized as follows. In Section 2 we explain how SP can support Disaster Operation Management (DOM). In Section 3, we show the proposed two-stage SP model, provide the notation, and present the problem formulation. Then in Section 4 we present the description and the results of a case study on the city of Bogotá. Finally, Section 5 concludes the paper and gives suggestions for future work.
2. Literature review

We review the literature of location-allocation models applied to emergency supplies and first discuss Linear and Stochastic Programming. Then we show how risk measures are incorporated by different authors to these models.

DOM is mainly divided into four phases: mitigation, preparedness, response, and recovery. Our formulation involves more than one phase, for this reason we use a two-stage structure. The first stage is related to the strategic location of response facilities (preparedness), and the second stage refers to the allocation of resources once the disaster strikes (response). The location-allocation problems can be solved assuming deterministic post-disaster demands, for instance in Iakovou et al. [12]. However this can be considered an unrealistic assumption, making a growing body of literature devoted to use tools such as SP to deal with uncertainty [9].

A considerable number of two-stage SP problems are proposed for disaster management. Balcik and Beamon [1] develop a variant of the maximal covering location model to determine the number and locations of distribution centers in a relief network, and the amount of multiple relief supplies to be stocked at each distribution center. They create scenarios for disaster demand based on historical information and maximize the total expected demand covered. On the other hand Mete and Zabinsky [14] minimize the total expected costs and a use a penalty cost for the unsatisfied demand. They also determine a detailed transportation plan using a predefined set of routes instead of using a vehicle routing problem. Rawls and Turnquist [17] include the transportation network connectivity and number of pre-stocked units available following a disaster. They minimize the total expected cost and show a heuristic solution algorithm to solve very large instance problems. In contrast with the previously mentioned models, Döyenko et al. [5] consider facility location decisions before and after a disaster happens. This kind of formulation involves a high number of binary decisions variables, for this reason they use a heuristic method to solve the problem.

The models mentioned above minimize the total expected cost or maximize the expected coverage. These are reasonable objectives when decision makers are indifferent to risk, because the expected values do not measure risk. These risk-neutral models may perform poorly when there are scenarios with extreme demands. Therefore, for high variable demands a risk-averse approach that considers these effects would provide more robust solutions [16]. Examples of risk averse models are few in DOM. Two extensions of the model proposed by Rawls and Turnquist [17] are proposed in Rawls and Turnquist [18] and by Noyan [16]. In the first extension, they introduce a lower bound on the probability that the network demand is satisfied and the resulting average total shipment distance is less than a specified threshold. However they use one binary variable per each scenario increasing their number and making the problem harder. Noyan [16] incorporates in the objective function a conditional value at risk (CVaR) to reduce the total expected cost and the risk of having high transportation costs. To solve the problem they used a methodology called L-Shaped [20] with a multi cut strategy. Finally, Hong et al. [11] focus on the probability of failing to satisfy the demand using chance constraints.

As an alternative to these models, we introduce a formulation done by Eppen et al. [7]. This formulation excludes the risk measure from the objective function and uses continuous variables instead of binary variables. Using this formulation brings an advantage with large instances and when the risk measure does not coincide with the units of objective function. To the best of our knowledge, this model has not been applied to DOM problems yet.

3. Mathematical Formulation

In this section we present a two-stage SP model to determine which of a set of pre-specified response facilities need to be operating in order to minimize the total expected cost. Also we decide an inventory level at each facility and the allocation of this inventory to the affected zones after the occurrence of a disaster. The optimal policy resulting from our SP model is a set of chosen of facilities and their inventory level from the first stage and a collection of response decisions for each possible realization of the demand.

In the first stage we determine which of a set of pre-specified response facilities need to be operating and decide how much to store in each. The model formulation of this stage includes a set of candidate locations for the response facilities (\(J\)) that are distributed around the city. Also we define three parameters for each facility \(j\). We denote the capacity by \(s_j\), the unit cost by \(c_j\) and the fixed cost of an operating facility by \(f_j\). Also for each facility there is a binary decision variable \(y_j\) that takes the value 1, if a facility \(j\) is selected to be operating and 0 otherwise. We include a variable \(z_j\) which indicates the inventory level of items to be stored in facility \(j\).

