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"Uncertainty Shocks in Colombia: Characterization and Effects"

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UNCERTAINTY SHOCKS IN COLOMBIA: CHARACTERIZATION AND EFFECTS

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Abstract. This research document proposes a measure of the level of uncertainty for Colombia's economy. With it, we calibrate a modified version of a DSGE model made for a small, open economy dependent on natural resources. Modifications include a second moments volatility shock to intermediate firm productivity, replicating an unexpected exogenous uncertainty shock. The second moments shock was solved using a third degree Taylor approximation, allowing for proper impulse response analysis. Findings show that an unexpected increase in local uncertainty, reduces local consumption, prices and worked hours. Inversely, wages, mid-term foreign consumption and the real exchange rate increase.

Este documento de investigación propone una nueva medida de incertidumbre para la economía colombiana. Con ésta, se calibra una versión modificada de un modelo dinámico estocástico de equilibrio general, desarrollado para una economía pequeña, abierta, con dependencia de recursos naturales. Las modificaciones realizadas incluyen un choque de volatilidad de segundos momentos a la función de producción de las firmas intermedias, buscando replicar un choque de incertidumbre exógeno. Este choque fue solucionado usando una aproximación de Taylor de tercer grado, lo cual permite hacer un análisis de impulso respuesta. Los resultados muestran que un aumento exógeno en la incertidumbre local reduce el consumo local, el nivel de precios y el número de horas trabajadas. De otro lado, los salarios aumenta junto con el consumo de bienes extranjeros y la tasa de cambio real.
JEL Classification codes/Códigos JEL: C32, C61, E32, F41

"Uncertainty is worse than knowing the truth, no matter how bad" (The Magazine of Wall Street, November 30, 1929, p. 177)

1. Introduction

Looking for a proper definition of uncertainty, one will almost always find the very short and bitter: *the state of being uncertain.* Some other times the not so bitter but even shorter *lack of certainty* will pop up. In any case, although there isn't a proper dictionary definition of uncertainty, it is a latent part of our economic and social systems, and so it should be studied.

This document's main objective is to try and answer the following question: do local uncertainty shocks have an effect on the behavior of macroeconomic variables in Colombia? The following steps are taken. First, a measurement of the uncertainty perceived by agents in Colombia needs to be built. Then, using structural vector auto regressions, a series of exogenous unexpected innovations on our uncertainty measure is captured, so that it can be later introduced as a shock in a dynamic stochastic general equilibrium model. This will allow for impulse response analysis and the study of the policy implications arising from these shocks.

In the same way Basu and Bundick (2012); Uribe and Yue (2006); Bloom et al. (2012b) do it, the hypothesis that unexpected increases in uncertainty levels replicate business cycle movements will be tested by looking at the effects on key macroeconomic variables, implying changes in agents' behavior. On this subject, Bloom et al. (2012b), show that periods of increased uncertainty are countercyclical, but the authors are unable to determine the causality of these events. According to the author, under a general equilibrium context virtually every macroeconomic variable is endogenous.

Similar to previous literature on the subject, motivation for this paper lies in the 2007-2008 financial crisis, as it became a starting point for many authors who tried to understand the crisis, and how to deal with future ones. Most of them (Basu and Bundick (2012); Bloom et al. (2012b); Bloom (2009); Guerrón et al. (2011); Uribe and Yue (2006) to name a few) came to the consensus that increased levels of volatility in financial markets were present at the time. While studying the effects of uncertainty, some of them found that changes in uncertainty levels would have aggregate consequences. This type of study has not yet been done for Colombia.

Different kinds of recoveries that came afterwards in different countries reflected the way economies react to different kinds of stimuli. But the interaction between countries was clearly one of the channels through which the crisis came about in the first place[1]. This interaction is something that needs to be taken into consideration when studying uncertainty since, in a way, it reflects the complexity of the world economy.

There seems to be a link between high levels of uncertainty and the fact that the economy stumbled into a recession. There is something else around uncertainty that interests economists other than providing insights into how agents will behave during the next financial meltdown. If, as Bloom (2009) and others show, a big enough shock in uncertainty will impact important macroeconomic variables, then it should be considered when implementing monetary and fiscal policy.

1.1. **Some Background on Uncertainty.** Before 2007, the literature on uncertainty studied the 1930's Great Depression and uncertainty in general. After 2007, several authors looked for commonalities between the financial crisis and what happened almost 80 years ago. Uncertainty shocks (unexpected exogenous innovations to an uncertainty measure) however haven't seen much development except for the last couple of years. Most studies have been focused on uncertainty in the U.S. economy (Bloom et al. (2012b); Bloom (2009); von Furstenberg (1988); Simsek (2010); Henry and Olekalns (2000)), although some efforts for other countries have also been published (Guerrón et al. (2011); Uribe and Yue (2006) and García-Cicco (2010); García-Cicco and Cruces (2012)) .

In "The Impact of Uncertainty Shocks", Bloom studies the effects that major political and economic events have on the U.S. economy. To do this, he uses a model that simulates a macro uncertainty shock, replicating a rapid drop and subsequent rebound in several aggregate macroeconomics variables. In the medium term, the volatility generated by the shock induces an overshoot effect on output, employment and productivity, generated by an economy with low temporary productivity and high volatility of business conditions, which activates the hiring threshold companies have resulting in labor falling back to its long-run path. Similar behaviors can be described for productivity and aggregate output.

[1]For a deeper understanding of financial contagion, see Allen and Gale (2000); Lagunoff and Schreft (2001); Kaminsky and Reinhart (2000).

Some results vary if the model assumes that firms have convex adjustment costs with similar conclusions.

In a subsequent study, Bloom et al. (2012b) propose that uncertainty shocks can drive business cycles. A properly calibrated dynamic stochastic general equilibrium model can explain drops and rebounds in GDP of around 3%. Two steps are considered: in the first, the authors develop new empirical measures of uncertainty using detailed Census micro-data for the period 1972-2010. Then Findings show that plant-level uncertainty shocks are highly countercyclical, that industry-level shocks are also countercyclical, but, that this industry level increase in variance during recession periods does not appear to be due to the slowdown itself. Lastly, large plant Total Factor Productivity shocks are highly correlated with more volatile daily parent stock returns. That is, "a popular high-frequency financial method of measuring uncertainty, is tightly linked to yearly plant TFP shocks, our low frequency real measure of uncertainty", (Bloom et al., 2012b)

With these results they simulate uncertainty shocks, using the mentioned DSGE model, and find that these shocks reduce hiring, investment and output. These shocks also reduce productivity growth because they reduce the degree of reallocation in the economy. This result, also relevant at the microeconomic scale, was partially shown by Foster et al. (1998, 2002).

Basu and Bundick (2012) do a similar analysis and arrive at a similar conclusion: after an uncertainty shock, there is a decrease in output and several other macroeconomic variables. The way they show this is by using a one-sector, representative agent dynamic stochastic general-equilibrium model.

Two different sets of assumptions are being used in their model, an economy with flexible prices and one without. In the model using flexible prices, an uncertainty shock induces precautionary savings and lower consumption. This same uncertainty shock generates something the authors call precautionary labor supply, which, all else constant (including current technology and capital stock), competitive demand for labor remains unchanged, resulting in an increase in output, investment and hours worked, but a decrease in consumption.

Using sticky prices, the same reduction in consumption is present. And so is the precautionary labor supply, but since prices remain stable, the marginal cost of production is reduced, which in turn represents an increase in firm markups over marginal cost. A higher markup reduces demand for investment goods, and since output is demand-determined, output and employment must also fall.

Shocks used by Bloom et al. (2012b); Bloom (2009); Basu and Bundick (2012) come from different sources (depending on the way they are being measured). Stock and Watson (2012) study these shocks and their variance decomposition. Through a dynamic factor model with 200 variables they study the events in the 2007-2009 recession, and arrive at three main conclusions: First, they find that although most of the results of the 2007-2009 recession were unprecedented, most of the macro shocks experienced by the economy were larger-than-regular shocks, to which the economy responded in a historically predictable way.

Secondly, oil shocks played a role in the initial slowdown, but the shocks that caused the recession were associated with liquidity/financial disruption and heightened uncertainty; these last two were present together since the high correlation between them makes difficult their separate interpretations. Lastly, the slow recovery after the events in 2007-2009 can also be explained as resulting from a long-term slowdown in trend unemployment growth, which translate into slower long run GDP growth.

As far as studying the effects that local uncertainty has in other countries, Guerrón et al. (2011) look at the variance of the borrowing interest rate, i.e. the difference between the country's interest rate and the standing U.S. Treasury rate[2]. Their ma in result is that even when the country's interest rate is kept stable, an increase in the country spread persistently reduces consumption, investment (through seven quarters), and output (sixteen quarters). Labor increases at first but it subsequently falls because of the decrease in

[2]Done for Argentina, Brazil, Ecuador and Perú

investment. Also debt is reduced, as it is costlier to pay because of the rising higher spread.

