DESIGN OF A MODEL PREDICTIVE REAL-TIME CONTROL STRATEGY FOR URBAN DRAINAGE SYSTEMS

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ABSTRACT

Urban drainage systems (UDS) are dynamic systems that deal with the recollection, transport, storing, and treatment of wastewater and rain water inside cities and urban areas. Disproportionate growth of cities, as well as climatic phenomena, have increased the risk of overflows and flooding events in UDS over the last decades. Real-time control (RTC) appears as a reliable and cost effective solution to this problem.

This dissertation presents a general overview of RTC application to UDS, focusing on a control strategy known as model predictive control (MPC). A series of MPC strategies, including a novel approach for controlling nonlinear systems, are proposed and tested through a study case, obtaining positive results such as reduction of drainage overflows released to the environment.

Chapter 1

Introduction

Drainage systems are complex dynamic systems composed by several processes that include recollection, transport, storing, and treatment of wastewater and/or rain water, among others. For urban drainage systems (UDS), most of these processes take place in cities and urban areas. UDS have a considerable social, economic and environmental impact, so its correct and efficient management is extremely important, in order to prevent flooding and polluting discharges to the environment [1].

Two demographic phenomena have produced a considerable negative impact on UDS over the last decades. On one hand, population in cities has grown much faster than the infrastructure of its drainage networks. On the other hand, population growth in cities has required an increase in the construction of buildings, roads, and other civil structures. As a result, the soil in these areas has lost rain water absorption capacity, making cities more vulnerable to flooding in the presence of heavy rain events [2]. Additionally, weather phenomena such as global warming have increased the frequency, intensity, and duration of rain events in many geographic areas [3].

These phenomena have caused considerable increments in the risk of overflows of combined rain water and wastewater (also known as combined sewer overflows (CSO)), and flooding events. Minimizing the occurrence and magnitude of these events becomes an objective of great importance. To achieve this objective, the most obvious solution consists of enlarging the infrastructure of the drainage system (either by adding more channels, pipelines and storage tanks or by expanding the capacity of the existing ones), in order to transport water and sewage away from cities in a faster way and avoid damming. However, this option generally involves costs and implementation times that may be too high, making it not feasible in most cases.

Therefore, a the second alternative arises, which consists in the execution of a more efficient management of the sewer system existing infrastructure, requiring none or minimal extension of its capacity. Such objective can be achieved by applying control theory to the operation of UDS [4].

Control of this kind of systems can be applied either off-line (static rules) or online (real-time varying control actions). UDS are generally large and complex systems with features such as nonlinear dynamics, delays and dead times. Due to this dynamic nature, as well as the dynamic loading conditions under which UDS operate, off-line control may not be the most appropriate option to consider. Hence, real-time control (RTC) appears as a suitable alternative to operate and manage UDS.

RTC application to UDS is an interesting field that includes several features such as multi-variable and multi-objective control problems, operational constraints, stochastic disturbances, and distributed large-scale architectures, among others [5]. This subject has been studied by many researchers over the last years, showing RTC as a reliable and cost effective solution that improves performance of UDS and helps them to achieve operational objectives in a better way [6–8].

This work initially presents an overview of RTC application to UDS, and some of the main RTC techniques applied to these kind of systems. Then, one of the most applied techniques to drainage systems, known as model predictive control (MPC), is presented. This technique, and its application to UDS, are featured. Additionally, a series of MPC control strategies are formulated for the control of UDS. A nonlinear approach is proposed for the control of nonlinear systems through the use of linear MPC controllers. Finally, the proposed control techniques are tested through a case study, and some conclusions are stated.

The rest of the dissertation is organized as follows. Chapter 2 describes the characteristics of RTC when applied to UDS, and introduces some of the most used RTC techniques in these kind of systems. Then, Chapter 3 provides a general overview of the MPC technique, and presents some of its features when applied to UDS. A description and mathematical formulation of the proposed real-time MPC strategies is presented in Chapter 4. Chapter 5 shows a study case where the MPC techniques are applied to a portion of the Barcelona drainage system. Finally, concluding remarks are given in Chapter 6.

Chapter 2

Real-time Control of Urban Drainage Systems

2.1 General Overview

An urban water system is controlled in real-time if process variables are monitored in the system and continuously used to operate actuators during the process [8]. RTC algorithms consist of sets of rules that determine the control action that will be taken in response to the conditions in the sewer network [9].

The first RTC prototype for sewer systems was implemented by fall 1960s in Minneapolis-St. Paul (United States) [10]. Since then, an increasing number of RTC strategies have been designed, simulated, and implemented for sewer systems all over the world, especially in Europe and North America.

Historically, the main objective in the application of RTC to UDS has been the reduction of volume and/or number of CSO, without volumetric extension of the existing system [10]. Other objectives commonly taken into account include prevention of urban flooding and minimization of operation costs. More recently, further control objectives regarding water quality and pollution loads have also been greatly considered.

RTC algorithms may pursue more than one of these objectives simultaneously through the use of multi-objective control strategies. Additionally, operational objectives in a drainage system may change depending on the state of the sewer network. This can be the case in countries that have seasons or highly varying weathers, where dry and wet seasons have very different weathers.