In the second stage, we determine the amount of items to be shipped to an affected zone. In the aftermath of the disaster, there will be affected zones that are included in the model as set \((I)\). We define a cost \(c_{ij}\) per unit shipped to emergency zone \(i\) from facility \(j\). Uncertainty is modeled through the use of a set \((K)\) of discrete scenarios, each
with a probability of occurrence of $p_k$. In each scenario $k$ there are a forecasted demand $d_{ik}$ for each affected zone $i$. For each of these scenarios $k$ we define a variable $x_{ijk}$ which indicates the amount of items shipped to emergency zone $i$ from facility $j$ in scenario $k$. The proposed two-stage, risk-neutral mixed-integer program follows:

\[
\begin{align*}
\min & \quad \sum_{i \in I} \sum_{j \in J} (f_{ij} y_j + h_{ij} z_j) + \sum_{k \in K} p_k \sum_{j \in J} \sum_{i \in I} c_{ijk} x_{ijk} \\
\text{subject to,} & \quad \sum_{j \in J} x_{ijk} = d_{ik} \quad \forall i \in I, k \in K \quad (1) \\
& \quad z_j \leq s_j y_j \quad \forall j \in J \quad (2) \\
& \quad \sum_{i \in I} x_{ijk} \leq z_j \quad \forall j \in J, k \in K \quad (3) \\
& \quad y_j \in \{0, 1\}, z_j \geq 0 \quad \forall j \in J \quad (4) \\
& \quad x_{ijk} \geq 0 \quad \forall i \in I, j \in J, k \in K \quad (5) \\
& \quad x_{ijk} \geq 0 \quad \forall i \in I, j \in J, k \in K \quad (6)
\end{align*}
\]

The objective function (1) includes the total cost of operating the response facilities, the total cost of the inventory, and the expected allocation cost. Constraints (2) guarantee that all the demand is covered by response facilities. Constraints (3) ensure that the selected inventory level does not exceed the capacity of the facility. Constraints (4) state that the amount of units shipped from a facility cannot be over the quantity stored there. Finally, (5) and (6) ensure the non-negativity of the decision variables.

The model defined in (1)-(6) corresponds to a risk-neutral model that is concerned with the minimization of the total expected cost. The solution of this model deals with the strategic decisions of the first stage which are taken once. Then emergency demands are known and allocation decisions are taken.

One way to measure risk can be defined as the probability that the allocation cost is higher than a certain threshold $C_{\max}$. Therefore a binary variable $\gamma_k$ such that it is equal to 1 if allocation cost in scenario $k$ is greater that $C_{\max}$, otherwise equal to 0. This can mathematically be expressed as follows where $M$ is a sufficient large positive parameter.

\[
\begin{align*}
\sum_{j \in J} \sum_{i \in I} c_{ijk} x_{ijk} & \leq C_{\max} + M \gamma_k \quad \forall k \in K \quad (7) \\
\sum_{j \in J} \sum_{i \in I} c_{ijk} x_{ijk} & \geq C_{\max} - M(1 - \gamma_k) \quad \forall k \in K \quad (8)
\end{align*}
\]

In the previous risk constraints a binary variable is required for each scenario $k$. Thus, the model complexity will be very large as the number of scenarios increases. To avoid these binary variables, we can use an alternative to measure risk introduced by Eppen et al. [7]. The main idea is to identify the amount of the total allocation cost is over $C_{\max}$ for each scenario $k$. Therefore, when the total allocation cost is greater than the limit $C_{\max}$ a positive continuous decision variable $\beta_k$ as the difference between these two values, and 0 otherwise. This can mathematically be expressed as follows.

\[
\beta_k \geq \sum_{j \in J} \sum_{i \in I} c_{ijk} x_{ijk} - C_{\max} \quad \forall k \in K \quad (9)
\]

Using the previous constraints to replace (7) and (8) reduce the number of constraints in $k$ and eliminates the binary variables, however $\beta_k$ make the problem unbounded. Therefore, we can include these decision variables in the objective function (1) or limit the expected value of these variables to be at most a parameter $B$. We select the second option because it is difficult to define the weight of risk in the objective function, for this reason we add the next constraint.

\[
\sum_{k \in K} p_k \beta_k \leq B \quad (10)
\]

To change the model from risk-neutral to risk-averse we add the constraints (9) and (10). The parameters $C_{\max}$ and $B$ defined in these constraints limit the allocation costs. When $C_{\max}$ is near zero the probability of having allocation costs greater the limit increase, therefore more $\beta_k$ variables will be different than zero. The expected value of these variables is limited by the parameter $B$, and as a consequence the relation between these two parameter force the risk-averse model to find a suitable combination of first stage decision to fulfil the requirements of the
constraints (9) and (10). These requirements can be accomplished by opening new facilities because when more facilities are opened the allocation cost is reduced. Indeed the number of facilities opened also has a direct impact on the variability of the allocation cost. For these reason the risk-averse solution will have good performance in extreme scenarios. To demonstrate how adding these constraints effects the optimal decisions, we consider the following case study and show the results from both risk-neutral and risk-averse models.