The literature on the effects of uncertainty on macro aggregates has highlighted important consequences for the U.S. economy, mostly during and after the financial meltdown of 2007-2009. At a microeconomic scale, uncertainty produces a slowdown in consumption, output and productivity. Regarding the literature that focuses on other countries, Guerrón et al. (2011) (and in some way Uribe and Yue (2006)) show that not only are smaller economies subject to changes in their own uncertainty, but also that changes in the global uncertainty also affect them. This paper contributes to the related literature by modeling both a local and an external uncertainty shock on a open small economy such as Colombia's. Local shocks have a larger effect on local variables (consumption, wages and labor), while external shocks affect trade related variables, as well as having an impact on the real exchange rate.

The rest of the document is organized as follows. Section 2 motivates the discussion around uncertainty, builds the uncertainty index for Colombia's economy, and extracts the exogenous shock. Section 3 describes the original model and the modifications made to reflect Colombia's economy, as well as the solution methods. Section 4 presents the main results and some possible limitations. Section 5 concludes.

2. Uncertainty

An uncertainty shock is an unexpected exogenous innovation on a certain uncertainty measure. Although they are few and far between, uncertainty shocks have a certain persistence over time. In other words, an unexpected increase in uncertainty takes several periods to diminish to the point just before the shock. And since it is unexpected, households and firms are not able to plan ahead, but only react; in this sense, policy measures should be taken to reduce the effects uncertainty has once it has reached the economy.

2.1. **The expected effects of uncertainty.** Building on the related literature, it is important to understand how the economy perceives uncertainty, and how uncertainty shocks are amplified. The literature has centered on three different issues: firms, consumers and asset prices.

Regarding firms, non-linearities in their objective function are the main source of uncertainty. If a firm's objective function F is strictly linear, when maximizing $V(\mathrm{E}[F_{t+1}])$, the present value of the expected future production, one could use the means of the variables as there is no interaction between inputs, so uncertainty about the future will not matter. Bloom et al. (2012a) states that the sources for these non-linearities come from the production function, factor-output adjustment costs, risk aversion and the cost of external finance.

Uncertainty affects the way agents behave in preparation for unforeseen future events, i.e. inputs are adjusted accordingly. Investment is one of the firm's inputs that can be modified and have inter-temporal consequences, given a certain structure of adjustment costs. The effects that uncertainty have on investment are well documented, both at the microeconomic scale, and at the macroeconomic one. Micro literature revolves around Leahy and Whited (1996)'s basic approach, which is to compare firm investment with the volatility of the firm's share price. They found some reverse causality, as lower investment increases the uncertainty of future cash flows; Bloom et al. (2007) later showed that, using lagged variables as a proxy, increased levels of uncertainty decrease investment in the following periods.

Inspired by Romer (1990), who analyzed consumption after the Great Depression, agents are regarded as risk-averse. She found that an increase in uncertainty discourages the acquisitions of durable goods, since their consumption is irreversible.

Lastly, uncertainty affects asset prices through their pricing mechanisms in two ways, a long run trend, and fluctuating economic uncertainty (Bansal and Yaron, 2004). This uncertainty, understood and studied as volatility, distracts markets from high equity prices and small price-dividend ratios, increasing the economy's averse sentiment to it. Linked to consumption, the authors attribute a large part of share price volatility to consumption volatility, which reduces the expected future price of a stock, which makes it more likely to be acquired by individuals in their "shopping rants".

2.2. **Uncertainty and Volatility.** Following previous work done by Bloom et al. (2012b) and several others (Bloom (2009); Uribe and Yue (2006); Guerrón et al. (2011); van Robays

(2012); Drèze (1999); Simsek (2010)), the purpose of this section is to explain why volatility in certain markets and variables can be used as a measure of uncertainty. Since it is intrinsically unobservable, subjectively perceived by agents, uncertainty is hard to measure, but as it will be argued, volatility is a proxy used throughout the literature.

One measure of volatility that has been proposed is industry uncertainty, which is linked to the macroeconomic uncertainty of GDP through intermediate goods markets (Basu and Bundick, 2012; Bloom, 2009, see). In this sense, Chen, 2012 in Bloom et al. (2012a) (p. 4) shows that "the dispersion of cross-industry stock indices rises in recessions". Sectors within the economy react negatively when the future does not look clear. This same behavior can be seen at an industry level, firm and further down to plant level data (Bloom et al., 2012b). In line with this argument, Berger and Vavra (2011) show that intermediate goods prices present increased levels of volatility in times of crisis. Storesletten et al. (2004) provides evidence that labor markets exhibit high levels of uncertainty contemporaneously with recessions; both perceived incomes and nominal wages exhibited increased dispersion during the 2001 *dotcom* crisis.

These measurements are made using United States data, a textbook large open economy. However, Engle et al. (2008) and Ramey and Ramey (1995) show that besides being subject to local levels of uncertainty, emerging economies are also subject to global volatility. On the one hand, Engle et al. (2008) develop a model that forecasts volatility in financial markets using economic data (something that was not very common at the time), concluding that periods of lower growth, high price levels or high interest-rate variability, increase his volatility measure. As expected using time series, the author does not claim causal implications, but rather a significant correlation between variables. On the other hand, Ramey and Ramey (1995) use volatility in growth rates for 92 countries, and find that higher volatility reduces growth in the long run, and that emerging markets are subject to higher volatility than more developed economies.

2.2.1. *Using Volatility.* This section presents an overview on methods for using volatility as a measure for uncertainty. The main difference between uncertainty and volatility is that uncertainty captures the expectations agents have about the future (it is a forward looking concept), while volatility is a realized measure of how agents act over these expectations (Bloom et al. (2012a)). Consider the following example: the following is a simple brownian motion process,

$$dX_t = \mu + \sigma_{t-1} dW_t,$$

where $dW_t \sim N(0,1)$. The uncertainty of a future moment in time, i.e. dX_{t+1}, is represented by σ_t, and its volatility is the variance of the last s periods, $\mathrm{var}(dX_{t-1}, \cdots, dX_{t-s})$. If this uncertainty persists over time, a good measure of how dX_t changes in the future, is $\mathrm{E}[\mathrm{var}(dX_{t+1})] = \mathrm{var}(X_t) = \sigma_t$. If there is no persistence, a measure of the uncertainty of future events is the implied volatility, which in financial markets is understood as the expected volatility that a certain process will have. *A small clarification:* The research developed here shows that uncertainty is persistent over some periods of time, which validates the first approach in measuring it.

2.3. **Vector Auto Regression.** As presented in Kilian (2011), structural auto-regressions have four main applications in studying macroeconomic models through time series. These are:

- To study the average response of the model to a given true structural shock,
- To allow for the construction or forecast error variance decompositions that quantify the average contribution of a given structural shock to the variability of the data,
- To provide historical decompositions that measure the cumulative contribution of each structural shock to the evolution of each variable over time, and
- To allow for the construction of forecast scenarios conditional on hypothetical sequences of future structural shocks.

One of the most commonly used structural vector auto-regressions in the literature uses identification through short-run restrictions. These restrictions (Christiano et al. (2005), and several others Galí (1999); Galí and Monacelli (2005); Galí (2008)), as opposed to long-run restrictions, restrict movements in certain variables in the DSGE model, so deep parameters can be established and the shocks can be properly identified Collard et al.. Such models, with their respective solutions, can be represented using the following linear approximation:

$$Ax_{t+1} = Bx_t + C\nu_{t+1} + D\eta_{t+1},$$

where each of A, B, C or D is a function of the structural parameters of the model, μ. A Solution to this equation can be expressed as

$$x_{t+1} = F(\mu)x_t + G(\mu)\nu_{t+1}.$$

A simpler model expression, the one used in this document, is $Ax_{t+1} = Bx_t + \nu_t$, where the solution that we seek is expressed as

$$x_{t+1} = Fx_t + \xi_{t+1}.$$

This means that the behavior of the time series x_{t+1} can be parameterized in terms of a lag, and of ν_{t+1}, a vector of exogenous innovations, or as commonly known in the literature, *structural shocks*. The error term ν_t, captures the innovations, and is theoretically built so that $E(\nu_t\nu_t')\Sigma_\nu = \mathrm{I}_K$, implying there are as many structural shocks as there are variables in the system; by definition they are mutually uncorrelated and their variance is normalized to unity. This doesn't imply a loss of generality, since the elements of A remain unrestricted. A possible solution for A is to multiply the basic form of the system by A^{-1}, so that now $\xi_{t+1} = A^{-1}\nu_t$. Studying the variance of ξ_t gives the following results:

$$E(\xi_t\xi_t') = A^{-1}E(\nu_t\nu_t')A^{-1'}$$

$$\Sigma_\xi = A^{-1}\Sigma_\nu A^{-1'}$$

$$\Sigma_\xi = A^{-1}A^{-1'}(\text{since } \Sigma_\nu = \mathrm{I}_K)$$

Now, Σ_ξ can be thought of a system of nonlinear equations in the unknown parameter A^{-1}. Fortunately, it can be consistently estimated using numerical methods such as the Cholesky decomposition for variance-covariance matrices. Performing decomposition through recursive methods, a matrix P such that $PP' = \Sigma_\xi$ is obtained. It is important that this matrix P is economically coherent[3].