It has been shown that the application of RTC techniques is a reliable, adaptable and cost effective solution that allows significant reduction of CSO volumes, amongst other benefits that improve the performance of UDS [8], [11]. The two main reasons of why RTC improves operation of existing drainage networks are [10]:

- Most parts of the urban wastewater systems are historically designed according to static design rules. The whole system is, however, operated under dynamic loading conditions.
- Climate change makes it necessary to adapt sewer systems with a life expectancy of tens of years to new loading situations that are developing. Singular climatic phenomena and events such as global warming increase urgency on this subject.

However, RTC implementation on existing drainage systems usually requires considerable investments and resources. RTC implementation includes several aspects, such as hydraulics, instrumentation, remote monitoring, process control, software development, mathematical modeling, organizational issues, and forecasting of rainfall or flows. Implementation and acquirement of all these aspects may be very expensive, depending on the characteristics of each system. For this reason, RTC potential and benefits must be identified in a drainage network before any implementation to justify the related investments.

There is no single set of criteria for determining whether or not a drainage system is suitable for RTC implementation; every RTC application will face site-specific challenges. Some important aspects to consider in this decision making process are, among others [9]:

- Online measurements are the foundation of the RTC system. Processes that are unable to keep online sensors up and running may not be suitable for the implementation of RTC. Experience with instrumentation (*i.e.*, determining whether the RTC system will use existing instrumentation or if mostly new instruments need to be installed, and how well established are the processes and the funding for maintaining those instruments) plays an important role.
- Size and complexity, overall hydraulic conditions, and dynamics of the sewer network.

- Topology of the sewer network and the general flow pattern ("looped" flow with many interconnections or "dendritic" flow pattern without many interconnections).

- Inline storage opportunities, including the location of the major storage spots in the network and their topology (centralized and/or distributed).

- Organizational issues, including the in-house expertise and resources available for hydraulic modeling, RTC development and implementation, and future maintenance/support of RTC.

- Overall information technology (IT) maturity of the organization interested in implementing RTC (*i.e.*, how stable is the SCADA system).

- Number, complexity, flexibility, and operational experience with the final control elements (*e.g.*, gates, valves).

Efforts have been made to establish basic standard aspects to be taken into account, when considering RTC implementation. One example of this is the working group "Real-time Control" of the German Association for Water, Wastewater and Waste (DWA in German), which prepared a guideline document on planning of RTC systems for urban drainage catchments (DWA M180) [12]. Software tools have also been designed to help in this decision process. One example of this is the planning tool named PASST (Planning Aid for Sewer System RTC) [12, 13].

2.2 RTC strategies applied to UDS

There are numerous and very different kinds of RTC strategies, and there are also many ways to classify them. Five different ways found in the literature for classifying RTC algorithms are presented below.

- The literature distinguishes between RTC strategies that are model-based and those that are not [9]. Among the most used control strategies for sewer network systems, there are control strategies based on the system model such as model predictive control (MPC) and the linear quadratic regulator (LQR). This type of algorithms requires a mathematical model that describes the dynamics of the plant with great precision. On the other hand, some decision making strategies do not require a model of the system, but a complete knowledge on the system behavior. Some examples of these strategies are fuzzy control and rule-based control (RBC).

- Another way to classify RTC strategies is into control algorithms based on optimization versus systems that use automated rules (rule-based algorithms) [9]. Rule-based systems consider the possible scenarios that can appear during the operation of the process, and establish rules to determine the appropriate control action. This kind of systems are usually transparent and can be easily understood by the operators [9]. Optimization algorithms usually require more computational efforts and a mathematical representation of the system dynamics, but they are less dependent on expert knowledge of the system, and can generate control actions that produce an optimal performance.

- Regarding the complexity of an RTC system, distinctions of the 'class' or 'level' of control implemented in UDS can be made. A system is operated on a local control level if the actuators are not remotely operated from a control room and if process measurements are taken directly at the actuator site. A system is operated on a global control level when sensors communicate their data to actuators located in other parts of the system. There exist many different configurations and communication architectures for this kind of control.

  One of the most commonly used configurations of global control is the centralized control, in which a central control room receives all the measurement data of local sensors and centrally operates the actuators in a coordinated way. Local control may represent a suitable solution in the case of one or few actuators only, but if the system is more complex or if all actuators have to be operated jointly, global control level is required [8].

  In large-scale systems and complex systems it is common to have both global and local levels of control. In this case, there can be up to three control levels. In first place there is a management level, which provides the operational objectives and the performance index for the control system. Then, the global control level takes this information into account to produce the set points for the local controllers placed at different parts of the system. At the global control level the information from the system is gathered, including measurements at different points of the drainage network and measurements of disturbances of the system such as rain, if available. Finally, local control level receives the determined set-points, and operates the actuators accordingly [5]. Figure 2.1 shows this hierarchical structure.

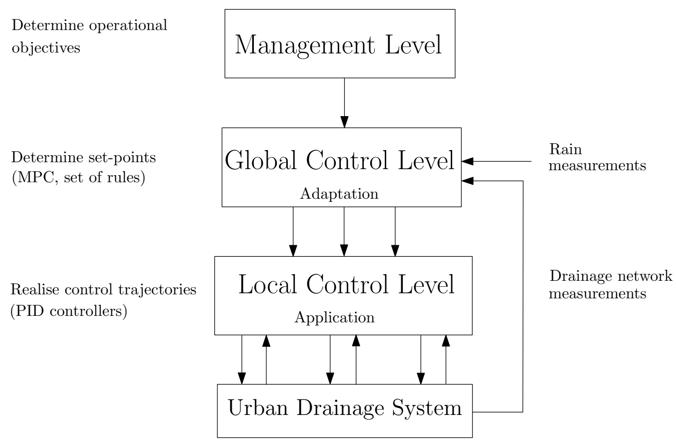


Figure 2.1: Hierarchical structure of an RTC system. Adapted from [5].