## 4. Case Study - City of Bogotá

In this section we implement the proposed SP model to a case study and choose facilities to respond to technological emergencies in the City of Bogotá, minimizing the total expected cost. Bogotá is the capital and most populated city in Colombia with more than 8 million residents according to the 2005 national census [3]. Bogotá’s development plan does not contain adequate planning and management of disaster response. This causes a problem because in many parts of the city residential and industrial zones are adjacent or share territory. Figure 2 shows the distribution of the population in the city, divided into its 19 districts.

FOPAE (The Prevention and Attention of Emergencies Fund) is a governmental entity dedicated to developing emergency plans such as Emergency Prevention and Attention Plan for the City of Bogotá to respond to different kinds of disasters [8]. For this reason, FOPAE has identified a set of nine feasible response facilities to locate in different parts of the city to respond to the potential technological emergencies. In Figure 1 is shown these feasible facilities and the potential technological emergencies. The allocation cost between facilities and all the emergencies were calculated as the Euclidean distance between them. The demand of each emergency is not equal because the population of the city is not equally distributed, as shown in Figure 2. To include uncertainty in the demand we estimate the demand according to the district in which the emergency happened, then we use these values as the mean of a probability function to generate the set of 50 scenarios. To be available to use any probability distribution we restrict the domain of the distribution to be any positive value, when the value is negative we add this probability to the probability of having zero demand. Also using different probability density function let us to simulate any type of circumstances from daily to large-scale emergencies.

![Figure 1: Feasible warehouse and technological emergencies locations.](image1.png)

![Figure 2: Distribution of population.](image2.png)

The optimal solution of the risk-neutral model is shown in Figure 4. This solution suggests to select four facilities from the nine possible locations available and is able to cover all the demand while minimizing the total expected cost. In order to illustrate how a risk-averse optimal solution differs from the previous solution we made...
an example setting the value of $C_{max}$ to $1600$. This value let us calculate the expected value of variables $\beta_k$ of the risk-neutral solution, which for this example is $2.98$. Then to find another solution different form the risk-neutral solution we set the value of $B$ to be less than the calculated expected value. The risk-averse solution is shown in Figure 3, where the optimal solution is to locate one extra facility in the farthest north location, since it is the best option to reduce the allocation cost.

Figure 3: Risk-averse solution.

Figure 4: Risk-neutral solution.
In addition the histograms of the total costs of risk-neutral and risk-averse solutions are shown in Figure 5. In this figure it is clear that the risk-neutral model’s total cost for all scenarios is less than the risk-averse model. However, the standard deviation of the allocation cost of the risk-neutral solution is greater than the risk-averse model, due to the trade-off between the costs and risk. The gap between the expected values of total cost shows how changes in the location decisions have great impact on the total costs.

![Histogram of total costs](image)

**Figure 5: Total cost histogram**

5. Conclusion and Future work

In this section we have proposed a SP model to improve the preparation and response phases in DOM, through identifying the optimal location and storage of facilities in order to respond to technological emergencies. The proposed model incorporates a set of expected downside risk constraints, which help to reduce the risk of having high allocation cost in the response phase. This methodology was first proposed by Eppen et al. [7] in a general context, however we applied it to DOM as an alternative to other recent works that consider risk.

To illustrate how this model can be applied in a practical context, we show a case study in the city of Bogotá. The case study shows the difference when the expected downside risk constraints are added to the model. In the base case or risk-neutral approach, only four facilities were opened. The risk-averse model requires more facilities to be operating and more widely dispersed in the network to reduce possible outcome of the scenarios.

Further work needs to be done in order to analyse the effects of various sources of uncertainty, to better understand their different impacts. Also, a further side by side comparison between our model and other risk-averse models would help to set clearly the advantages and disadvantages in DOM. Finally, a bi-objective model where total costs and risk are optimized simultaneously can be an alternative to the solve the problem.

References