2.4. **Uncertainty Shocks.** The approach used to measure uncertainty in this document relies on the volatility of financial markets. The steps to achieve this measure are briefly

[3] *"The distinguishing feature of "orthogonalization" by Cholesky decomposition is that the resulting structural model is recursive ([i.e.] conditional on lagged variables). This means that we impose a particular causal chain rather than learning about casual relationships from the data. In essence, we solve the problem of which structural shocks causes the variation in ξ_t by imposing a particular solution. This mechanical solution does not make economic sense, however, without a plausible economic interpretation for the recursive ordering."* Kilian (2011)

the following: Colombia's uncertainty is proxied with the Índice General de la Bolsa de Colombia - IGBC[4]'s monthly volatility. Afterwards, an external uncertainty measure is built, using as proxy the Standard & Poor's 500 monthly volatility. These two uncertainty measures are then analyzed using a vector auto regression and their respective shocks are estimated. **?** uses the Chicago Board Options Exchange Market Volatility Index to measure uncertainty rather than volatility, however, using the results from the previous section, volatility of the IGBC will be used instead. Also, since Colombia's financial derivates market is underdeveloped, there is no real VIX equivalent available.

2.4.1. *On the empirical strategy.* Regarding the methodology to be used, there are certain documents that try to do so for a different set of variables. Kilian (2009) uses a structural vector autoregression to identify the shocks that move the price of oil. A similar approach is used by Romer and Romer (2004), where the authors develop a measure of monetary policy shocks, relatively free of endogenous and anticipatory movements. Several other papers, like Mertens and Ravn (2010) deal with finding shocks in key macroeconomic variables. The resulting time series provide a fresh way of measuring a variable that could not otherwise be measured. Also, Uribe and Yue (2006), determine the effect that changes in the economy's interest rate have on aggregate activity in emerging economies.
The proposed VAR is as follows:

$$A_0 z_t = \alpha + \sum_i A_i z_{t-i} + \epsilon_t$$

where

$$z_t = \begin{pmatrix} \nu_t \\ \mu_t \end{pmatrix}$$

and ν_t represents the local uncertainty measure, μ_t represents the external uncertainty measure, and ϵ_t stores the exogenous shocks to each of these variables.

There are very good ideas for measuring shocks, and the effects they have on the economy. There are also a several ways of determining the real and true effect a policy or program has on a certain variable, free of interactions and causalities. The link between these two concepts further motivates this document, and provides tools for determining

[4]Colombia's main stock market index

the true effect of local uncertainty on macroeconomic variables.

2.5. **Results.** Figure 1 shows the normalized behavior of both the IGBC (Figure 1a) and the Standard & Poor's 500 (Figure 1b) starting on 2001-06-03 all the way to 2013-04-05. As we can see, the IGBC has shown important growth (of around 1600%) since the beginning of the series, because it was created in 2001, merging the indices for Bogotá, Medellín and Cali. On the other hand, the S&P 500 oscillates between 60 and 130 units as if it had achieved certain stability 55 years after conception.

Measuring uncertainty equates to calculating monthly standard deviations of daily returns for both indices. These measurements are presented in Figure 2. An important event occurred in 2006 in Colombia that sharply affected the uncertainty, and that increased level of uncertainty persisted for a few months. This event only affected Colombia, as the uncertainty measure for the United States wasn't affected. There are also a higher number of increases in uncertainty for Colombia, which would provide evidence for Engle et al. and Ramey and Ramey's hypothesis, that emerging economies are prone to uncertainty due to economical instability. In 2008 there is a sharp increase in uncertainty, higher for United States than for Colombia, showing the moment the financial crisis kicked in.

Though these measurements of uncertainty are certainly interesting, one more step to accomplish this document's first objective is needed: to characterize the exogenous, unexpected uncertainty shocks for the Colombian economy. The series of innovations are presented in Figure 3; it can be seen that the shocks are different for both economies. Colombia has four important shocks: 2004Q1, 2006Q2, 2008Q1 and 2008Q3. 2004Q1 was almost as big as 2008Q1, but the effect it had on uncertainty was smaller in magnitude, which could imply that were structurally different. 2006Q2 is the highest of the four, and as shown in Figure 3a, is seen as a local shock (as well as 2004Q1 and 2008Q1).
For ease of exposition, Figure 4 and Figure 5 show local and external uncertainty with their respective shocks, so that they can be better understood[5].

[5]The dates are the same for both graphs. The same applies for the next figure.

This first result appears somewhat enlightening, as it shows that there are shocks that affect local uncertainty that do not appear to have an effect on external uncertainty. This result is relevant for the Dynamical Stochastic General Equilibrium model which replicates an uncertainty shock for a small, open economy, endowed with a high level of natural resources (oil). An important part of the analysis is understanding how the economy reacts and what are some policy measures that can be taken.

3. Dynamic Stochastic General Equilibrium

Dynamic stochastic general equilibrium models became theoretic tools for economist to study policy implications at a macroeconomic scale. They started using real data as background information to properly calibrate the models, so that results would reflect the real behavior of the economy. The main purpose of this section is to modify an existing DSGE to include exogenous local and external uncertainty shocks and estimate their effect on a small open economy, so impulse response analysis can be made, as well as a base for policy implications analysis. In the previous chapter, conditions for using structural vector auto regression analysis on DSGE calibration were exposed. This section is then divided into two parts. The model to be used, a DSGE model for a small open economy (following Medina and Soto (2005)), and the changes made to it (following Basu and Bundick (2012)) by introducing uncertainty shocks.

3.1. **Small Open Economy General Equilibrium Model.** The model used to represent a small open economy with a large endowment of exploitable natural resources (oil) is Medina and Soto (2005)'s [6] DSGE model in "Oil Shocks and Monetary Policy in an Estimated DSGE Model for a Small Open Economy". The author's main objective is to analyze the impact of an oil-price shock using a dynamic stochastic general equilibrium estimated with bayesian methods. This section briefly explains their model:

3.1.1. *Some clarifications.* Wage and price rigidities are introduced not only because they increase the fit of the model but because they generate a stronger trade-off between inflation

[6] Galí and Monacelli (2005) may be a viable alternative as well.

and output fluctuations. Domestic households consume domestically-produced goods, imported differentiated goods, and fuel. Home goods are partly sold domestically and partly exported abroad. Households supply a differentiated labor service and receive the corresponding wage compensations. Furthermore, households are the owners of firms producing home goods, and therefore, they receive the income corresponding to the monopolistic rents generated by these firms.

3.1.2. *Households.* Households maximize the present value of their utility:

$$U_t = E_0 \left\{ \sum_{i=0}^{\infty} \beta^i \left[\log\left(C_t(j) - h(1+g_y)C_t\right) - \frac{\zeta_t}{1+\sigma_L} l_t(j)^{1+\sigma_L} \right] \right\} \tag{1}$$

where

- $l_t(j)$ is labor effort
- $C_t(j)$ is total consumption
- σ_L is the inverse elasticity of labor supply with respect to real wages.
- ζ_t preference shock that shifts labor supply
- h is a habit formation constant ($h = h(1+g_y)$), where h corresponds to the habit formation parameter in an economy without steady state growth.

Consumption is split between oil ($O_{C,t}(j)$) and core goods ($Z_t(j)$), which are in turn split into local ($C_{H,t}(j)$) and foreign goods ($C_{F,t}(j)$). Demand for these goods is represented by the fraction of the aggregate demand multiplied by their respective price elasticity.

Households are firm owners, and they receive income as wages according to $W_t(j)$; they are able to update and optimize this wage every period with probability $(1-\phi)$, following Calvo (1983). The aggregate wage index is defined as:

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_L} dj \right)^{\frac{1}{1-\epsilon_L}} \tag{2}$$

Aggregate labor is determined by

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\epsilon_L - 1}{\epsilon_L}} dj \right)^{\frac{\epsilon_L}{\epsilon_L - 1}} \tag{3}$$

3.1.3. *Domestic Production and Exports.* Local production is done by household owned firms acting as monopolies in the production of a single variety of goods. In the same way as households, each firm chooses a price $P_t(j)$ that maximizes its profit according to the available technology:

$$Y_{H,t}(Z_H) = A_{H,t}\left[(1-\alpha)^{\frac{1}{\omega}}(L_{H,t}(z_H))^{1-\frac{1}{\omega}} + \alpha^{\frac{1}{\omega}}(O_{H,t}(z_H))^{1-\frac{1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \quad (4)$$

Where $L_{H,t}$ is the labor input utilized and $O_{H,t}$ is the amount of oil used.

Price setting. Following (Calvo, 1983), it is assumed that only a fraction ϕ_θ of producers are able to set their prices each period. Those who are not able to adjust prices correctly, maximize their earnings subject to the demand for the variety they produce.

Foreign sector. Suppose the economy exports two kinds of goods, local production, and natural resource endowments which is not consumed domestically. Since domestic firms cannot price discriminate across markets, the law of one price holds for local goods sold externally. (Medina and Soto, 2005) assume that the real exchange rate is built using the price of foreign goods, which supposes that the percentage of local goods on foreign markets (through the domestic consumption basket) is small.