- Distinctions between reactive systems and predictive systems can also be made. Reactive systems react to present (and possibly past) external events. Predictive systems have forecasting mechanisms and methodologies to estimate or "predict" future events, and take these future events into account (as well as present and past events) to choose a control action. Introducing forecasting in control systems may improve performance, but it will add complexity to the algorithm, since additional calculations and computations will be required [9]. In the case of sewer networks, forecasts of variables like rainfalls and volumes may give important and useful information about the system, although they deteriorate with the length of the prediction horizons. For these reasons, benefits over the simpler reactive system should be identified in order to justify the additional complexity and expense of implementing predictive systems.

- The type of controlled variables is also an important criterion to classify RTC algorithms. Regarding this, the literature distinguishes between three different types of RTC: volume-based RTC, pollution-based RTC (PBRTC), and water quality-based RTC (WQBRTC) [10].

  Most RTC for UDS projects developed have focused on wastewater volumes only (volume-based RTC). In the last decade, the other two approaches have been taken more into account. Both PBRTC and WQBRTC require knowledge of the dynamics of not only the sewer network, but also the WWTP and the water bodies where the sewage is released. This means that an integrated model of the whole sewage system is needed [14], [8], [15].

Depending on the type of RTC strategy chosen, different components are required for its implementation in UDS. Table 2.1 shows some of the components needed for the implementation of different kinds of control in sewer systems. A detailed description of the measurement and control components needed for applying RTC to UDS can be found in [16]. Next, some of the most commonly used RTC techniques are introduced.

2.2.1 Heuristic Algorithms

The main characteristic of heuristic algorithms is that these techniques are experience or knowledge based. This means that a model of the system is not required to find a solution of the control problem. On the other hand, because of the heuristic nature of these algorithms any solution found is not guaranteed to be optimal. Heuristic algorithms are usually developed to have low complexity, and they are generally used for problems that are complex or cannot be easily solved [17].

In the case of UDS, the design of RTC heuristic algorithms does not need a control-oriented or mathematical model of the system. However, simulation oriented models of the drainage network are desired in order to evaluate the performance

Table 2.1: Components Required for Different Control Configurations [9].

CONTROL MODEL	Instrumentation	PLCs	SCADA/ communic.	Central SCADA server	Active operator	Central RTC server	Rainfall forecasting	On-line model
Local manual control	X				X			
Local automatic control	X	X						
Regional automatic control	X	X	X	X				
Supervisory remote control	X	X	X		X			
Global aut.- rule-based	X	X	X	X		X		
Global aut. - optimization	X	X	X	X		X	X	X

of these controllers in the system, before its implementation. Additionally, the ability to simulate the system behavior with an adequate precision may improve the parameter tuning of this kind of controllers.

One of the most broadly RTC heuristic algorithms used in drainage systems over the last decades is rule-based control [18]. A particular rule-based strategy, known as fuzzy logic control, has gained popularity in its application to UDS over the last two decades. A brief description of both conventional rule-based control strategies and fuzzy logic control is presented below.

2.2.1.1 Rule-Based Control

Conventional rule-based control (RBC) is one of the simplest RTC strategies that have been used in drainage systems. RBC for real-time flow control has been widely used in several UDS over the last decades. The hydraulic conditions at several points in a sewer system or at the wastewater treatment plant are important for the control of the system, and control actions must be taken based on the values of these conditions. Therefore, control systems are based on a large number of rules. For example, the CSO of a tank can be adjusted as a function of the water level in the storage tank [18].

RBC strategies are generally determined off-line. This means that the control rules are specified before the process start and are represented in a way that allows quick selection of the control action to be taken, depending on the current state of the system. Examples of this kind of representations include "if-then" rules (where the "if" part depends on the current state of the system and the "then" part represents the corresponding control action), scenarios, and decision matrices (a list of all possible combinations of inputs and current state variables in the process, relating them to the appropriate control actions) [19]. Even if the rules of the control strategy are previously defined off-line and do not change, control actions depend on the current state of the drainage system at each time. Therefore, RBC can still be considered as RTC.

Despite being one of the simplest RTC algorithms to implement, understand, and operate, RBC has some disadvantages. In the first place, there is not a conventional methodology to establish the control rules for RBC. Rules are usually set using the expert knowledge available about the system characteristics and behavior, so the quality and performance of the rules and the controller highly depend on this expertise. Additionally, for large and complex systems the strategy may demand a huge number of rules and scenarios.

2.2.1.2 Fuzzy Logic Control

Instead of conventional RBC systems, it is possible to use control strategies based on fuzzy logic. Fuzzy logic control (FLC) is a control technique derived from fuzzy set theory. In contrast with classical binary logic where the variables can only have two values ('0' or '1'), variables in fuzzy-logic are allocated to so-called degrees of membership ranging between 0 and 1 [20].

FLC combines the simple rules of an expert system with a flexible specification of output parameters. Conventional controllers adjust the control sizes of the system based on a set of differential equations that represent a model of a dynamic system. In fuzzy controllers, the control values are obtained on the basis of fuzzy rules, which are similar to the model of human reasoning [21].