3.1.4. *Aggregate equilibrium.* The equilibrium in the home goods market and labor market is represented by

$$Y_{H,t} = C_{H,t} + C^*_{H,t}, \quad (5)$$

where $C^*_{H,t}$ is foreign demand for local goods.
GDP, is expressed as

$$\frac{P_{Y,t}}{P_t}Y_t = C_t + \frac{P_{X,t}}{P_t}X_t - \frac{P_{M,t}}{P_t}M_t \quad (6)$$

i.e., total production equals home consumption plus foreign consumption of local goods minus local consumption of foreign goods.

3.2. **DSGE Modifications.** The original model by (Medina and Soto, 2005) presented in the previous section does a great job at representing an open economy with natural resources. However, arguing that firms use oil as a production factor (hence their focus on oil shocks), capital was left out of the model. These are the main modifications made to the original model. Also, the steady state parameters were calibrated so that they could reflect the behavior of Colombia's economy.

3.2.1. *Capital.* Capital, as presented in the neoclassical context, is one of the three productive factors together with labour and land. It was left out of the base model in (Medina and Soto, 2005), and was replaced by oil. In contrast with this set up, in Leduc and Sill (2010)'s approach oil is a function of the capital utilization rate for intermediate firms. Looking to capture both types of behaviors, this document includes both factors of production, jointly, oil and capital. As a separate production factor, capital provides a new tool for understanding household behavior: since households own capital they will invest more or less of it in their intermediate firms according to the level of uncertainty in the economy.((Sbordone et al., 2010; Fernández-Villaverde and Rubio-Ramírez, 2010).

Acquiring capital is a 'safe' way for households to react to uncertain times. They are able to expand capital, be it for accumulation (high interest rates will increase capital returns) or compensating a reduction in other production factors - production highly depends on prices, which in turn depend on costs, which in turn depends on the elasticity of production factors, and capital is relatively less expensive to increase (Christiano et al., 2010). This another reason to include capital in general equilibrium models: when production factors are increased with small marginal costs, inflation will not rise as fast after a monetary policy shock[7].

However, as capital is linked to investment, there are some considerations when increasing capital: adjustment costs, which the literature regards as way of making decisions more accurate since they reflect irreversible investments. The way of including these adjustment costs in the capital accumulation equation is standard[8]:

$$K_{t+1} = \left(1 - S\frac{I_t}{I_{t+1}} - 1\right)^2 + (1-\delta)K_t,$$

S is such that $S = S' = 0$ and $S'' > 0$. This equation states that the amount of capital available in the following period equals what is left after depreciation plus the amount needed to keep up with the investment rate. (Christiano et al., 2010) consider this a useful way of describing convex adjustment costs, since using production either for consumption or for acquiring capital goods, involves a learning by doing process. This process is expensive

[7]The curvature of the capital utilization function in the payoff for capital determines the cost of acquiring additional capital. If curvature is small, households will acquire capital at little cost.

[8]Following Christiano et al. (2010), following Christiano et al. (2005)

to firms, especially at low capital goods accumulation. As an example, the authors mention that housing market models without convex adjustment costs as proposed above are not precise since they do not reflect the slow dynamics of housing construction.

3.2.2. *Uncertainty shock.* The structural vector auto regression of section 2 suggests the existence of an exogenous shock with some short run effects on the level of macroeconomic uncertainty. We would like to determine the dynamic effects of these shocks on the economy through our model. Understanding how these shocks came to be is outside of the scope of this paper, however, a quick search provides some insight on why the shock occurred: There were high expectations that the Federal Reserve would increase interest rates after the European Central Bank increased interest rates 25 percentage points on its last meeting. This triggered the uncertainty from local (and several other large markets such as Frankfurt, Paris and London) investors and traders, sending the IGBC's levels to record lows (see Figure 3a). Although this unexpected decrease in Colombia's main index was due to an expectation on external agents, there was not a similar behavior in large United States markets, hinting that the shock was produced by local uncertainty; performing exogenous shock analysis using Europe's large stock markets on intraday behavior yields no results as well.

Medina and Soto's model also includes exogenous shocks as a policy analysis tool, like most DSGEs do. Following Basu and Bundick and Bloom et al.'s research on uncertainty, an exogenous shock on productivity is included in the model, assuming that an unexpected increase in uncertainty will affect local intermediate firm behavior first and then spread to households[9]. Decisions and reactions will then be considered to take place in this order. Authors also include a preference shock affecting household consumption (negatively), but it will not be used in this model as there was no data to accurately calibrate the model. Further research could focus on Fedesarrollo's Consumer Confidence Index. The equations used to describe the uncertainty shock are:

$$\ln(Z_{t+1}) = \rho_z \ln(Z_t) + \sigma_t^z \epsilon_t^z, \quad \epsilon_t^z \sim N(0,1)$$
$$\ln(\sigma_{t+1}^z) = (1-\rho_{\sigma_z})\ln(\sigma^z) + \rho_{\sigma^z}\ln(\sigma_t^z) + \sigma^{\sigma^z}\epsilon_t^{\sigma^z}, \quad \epsilon_t^{\sigma^z} \sim N(0,1)$$

[9]For more information on the development of the Real Business Cycle model Basu and Bundick use, see Ireland (2011, 2003).

These equations seek to capture the effects of independent changes in the level and volatility on the uncertainty shock process. ϵ_t^z is a first moment shock capturing the innovations to the level of the stochastic process, and $\epsilon_t^{\sigma^z}$ is considered the uncertainty shock (second moment), capturing the volatility of the exogenous process of the model. Increasing the volatility of the shock increases the uncertainty of the future time path of the stochastic process. These shocks are independent random variables.

3.3. **Solving the model: Perturbation methods - a small caveat.** Given the proposed setup, our main interest is to understand the effects of an unexpected increase in uncertainty, that is, an increase in the volatility shock. In this case, finding the equilibrium of our model through the standard first order and log-linear approximations will not yield the desired results, as the policy functions are not affected by the volatility of the equations that describe the shock (Basu and Bundick, 2012). Using a second order approximation, second moments relate to the other second-order approximation to the policy functions of other state variables as cross-products, so the effects would not be direct. Third order approximations are then needed. These allow impulse response analysis of an uncertainty shock holding the levels of state variables constant.

Basu and Bundick (2012) propose a third order approximation to solve the problem, which according to Guerrón et al. (2011), is sufficient to capture the dynamics of the basic model. They also argue that higher than first-order approximations shift the ergodic distributions of the endogenous variables away from their deterministic steady-state values. A small caveat: results presented in this document are expressed as deviations from the variable's ergodic mean[10].

3.4. **Model calibration.** Since the original model was developed for the Chilean economy, there were some (further) modifications, regarding the calibration of parameters that needed to be made. The parameters for the steady state of C_t, n_t and $\frac{s_t p_t^*}{p^c}$ were calibrated,

[10]The ergodic mean of a process is deduced from a single, sufficiently long sample (realization) of the process. See also (Fernández-Villaverde and Rubio-Ramírez, 2006)

using the calibration for Colombia's PATACON Model (González et al., 2011)[11]. The following values were used:

The rest of the steady state values were calibrated using the equilibrium equations.

C_t	0.68
n_t	0.3
$\frac{s_t p_t^*}{p^c}$	1.19

Regarding the shares of the different levels of consumption and relative participations, the values used are presented in Table 2.

4. Results

In this section we analyze the results of an exogenous increase in uncertainty at firm productivity and an uncertainty shock in the international exchange rate, and their effects through the rest of the model's variables. Both shocks are included to maintain coherence between the developments of Section 2. The main purpose of this section is to analyze the reaction of the economy to the shocks.

Section 2 provided evidence of the existence of uncertainty shocks, both local and external, that existed independent of each other. This allows for an individual analysis of the local uncertainty shock, which affects productivity, and an external uncertainty shock, that affects the external interest rate equation.

4.1. **Local Uncertainty Shock.**

An exogenous local uncertainty shock of one standard deviation from the mean, represented by $z_{u,t}$, is presented in Figure 6, and the reaction in Figures 7 through 10. The variables that I focus on are presented in Table 1. The vertical axis in the graphs represent distance, in percentage points, from the standard deviation of the steady state.

[11] `http://www.eafit.edu.co/escuelas/economiayfinanzas/departamento-economia/Documents/Presentaci\unhbox\voidb@x\bgroup\let\unhbox\voidb@x\setbox\@tempboxa\hbox{o\global\mathchardef\accent@spacefactor\spacefactor}\accent19o\egroup\spacefactor\accent@spacefactorn1.pdf`

Section 2 provided some intuition on how should the economy react; the results from the model appear to be consistent with it. Uncertainty enters the model through the intermediate goods production function, and behaves as a technology shock (Basu and Bundick, 2012). Households reduce consumption, perceiving the increase in uncertainty in the economy. Foreign goods consumption increases, but only because their share in the consumption basket is significantly less than domestic consumption - the uncertainty shock affects the production of locally produced goods, so the rise in foreign consumption is perceived as higher than it would have been otherwise.

After the shock, firms exhibit a more precautionary behavior, demanding less work, and increasing the real wage. This increases the marginal cost of production in intermediate firms which forces the firms to increase domestic product prices. These effects combine to produce a fall in total output, consumption, hours worked and product mark-up (regarding the mark-up, some periods after the shock, final product prices increase, increasing mark-up momentarily before falling back to the steady state level). Also, household marginal utility is reduced as a consequence of lower levels of consumption and hours worked.

In response to the shock, the real interest rate falls, there is a higher price distortion (although small) This ultimately results in a reduction of traditional exports, their price and real exchange rate.

Notice that although the shock is introduced at the production function for intermediate firms, consumptions falls almost instantly. A possible explanation might be: when firm productivity is affected, and investments are likely to remain unchanged, the easiest production factor to modify is labor. This explains the decrease in the number of worked hours. Less hours means less salary, which means less expending money. As exposed before, households perceive this uncertainty and reduce consumption locally. This hypothesis was empirically considered by toggling the parameter that represents household sensibility to labor income. Higher sensibility yields a larger dip in consumption.

4.2. **External Uncertainty.** An exogenous external uncertainty shock of one standard deviation from the mean,$z_{i^*_t}$ represented by the volatility affecting the external interest rate, i_t^*, is presented in Figure 11, and the reaction of the economy in figures 12 through 15. Just as before, the variables that compose the analysis are presented in Table 1.

The intuition on how this particular shock should affect the economy, is stated in Guerrón et al. (2011), and to some extent in Section 2. The results from here appear to be consistent with this intuition. Uncertainty ($z_{i_t^*}$) enters the model through the external interest rate.

Consumption is reduced in the first periods, although it sees a relative increase 6 periods into the shock. Showing the inverse relation as in the local uncertainty shock, local consumption increases and foreign decreases, the latter reaching 0 after 4 periods (explaining the lump shape of general consumption). Since the external interest rate increases, borrowing money from external sources becomes more expensive, local banks see an increase in their savings, even if their interest rate remains unchanged. Also, oil consumption decreases, reflecting the increase in oil based goods price, $\frac{p_t^o}{p_t^c}$.

Another consequence of this higher interest rate, is an increase in the real exchange rate. This is a result of the higher lending rate outside the economy, which increases demand for foreign currency. This in turn increases exports as a way of acquiring other currencies. This pushes local companies to increase production, increasing the number of hours worked and decreasing wages, both optimal and real.

4.3. **Some potential issues.** One potential criticism of the model used in this document is that it lacks a realistic financial sector, which according to some of the literature that came after the financial meltdown, was important to understand the crisis. The only way the financial system enters in the model is through the uncertainty measure. A refutal to this criticism, is that although the crisis might start at the financial system, it is merged with every sector in the economy. This means that it has an various channels through which it might impact the economy; Studying these channels is important and one of the main goals of this paper.

The actual measure of uncertainty may also be an issue. Due to the relatively shallow financial market in Colombia, and the fact that the companies that compose the leading index (IGBC) belong to a couple of investment groups may bias the actual reaction to events that increase uncertainty. Another issue is the speed at which companies are able to react to this events. Increased uncertainty could be measured as changes in the price of a stock on a small time window (the approach I take), but this could only reflect how traders reacted to news and not reflect structural changes in the company. On the other hand, looking only at changes in a certain stock due to structural changes in the company may be even more biased, because they reflect management decisions and not reactions to uncertainty. Given this, the first approach was considered for this document, rather than the second one.

5. Conclusions

This document provides a measure of uncertainty for the Colombian economy using the volatility in financial markets. Then, using structural vector auto regressions, a series of local, exogenous, unexpected, uncertainty shocks are recovered, free from interactions coming from external measures of uncertainty. This series of innovations is then used as an input for shocks in a dynamic stochastic general equilibrium model adapted for a small, open economy like Colombia. Besides local uncertainty shocks, an external uncertainty shock is analyzed.

The behavior of the model's variable show that an unexpected rise in local macroeconomic uncertainty, decreases domestic consumption, output, the number of hours worked, prices and inflation, firm mark-up, although only in the first periods after the shock. It increases real, oil consumption and mark-up in the late periods after the shock.

The effect of an external uncertainty shock can be summarized as one local consumption, the real exchange rate, production, number of hours worked and exports. In turn, oil consumption, foreign goods price and consumption and marginal costs are reduced.

This result indicates that after an uncertainty shock, the Colombian economy behaves similarly as to going through an economic cycle, with some important differences in firm behavior depending on the origin of the shock. This is a somewhat standard result in the literature surrounding uncertainty shocks, however this is the first time it has been exposed for a small open economy with a high endowment of a natural resource such as oil (although

García-Cicco et al. (2013) have made some advances in this sense). Further research should be focused on developing more sound and robust measurements of uncertainty, like the ones in Jurado et al. (2013), and including financial frictions in the model, although this could obscure the mechanisms through which uncertainty affects the economy.

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6. Annex

6.1. **Perturbation methods.** The concept of using perturbation methods for solving non-linear economic models started with Judd (2003), seeking a way of applying numerical methods to complex economical problems. The basic idea is that most complicated problems have particular cases that are easy to solve, and this particular (and easier solution!) can be used as building block for the general solution. Complex sets of equations are written in terms of a small perturbation parameter, problem is solved for this perturbation parameter and the solution is used to approximate the solution of the original problem. Let's consider a simple example: searching for the root of the cubic equation

$$x^3 - 4.1x + 0.2 = 0$$

such that $x < 0$. The steps to find a solution are

(1) Small perturbation: $x^3 - (4+\epsilon)x + 2\epsilon = 0$, where $\epsilon \equiv 0.1$
(2) Index the solution in terms of the parameter x:

$$g(\epsilon)^3 - (4+\epsilon)g(\epsilon) + 2\epsilon = 0$$

$\epsilon = 0$ is an easy solution: $x^3 - 4x = 0$, $x = -2, 0, 2$. Since the problem asked for $x < 0$, $g(0) = -2$ should be our starting point.

(3) Building an approximate solution using Taylor's Theorem.

Using Taylor's theorem, $x = g(\epsilon)|_{\epsilon=0} = g(0) + \sum_{n=1}^{\infty} \frac{g^n(0)}{n!}\epsilon^n$. Substituting the solution into the problem and recovering the coefficients for $n = 0, 1, \cdots$ should take us closer to the solution.

- **0-order approximation.** This is just taking $\epsilon = 0$, so $g(0) = -2$. Substituting into the equation gives

$$x^3 - 4.1x + 0.2 \Rightarrow -8 + 8.2 + 0.2 = 0.4$$

which is relatively close to 0.

- **1-order approximation.** The first derivative with respect to ϵ gives:

$$3g(\epsilon)^2 g'(0) - g(\epsilon) - (4+\epsilon)g'(0) + 2 = 0$$

Setting $\epsilon = 0$ and knowing that $g(0) = -2$, one arrives at $8g'(0) + 4 = 0$, so $g'(0) = -\frac{1}{2}$. By Taylor's Theorem,

$$x = g(\epsilon)|_{\epsilon=0} \simeq g(0) + \frac{g^1(0)}{1!}\epsilon^1 = -2 - \frac{1}{2}\epsilon$$

In our example $\epsilon \equiv 0.1$, so x=-2.05. Substituting this solution into the equation gives

$$x^3 - 4.1x + 0.2 \Rightarrow -8615125 + 8.405 + 0.2 = -0.010125$$

which is even closer to 0, but not quite there yet.

- **2-order approximation.** The second derivative with respect to ϵ gives:

$$6g(\epsilon)\big(g'(\epsilon)\big)^2 + 3g(\epsilon)^2 g''(0) - 2g'(0) - 4g''(0) = 0$$

Setting $\epsilon = 0$ and knowing that $g(0) = -2$ and $g'(0) = -\frac{1}{2}$, one arrives at $8g''(0) - 2 = 0$, so $g''(0) = \frac{1}{4}$. Using Taylor's Theorem, $x \simeq -2 - \frac{1}{2}\epsilon + \frac{1}{8}\epsilon = -2.05875$. Substituting this solution into the equation gives

$$x^3 - 4.1x + 0.2 \Rightarrow -8.59937523242188 + 8.399875 + 0.2 = 4.997675781240329x10^{-4}$$

A margin of error that poses as acceptable. The numerical solution to the problem is $x = -2.04880884817015$.

Onto a more complex example, a basic Real Business Cycle problem:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \{\log(c_t)\}$$

subject to:

$$c_t + k_{t+1} = e^{z_t} k_t^{\alpha} + (1 - \delta)k_t$$
$$z_t = \rho z_{t-1} + \sigma\epsilon_t, \quad \epsilon_t^z \sim N(0, 1)$$

Solving this problem is basically searching for policy functions for consumption and capital, these being $c_t = c(k_t, z_t)$ and $k_{t+1} = k(k_t, z_t)$.