The way in which fuzzy controllers produce control actions can be summarized in three steps. In the first step, the scalar inputs are transformed into memberships of fuzzy sets by *fuzzifying* functions. This information is then given to the inference engine. Finally, the membership values are transformed into required scalar output variables by a *defuzzification* step [18]. This process requires the fuzzy functions to be already defined, in order to establish the degrees of membership of the inputs. FLC has been studied for reduction of CSO in UDS, and also for control in WWTP [22].

The control processes of conventional RBC and FLC are different in many respects. Even the use of an identical rule base for both systems leads to different inference values [18]. An example of this can be seen in Figure 2.2, which shows the use of both techniques to regulate a storage tank of outflow depending on its water level. This figure shows how, despite of having the same rule base (second column), both algorithms produce different outputs (third column), with more flexibility in the sizing of the outflow given by the FLC technique.

RBC and FLC have been studied in several applications involving UDS. In [23], a rule-based fuzzy algorithm is used to reduce overflows and the volume of CSO in the drainage system of Wilhelmshaven (Germany), achieving both control objectives. In [24], rule-based and fuzzy principles are used in Taiwan for the control of pumping operations in Taipei City sewage system, achieving a more effective draining of rain water in order to avoid flooding in the city. Other applications of heuristic RTC techniques in UDS include the use of fuzzy expert systems to establish rehabilitation priorities of sewer networks in Laval (Canada) [25], and the use of FLC in urban drainage tunnels with nonlinear dynamics and random interferences, obtaining positive results such as improvement of the drainage system efficiency, extention of the pump work life, and a reduction in energy consumption [26].

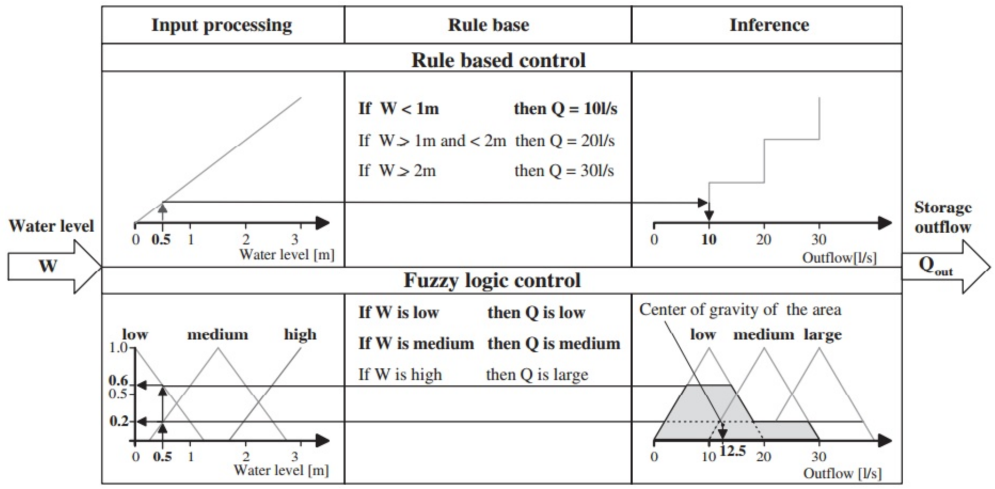


Figure 2.2: Application of RBC and FLC to a storage tank [18].

2.2.2 Optimization-Based Algorithms

Optimization-based control algorithms involve an optimization problem that determines the desired behavior of the system. Based on the optimization problem and the measure (or estimation) of the current system variables, these algorithms seek the best possible (*i.e.*, the “optimal”) control action. In UDS, optimization-based algorithms deal with the problem of generating control strategies in order to minimize or maximize certain criteria, based on current and past readings of the telemetry system [4].

The criterion to be minimized or maximized is usually expressed mathematically as a scalar function $J(x)$ known as objective or cost function. As previously stated, there can exist many different control objectives when applying RTC to UDS. Some of them are:

- Minimization of flooding in streets.
- Minimization of the CSO to the receiving environment.
- Maximization of the treated sewage in the system.
- Minimization of operation costs (pump stations and treatment plants).

- Minimization of the water pollution released to the environment.

Regardless of the control objective for a particular drainage system, this should be expressed as a cost function to solve the optimization problem. It is possible for some algorithms to take into account two or more control objectives. This is known as multiple-objective control, and can be done in several ways. One of the most widely used multi-objective techniques is called scalarisation and consists of converting the problem into a single-objective optimization problem with a scalar-valued objective function [5]. This is done by forming a new objective function that is a linearly weighted sum of several single-objective cost functions. Thus, if there are N single-objective cost functions $J_1(x), \ldots, J_N(x)$, a scalar weight w can be assigned to each function, obtaining the new objective function

$$\mathcal{J}(x) = \sum_{i=1}^{N} w_i J_i(x). \tag{2.1}$$

There are different ways of assigning the weights w_i depending on the priority that each control objective has in a specific system. Other multi-objective techniques focus on the Pareto-optimal solution concept. A Pareto-optimal solution has the characteristic that one objective cannot be improved without worsening a different one [14]. Generally, there is more than one solution in a problem that satisfies this condition, generating sets of solutions known as Pareto sets. Additionally, several techniques use evolutionary approaches as well for solving multi-objective optimization problems. Examples of techniques based on these two notions can be found in [5, 14, 20, 27, 28].

Most research has focused on single-criteria optimization so far. For this reason, multi-criteria optimization is an ongoing field [20]. An extensive review of several multi-objective optimization methods can be found in [29].