Substituting the constraints in the equilibrium conditions, one arrives at

$$\frac{1}{c(k_t, z_t)} = \beta E_t \left(\frac{\alpha e^{\rho z_t + \sigma\epsilon_{t+1}} k(k_t, z_t)^{\alpha - 1}}{c(k(k_t, z_t), \rho z_t + \sigma\epsilon_{t+1})} \right)$$

and

$$c(k_t, z_t) + k(k_t, z_t) = e^{z_t} k_t^{\alpha}$$

This Euler equation is the equivalent to $x^3 - 4.1x + 0.2 = 0$ in the previous example. $c(k_t, z_t)$ and $k(k_t, z_t)$ are the equivalent to x. The corresponding steps to be taken are:

(1) Small perturbation: σ, the standard deviation
(2) Index the solution in terms of the parameter x: $c(k_t, z_t, \sigma)$ and $k(k_t, z_t, \sigma)$
(3) Building an approximate solution using Taylor's Theorem in the equilibrium conditions:

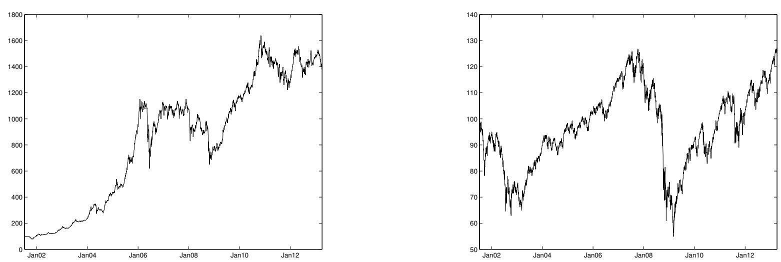
$$E_t\left(\frac{1}{c(k_t, z_t, \sigma)} - \beta \frac{\alpha e^{\rho z_t + \sigma \epsilon_{t+1}} k(k_t, z_t, \sigma)^{\alpha - 1}}{c(k(k_t, z_t, \sigma), \rho z_t + \sigma \epsilon_{t+1}, \sigma)}\right) = 0$$

$$c(k_t, z_t, \sigma) + k(k_t, z_t, \sigma) - e^{z_t} k_t^{\alpha} = 0.$$

Note: Further detail on how `dynare` uses this method to solve the equations, and general theory on perturbation methods can be found at (Fernández-Villaverde and Rubio-Ramírez, 2006; Guerrón et al., 2011; Schmitt-Grohe and Uribe, 2002; Judd, 2003)

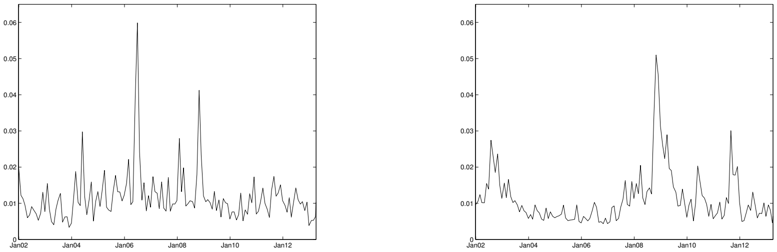
7. Figures

Figure 1. Normalized series



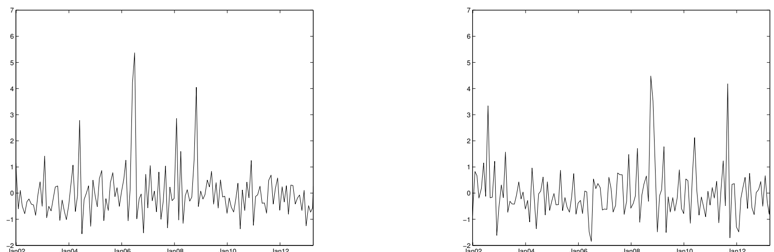
(a) IGBC (b) S&P 500

Figure 2. Uncertainty measure



(A) Local (B) External

Figure 3. Uncertainty shocks



(A) Local (B) External

FIGURE 4. Comparison - Local Uncertainty

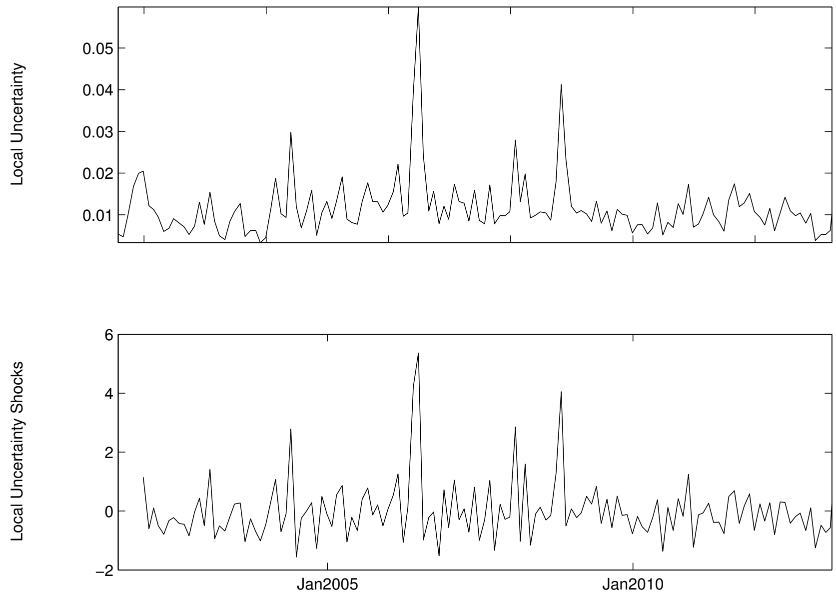


FIGURE 5. Comparison - External Uncertainty

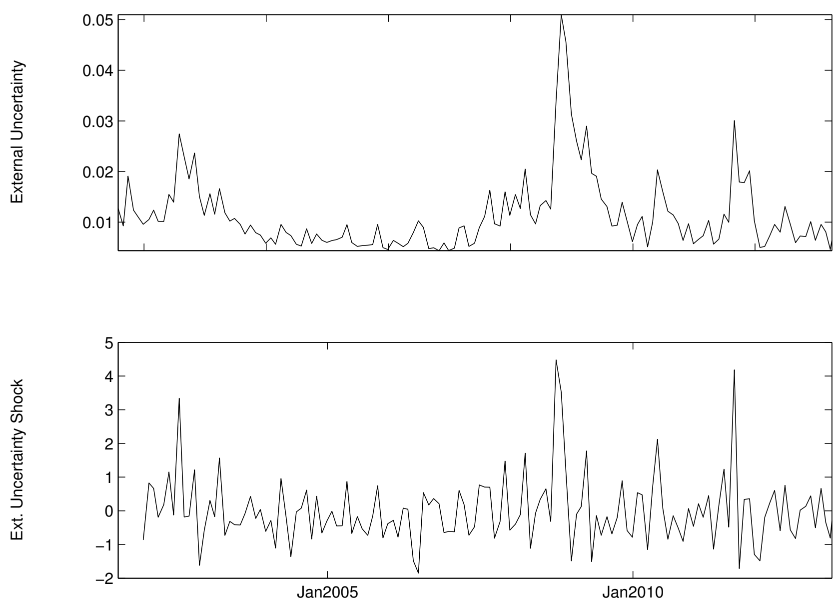


FIGURE 6. Impulse Responses of State Variables to Second Moment Local Uncertainty Shock - $b_t^*, s_{z_{u,t}}$

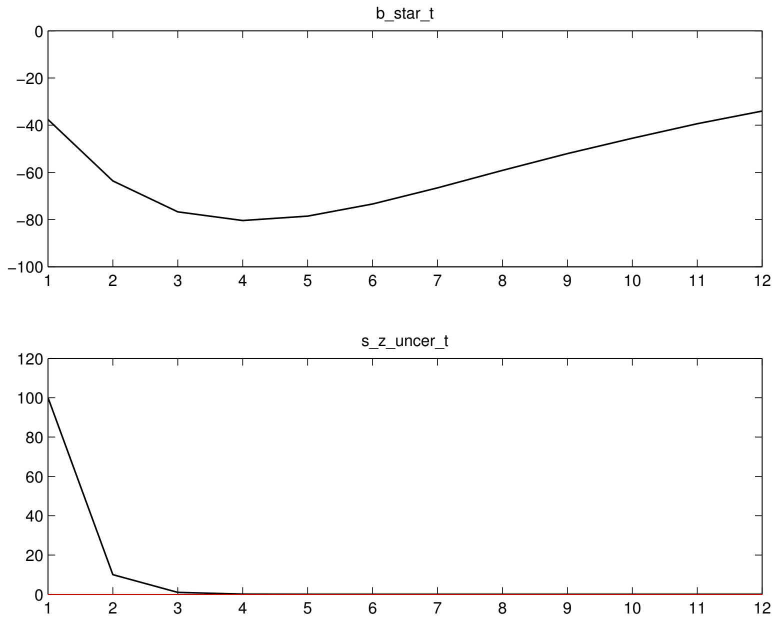


FIGURE 7. Impulse Responses of State Variables to Second Moment Local Uncertainty Shock - $C_t, C_t^h, C_t^f, C_t^z, C_t^o, \pi_t^c, \pi_t^z, \pi_t^h, \pi_t^f$

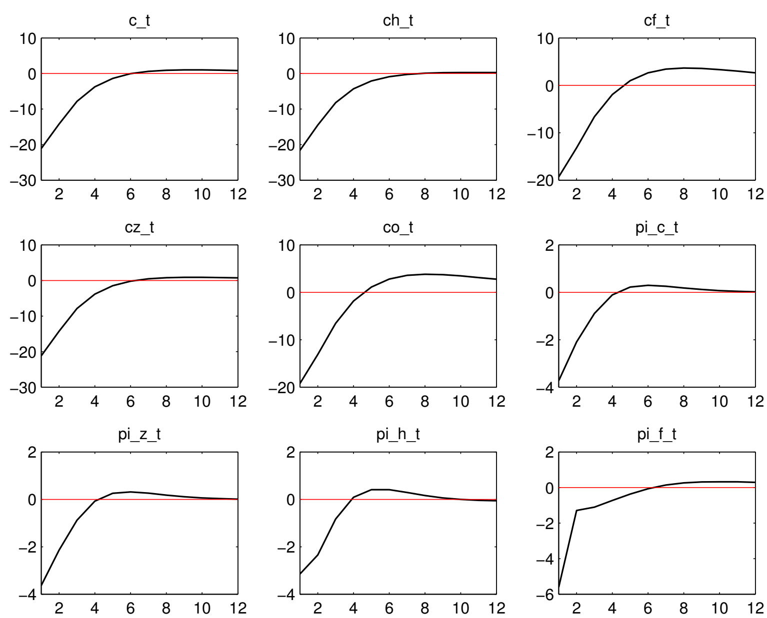


FIGURE 8. Impulse Responses of State Variables to Second Moment Local Uncertainty Shock - $\pi_{o,t}, \lambda_t, \frac{p^f}{p^c}, \frac{p^h}{p^c}, \frac{p^z}{p^c}, \frac{p^o}{p^c}, rm_t^o, \frac{s_t p_t^*}{p^c}, i^*$

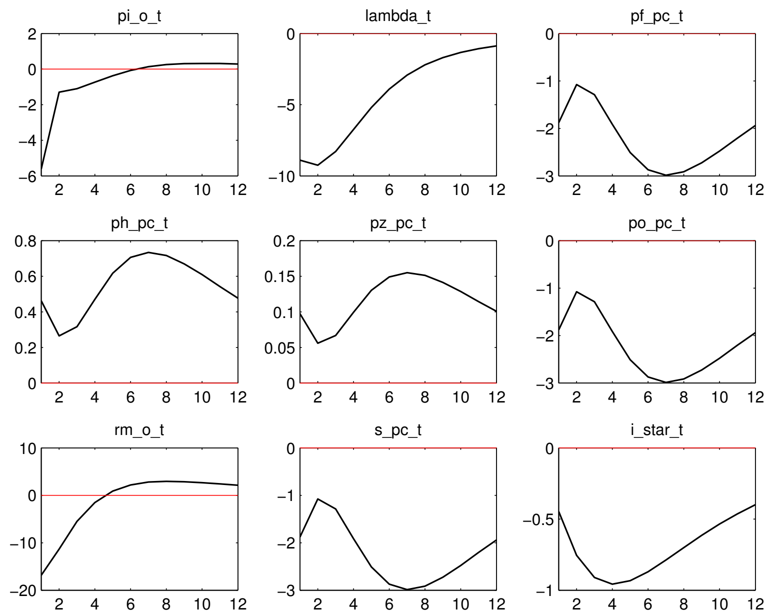


FIGURE 9. Impulse Responses of State Variables to Second Moment Local Uncertainty Shock - $\xi_t^q, w_t, n_t, f_t^1, f_t^2, w_t^{\text{opt}}, mc_t, Q_t^d, Q_t^s$

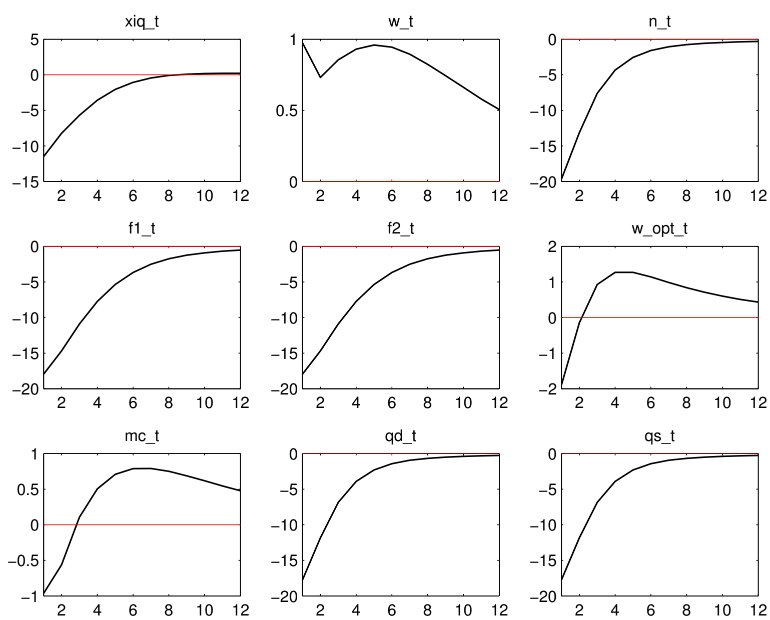


FIGURE 10. Impulse Responses of State Variables to Second Moment Local Uncertainty Shock - $p_t^{h\text{-opt}}, \varphi_t^{q_1}, \varphi_t^{q_2}, \nu_t^q, c_t^{h*}, \frac{p^{e*}}{p^*}, i_t, d_t, \frac{p^x}{p^c} x$

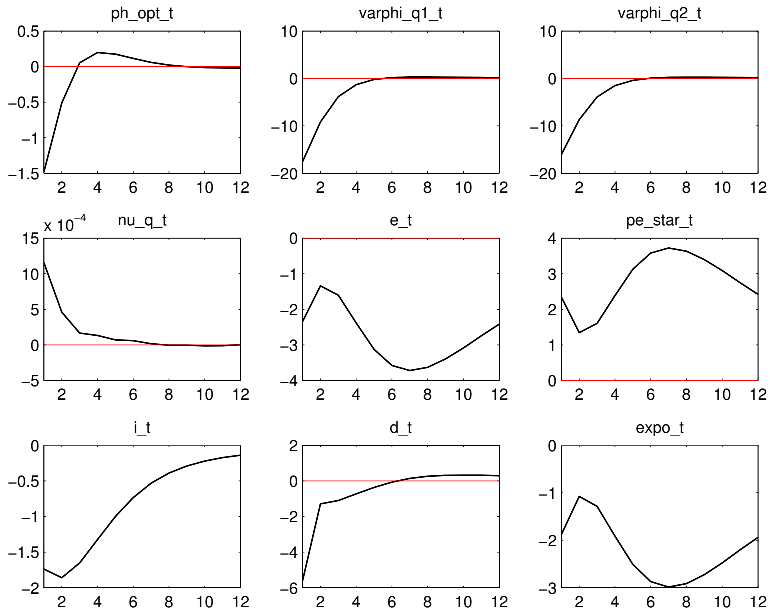


FIGURE 11. Impulse Responses of State Variables to Second Moment External Uncertainty Shock - $b_t^*, s_{z_{u,t}}$

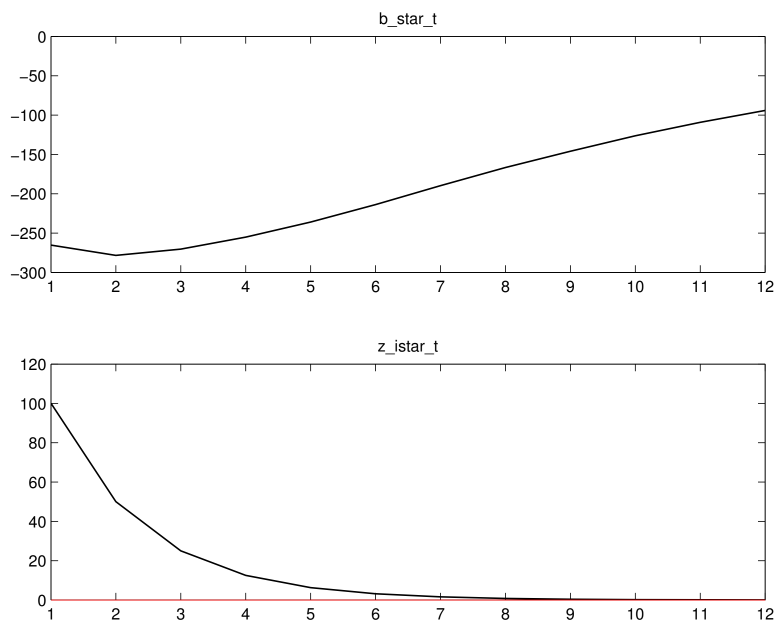


FIGURE 12. Impulse Responses of State Variables to Second Moment External Uncertainty Shock - $C_t, C_t^h, C_t^f, C_t^z, C_t^o, \pi_t^c, \pi_t^z, \pi_t^h, \pi_t^f$