One of the most used optimization-based RTC techniques in UDS applications is model predictive control, which will be discussed in detail in the next Chapter. Some of the other main optimization-based RTC algorithms are described below.

2.2.2.1 Linear Quadratic Regulator

Linear quadratic regulator (LQR) is an optimal controller that produces a linear control action in order to minimize an objective function J associated to the state variables norm (states x_i, $i = 1, ..., n$) and the energy (control outputs u_j, $j = 1, ..., m$). In general, the objective function J has the form

$$J = \frac{1}{2} \int (x^\top Qx + u^\top Ru) dt. \tag{2.2}$$

In (2.2) Q and R are weighting matrices that determine what is most relevant to reduce in the system. In the case of drainage systems, states x can be associated with volumes and/or flows in the system, and control actions u depend on the type of actuators available in the system. Objective function J and its parameters are set according the control objectives of the process.

For the LQR design, it is necessary to have a space state representation of the system given by

$$\dot{x} = Ax + Bu,$$

where x is a column vector with the n states of the system, u is a column vector with the m control actions (inputs to the system), and A and B are coefficient matrices. In order to minimize the objective function, LQR controllers produce the linear control law $u = -Kx$. K is a gain matrix that must be found by solving a quadratic, first-order, ordinary differential equation known as the Riccati equation [30].

In [31] multi-variable LQR is applied to sewer network flow control in Bavaria, Germany. In this case the control objectives are minimization of overflows in the system by using all available storage space in an optimal way, and emptying the network as soon as possible. Results of the study show positive results with respect to the uncontrolled case, presenting the LQR as a valid alternative for the control of drainage systems. LQR techniques have also been applied in the RTC of water delivery and irrigation channels, in order to improve their delivery service [32], [33].

2.2.2.2 Evolutionary Strategies

Evolutionary algorithms (EA) use and mimic evolutionary principles to search for optimal solutions. This kind of algorithms belong to the global optimization procedures, which do not make assumptions on the continuity of the objective function and do not require information on its derivatives, making EA suitable for solving a very wide range of optimization problems [14].

Unlike classical methods, EA use a population of solutions in each iteration instead of evaluating just one. Therefore, these algorithms do not reach a single optimal solution of the optimization problem, but a population of optimal solutions. The ability to find multiple optimal solutions in one single run makes evolutionary algorithms to be particularly suited for solving multi-objective optimization problems [14].

Most research in multi-objective optimization has mainly focused in Pareto optimization, a technique which requires large computational efforts. EA constitute therefore an important alternative, which can be more computationally efficient. In addition, EA allow the consideration of linear and non-linear constraints and the handling of complex optimization problems.

One of the EA that has been studied and applied in the context of UDS is fuzzy decision making (FDM). This strategy is a fuzzy-logic based, decision making tool that can be used for multi-criteria optimization. Decision making can be described as the selection of the best alternative from a given set of possible choices, given the information regarding the decision problem and the goals of the decision maker. Since decision making resembles the selection of the best available alternative, it can be described mathematically as an optimization problem [34].

FDM allows to transform a multi-objective optimization problem into a single-

Table 2.2: Comparison between real-time controllers [39].

Type of Controller	Optimization Based	System Non-linearities	Consideration of Constraints	Centralized or Distributed	Model Free	Degree of Implementation
RBC - FLC	No	Yes	No	C / D	Yes	Medium
LQR	Yes	No	No	C	No	Medium
EA	Yes	Yes	Partially	C / D	Yes	Low
MPC	Yes	Yes	Yes	C / D	No	High

objective problem by merging all partial objectives in one substitute quality criteria [20]. This technique has been applied in multi-criteria optimization of non-lineal and dynamic control systems, showing advantages such as transparent criteria weighting, low computational effort and optimal trade-off between performance criteria, among others [35]. Knowledge-based approaches have also been used to support decision making algorithms that aim to achieve environmental objectives in UDS, such as reduction of pollution in rivers due to wastewater discharges [36].

Other EA applied to UDS include genetic algorithms. This kind of algorithms imitate the natural genetic processes of evolution, deliberately keeping a range of good solutions to avoid being drawn into false local optima [37]. Genetic algorithms are usually used for solving complex and/or nonlinear optimization problems. Examples of the application of genetic algorithms to water quality management systems and control of UDS can be found in [38] and [28]. In [14], evolution strategies combining non-dominating sorting and self-adapting algorithms are applied to the control of integrated UDS, achieving an improvement in the recieving river water quality and lower investment costs.

Table 2.2 shows a comparison between the RTC techniques described in this document. Aspects such as the ability to deal with constraints and non-linear dynamics in the system were taken into account for the comparison. The configuration in which the control techniques can be implemented (centralized and/or distributed) was considered too, as well as the degree of implementation of these techniques in applications related to UDS.

Chapter 3

Model Predictive Control

3.1 General Overview

Model predictive control (MPC) is a model-based control strategy that uses a prediction of the system response to establish an appropriate control action [40]. This algorithm does not designate a specific control strategy, but a very ample range of control methods that make an explicit use of a model of the process to obtain the control signal by minimizing an objective function. MPC uses a mathematical model of the process to generate a sequence of future actions within a finite prediction horizon. These actions, known as the control law $u(t)$, aim to minimize a given cost function.