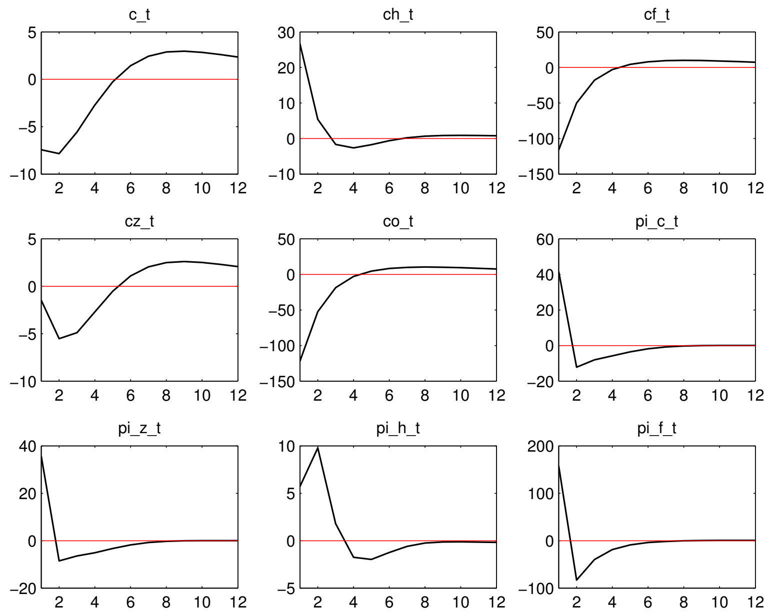


FIGURE 13. Impulse Responses of State Variables to Second Moment External Uncertainty Shock - $\pi_{o,t}, \lambda_t, \frac{p^f}{p^c}, \frac{p^h}{p^c}, \frac{p^z}{p^c}, \frac{p^o}{p^c}, rm_t^o, \frac{s_t p_t^*}{p^c}, i^*$

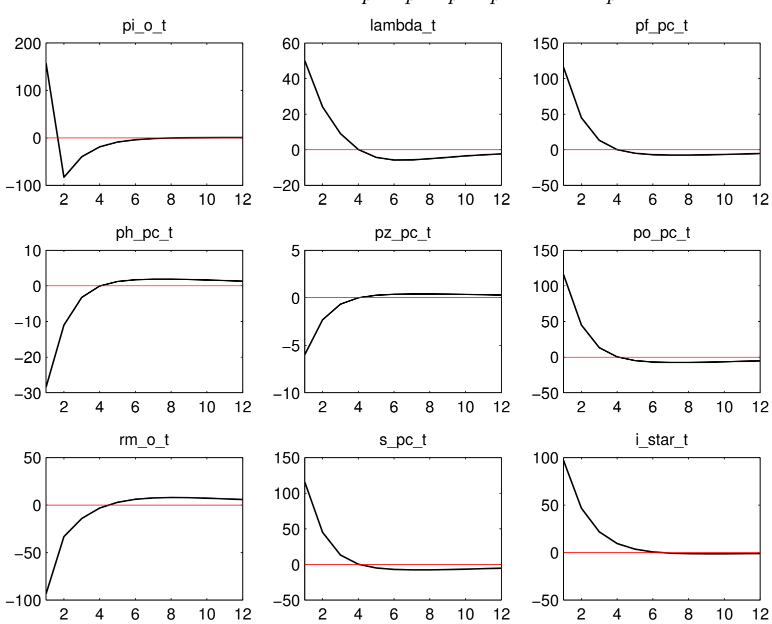


FIGURE 14. Impulse Responses of State Variables to Second Moment External Uncertainty Shock - $\xi_t^q, w_t, n_t, f_t^1, f_t^2, w_t^{\text{opt}}, mc_t, Q_t^d, Q_t^s$

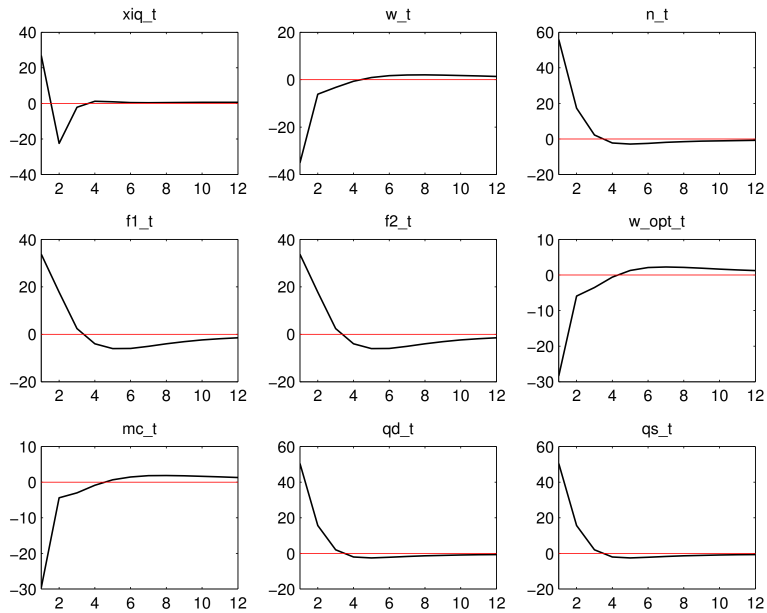


FIGURE 15. Impulse Responses of State Variables to Second Moment External Uncertainty Shock - $p_t^{h\text{-opt}}, \varphi_t^{q_1}, \varphi_t^{q_2}, \nu_t^q, c_t^{h*}, \frac{p^{e*}}{p^*}, i_t, d_t, \frac{p^x}{p^c}x$

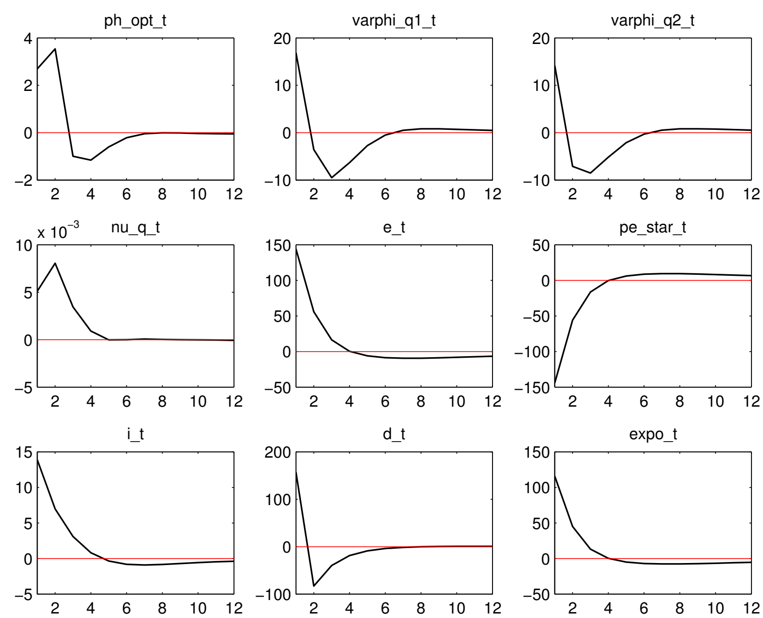


TABLE 1. Variable Names

C_t	Total comsuption
C_t^h	Domestic goods consumption
C_t^f	Foreign goods consumption
C_t^z	Foreign and domestic consumption basket (excluding oil)
C_t^o	Oil consumption
π_t^c	Total consumption inflation (incl. oil)
π_t^z	Foreign and domestic consumption inflation (excl. oil)
π_t^h	Domestic goods inflation
π_t^f	Foreign goods inflation
π_t^o	Oil inflation
λ_t	Marginal utility of consumption
$\frac{p^f}{p^c}$	Foreign goods price
$\frac{p^h}{p^c}$	Domestic price
$\frac{p^z}{p^c}$	Relative goods price (excl. oil)
$\frac{p^o}{p^c}$	Relative oil price
rm_t^o	Raw materials from oil
$\frac{s_t p_t^*}{p^c}$	Real exchange rate
i_t^*	External interest rate
ξ_t^q	Final goods Mark-up
w_t	Real wage
n_t	Worked hours
w_t^{opt}	Optimal wage
mc_t	Real Marginal cost
Q_t^d	Total domestic goods demand
Q_t^s	Total domestic goods supply
$p_t^{h\text{opt}}$	Optimal price
φ_t^{q1}	Final goods price 1
φ_t^{q2}	Final goods price 2
ν_t^q	Price distortion
c_t^{h*}	Traditional exports
$\frac{p^{e*}}{p^{ast}}$	Tradicional exports price
i_t	Intervention rate
d_t	Nominal exchange rate devaluation
$\frac{p^x}{p^c}x$	Total exports
b^*	External Deficit
K_t	Capital
$z_{u,t}$	Uncertainty
$s_{z_{u_t}}$	Uncertainty shock
f_1	Production function equation 1
f_2	Production function equation 2

Table 2. Variable Calibration

Consumption		
η_z	0.99	reflecting high elasticity
η_{tot}	0.99	reflecting high elasticity
α_z	0.20	reflecting a relatively small oil consumption
α_{tot}	0.05	reflecting a small foreign consumption
Product		
α_q	0.10	reflecting the participation of labor in the production function
η_q	0.99	reflecting high elasticity

Wage rigidities	
ϵ_l	2.53
ϕ_w	0.51
χ_w	0.4
Price rigidities	
ϵ_q	5.07
ϕ_q	0.32
χ_q	0.4
Imported goods price rigidities	
ϵ_f	5.07
ϕ_f	0.32
χ_f	0.4
Monetary policy	
ρ_i	0.85
$\rho_p i$	1.92
ρ_y	0.25