MPC control uses a receding horizon philosophy, where decisions are made based on the prediction of the system future behavior, within a finite prediction horizon [41]. At time instant t, the algorithm plans a sequence of future control actions $u^*(t), u^*(t+1), \ldots, u^*(t+H_p)$, where H_p is a finite prediction horizon that has been previously determined.

This sequence is determined by solving an optimization problem (OP) based on the system predicted outputs and the cost function to be minimized. In practice, the controller can only make the first action $u(t)$. At time instant $t+1$ the same procedure is repeated, moving the prediction horizon one step ahead in time. Receding horizon philosophy can be seen graphically in Figure 3.1.

An MPC controller is compounded by four main elements [41]:

- Mathematical model of the system: a control oriented-model that describes the system dynamics and allows the prediction of its future behavior. This model needs to know the current states of the system, information that is not

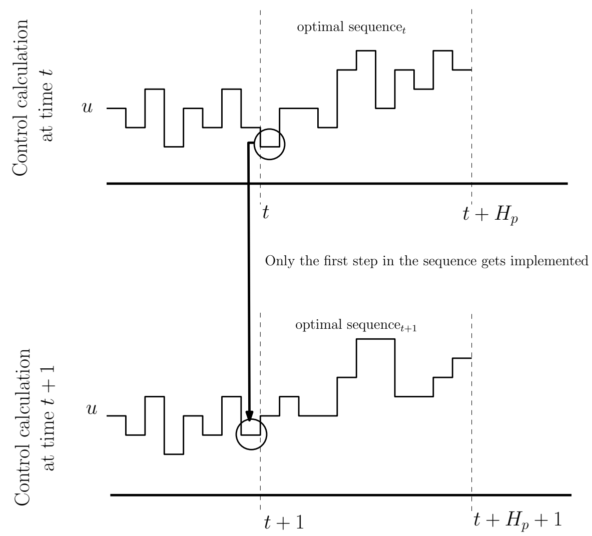


Figure 3.1: Receding horizon control. Adapted from [42].

always fully measurable or available. In case it is not possible to measure this information, a state estimator must be implemented in the control loop.

- Cost function: mathematical formula that expresses the control objective to be achieved through MPC. The cost function can take into account multiple objectives. Besides the cost function, there can also be set points and reference trajectories for some states or variables of the system.

- Restrictions: Set of constraints present in the system. They can be physical limits of the plant as well as restrictions on the control signals and system outputs.

- Dynamic Optimizer: Probably the main element of an MPC controller. Its goal is to minimize the objective function at each time instant based on the current states and future predictions provided by the mathematical model. Besides the cost function, the optimizer may consider the system set points and constraints.

Figure 3.2 shows the basic application of an MPC controller in a closed control loop.

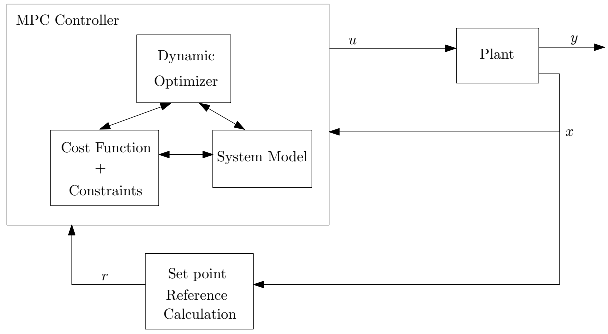


Figure 3.2: Closed loop MPC configuration.

The basic formulation of a linear MPC controller considers dynamic systems described by the space-state model:

$$x(k+1) = Ax(k) + Bu(k), \tag{3.1}$$

$$y(k) = Cx(k) \tag{3.2}$$

where x, u, and y are column vectors containing with system states, inputs, and outputs, respectively, and A, B, and C are coefficient matrices with suitable dimensions. In the most general case, the cost function of the optimization problem associated to the MPC algorithm may be a quadratic function of the form

$$J(x,u) = \sum_{j=1}^{H_p} \|\hat{x}(k+j|k)\|_Q^2 + \sum_{j=0}^{H_p-1} \|\hat{u}(k+j|k)\|_R^2. \tag{3.3}$$

Here $\widehat{x}(k+i|k)$ and $\widehat{u}(k+i|k)$ denote the prediction of the state $x(k+i)$ and the input $u(k+i)$, respectively, from knowing (or estimating) $x(k)$. The notation $\|x\|_Q^2$ denotes the quadratic form $x^\top Qx$.

In this cost function, H_p is the established prediction horizon, P and Q are positive semi-definite matrices with suitable dimensions and R is a positive definite matrix with suitable dimensions. Let $\mathbf{u}$ be the sequence of control actions given by

$$\mathbf{u} = [\hat{u}(k|k)^\top, \hat{u}(k+1|k)^\top, ..., \hat{u}(k+H_p-1|k)^\top]^\top, \tag{3.4}$$

the objective is to find the optimal sequence $\mathbf{u}^*$ that solves the following OP [41]

$$\min_{\mathbf{u}} J(x,u) \tag{3.5}$$

subject to

$$\begin{gathered} x(k+1) = Ax(k) + Bu(k), \\ x(0) = x(k), \\ x(k) \in X, \quad u(k) \in U, \quad k = 0, ..., H_{p-1}. \end{gathered}$$

Here X and U are the set of values that the system states and inputs can take, respectively. These two sets depend on any existing restrictions on $u(k)$ and $x(k)$, that must always be satisfied. In the case where there is an infinite prediction horizon ∞, and there are no restrictions in the system, the OP (3.5) is equivalent to a linear quadratic regulator (LQR) problem, and can be solved as one of these problems [27].

MPC theory has been developed into a quite matured stage. However, some problems and subjects remain open in this field. Adaptive MPC is one of the problems that has not been yet covered, especially in constrained systems [43]. Robust MPC is another field of interest great interest [44], [45], [46].

Additionally, decentralized and distributed MPC configurations have become a growing research field. Distributed MPC has been studied for different applications including control and coordination of power systems [47], [40] and urban traffic control [48]. Decentralized and distributed MPC strategies have also been studied in water related applications such as level control in tanks [49] and coordination in water supply networks [50]. A review of different distributed MPC configurations and future research directions in this field can be found in [51].

3.2 MPC Applied to UDS

MPC strategies have been successfully applied in an increasing number of industries during the last decades. In the case of UDSs, MPC techniques have been applied and studied in several cities such as Barcelona, where an MPC controller is simulated in part of the city drainage network and it is shown that significant reductions in flooding and combined sewer overflows (CSO) may be achieved, even in networks with limited control possibilities [5, 52]. Predictive control is also being currently studied in Haute-Sure, Luxemburg. MPC techniques are being developed for a drainage system that gathers the sewage of 23 villages and several small settlements located around the Haute-Sure reservoir, and directs it to one main WWTP [53, 54]. Other countries where MPC has been studied and/or applied in UDS include Canada [8], Germany [55], Colombia [56], and Netherlands [57].

In order to formulate the MPC problem in UDS it is convenient to take into account a different type of inputs for the system. These inputs, called disturbances, cannot be manipulated by the controller. Thereby, it is necessary to modify the system mathematical model. In order to include disturbances in the model, (3.1) must be modified, obtaining

$$x(k+1) = Ax(k) + Bu(k) + B_p d(k), \tag{3.6}$$

where $d(k)$ is a vector containing the system disturbances, and B_p is a coefficient matrix with suitable dimensions. These disturbances can be measured or unmeasured. Table 3.1 shows the physical meaning that the variables $x(k)$, $u(k)$, and $d(k)$ would have in a drainage system, according to the model proposed in [52].

Constraints in this case are given by the volumetric capacity of tanks and pipes, and by flow restrictions in channels and actuators. These variables have

Table 3.1: System variables.

Type of variable	Symbol	Description
System states	$x(k)$	Tank volumes
Control inputs	$u(k)$	Manipulated flow through pipes and sewers
Measured disturbances	$d(k)$	Rain inflow

maximum and minimum values, which must not be overstepped in order to ensure a proper system behavior. Moreover, the types of sensors and actuators used in the system may add extra restrictions. Regarding the cost functions, there are several options depending on the control objective that each drainage system may have. Cost functions do not have to be quadratic functions in the form of (3.3) necessarily.

Additionally, these functions can have different mathematical forms, and can take multiple control objectives into account, as it has been noted before. In the case of MPC with multiple objective functions, scalarisation can be applied, obtaining a new objective function of the form of (2.1). Other multi-objective control methods can be applied as well, such as lexicographic minimization [58].

The characteristics of MPC controllers have certain benefits in its application to UDS. Some of these benefits include:

- The optimization problem statement allows to explicitly and systemically expressing the constraints present in the system. MPC strategies allow controllers to be aware of input constraints, in particular saturation constants, and never generate input signals which attempt to violate them. This feature avoids problems such as the existence of integrator wind-up in the controlled system [59].

- The predictive nature of this algorithm allows it to anticipate the system response to future rain events. The quality of this forecast will depend on a good prediction of the future rain events.

- The algorithm allows the controller to consider non-ideal elements in the system such as delays and disturbances.

Other advantages of MPC include its suitability for multiple input multiple output (MIMO) systems, and for large systems with complex dynamics.

Chapter 4
Mathematical Formulation of the MPC Strategies

A basic formulation for linear, discrete-time MPC has been given in Chapter 3. Based on this approach, a series of MPC strategies have been designed and simulated for its application to UDS. The description and mathematical formulation of the proposed real-time MPC strategies are presented in this Chapter.

Section 4.1 describes the design of unconstrained and constrained linear MPC, as well as the modifications needed to take measured disturbances into account in the control system. Additionally, Section 4.2 describes two alterations of the linear MPC strategies, to make them suitable for the control of nonlinear systems.

4.1 Linear approach

4.1.1 MPC without restrcitions

The first MPC controller that has been developed is a linear controller without restrictions, following the methodology shown by Jan Maciejowski in [59]. This controller aims to minimize the following objective function

$$J_1(x, \Delta u) = \sum_{j=1}^{H_p} \|\hat{x}(k+j|k) - r(k+j)\|_Q^2 + \|\Delta\hat{u}(k+j-1|k)\|_R^2, \qquad (4.1)$$

where $\Delta u(k)$ is the change in the control action at time k defined as $\Delta u(k) = u(k) - u(k-1)$, $\hat{\Delta u}(k+j|k)$ denotes the prediction of $\Delta u(k)$ from knowing (or estimating) $x(k)$ and $u(k-1)$, and the vector $r(k)$ contains the set-points to be followed by the system in tracking applications.

It is important to note that cost function (4.1) considers the change in the input variables $\Delta u(k)$ instead of dealing with the input variables $u(k)$. Thus, the control objective in this case is not to find the optimal sequence $\mathbf{u}^*$, but to obtain the optimal sequence ΔU^* that minimizes $J_1(x, \Delta u)$. Once that ΔU^* is reached, $\mathbf{u}^*$

can be obtained through a discrete time integrator.

If the following vectors are defined

$$X(k) = [\hat{x}(k+1|k)^\top, ..., \hat{x}(k+H_p|k)^\top]^\top, \tag{4.2}$$

$$R(k) = [\hat{r}(k+1|k)^\top, ..., \hat{r}(k+H_p|k)^\top]^\top, \tag{4.3}$$

$$\Delta U(k) = [\Delta\hat{u}(k|k)^\top, ..., \Delta\hat{u}(k+H_p-1|k)^\top]^\top, \tag{4.4}$$

then Equation 4.1 can be rewritten as

$$J(x, \Delta u) = \|X(k) - R(k)\|^2_{\mathcal{Q}} + \|\Delta U(k)\|^2_{\mathcal{R}}. \tag{4.5}$$

Here $\mathcal{Q}$ is a matrix of dimensions $nH_p \times nH_p$ and $\mathcal{R}$ is a matrix of dimensions $lH_p \times lH_p$, where n is the number of state variables and l is the number of controlled inputs of the system. These two matrices are defined as

$$\mathcal{Q} = \begin{bmatrix} Q(1) & 0 & \dots & 0 \\ 0 & Q(2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q(H_p) \end{bmatrix}, \tag{4.6}$$

$$\mathcal{R} = \begin{bmatrix} R(0) & 0 & \dots & 0 \\ 0 & R(1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R(H_p-1) \end{bmatrix}. \tag{4.7}$$

State Equation 3.1 can be modified in order to express $X(k)$ in terms of $x(k)$ and $u(k-1)$, which is data that can be known and/or estimated by the controller at the time instant k, and $\Delta u(k)$. This leads to the following prediction model

$$X(k) = \Psi x(k) + \Upsilon u(k-1) + \Theta \Delta U(k), \tag{4.8}$$

where Ψ, Υ, and Θ are matrices defined as

$$\Psi = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{H_p} \end{bmatrix}, \tag{4.9}$$

$$\Upsilon = \begin{bmatrix} B \\ AB + B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix}, \Theta = \begin{bmatrix} B & \dots & 0 \\ AB + B & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_p-1} A^i B & \dots & B \end{bmatrix}. \tag{4.10}$$

A tracking error signal for the controller can be defined as

$$\mathcal{E}(k) = R(k) - \Psi x(k) - \Upsilon u(k-1). \tag{4.11}$$

Combining Equations 4.8 and 4.11, cost function J_1 can be rewritten as

$$J(x, \Delta u) = \|\Theta \Delta U(k) - \mathcal{E}(k)\|_{\mathcal{Q}}^2 + \|\Delta U(k)\|_{\mathcal{R}}^2. \tag{4.12}$$

By doing this it is possible to express J_1 in terms of a single variable ($\Delta u(k)$), and known data at the instant k such as $x(k)$ and $u(k-1)$. At this point it is possible to find ΔU^* by solving the following OP

$$\min_{\Delta U} J_1(x, \Delta u) \tag{4.13}$$

subject to

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ x(0) &= x(k). \end{aligned}$$

In the unconstrained case, the optimal solution of (4.13) is linear, and is given by

$$\Delta U^{*} = 0.5 * \mathcal{H}^{-1} * \mathcal{G}, \tag{4.14}$$

where

$$\mathcal{G} = 2\Theta^{\top}\mathcal{Q}\mathcal{E}(k), \tag{4.15}$$

$$\mathcal{H} = \Theta^{\top}\mathcal{Q}\Theta + \mathcal{R}. \tag{4.16}$$

4.1.2 MPC with restrcitions

In the case of constrained linear MPC, a series of inequality constraints are defined in [59] for the unconstrained controller described previously in this Section. Following this approach, a controller was designed and simulated in various case studies. Results of these simulations showed poor results, with feasibility problems in the solution of the optimization problem, and difficulties for satisfying the system constraints.

For this reason, the following approach was chosen. Going back to the original state-space model of the system

$$x(k+1) = Ax(k) + Bu(k), \tag{4.17}$$

$$y(k) = Cx(k), \tag{4.18}$$

a prediction model to calculate the future value of the system state variables can be stated as

$$X(k) = \Psi x(k) + \Lambda U(k). \tag{4.19}$$

In this case matrix Ψ is the same matrix defined in 4.10. On the other hand,

Λ is defined as

$$\Lambda = \begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{H_p-1}B & A^{H_p-2}B & \dots & B \end{bmatrix}. \tag{4.20}$$

For this case a new cost function can be defined as

$$\begin{aligned} J_2(x,u) &= \sum_{j=1}^{H_p} \|\hat{x}(k+j|k) - r(k+j)\|_Q^2 + \|\hat{u}(k+j-1|k)\|_R^2 \\ &= \|X(k) - R(k)\|_{\mathcal{Q}}^2 + \|U(k)\|_{\mathcal{R}}^2, \end{aligned} \tag{4.21}$$

where

$$U(k) = [\hat{u}(k|k)^\top, \dots, \hat{u}(k+H_p-1|k)^\top]^\top \tag{4.22}$$

and the matrices $\mathcal{Q}$ and $\mathcal{R}$ are defined in the same way as in the case without restrictions. The optimization problem to be solved by the MPC controller is now given by

$$\min_U J_2(x,u) \tag{4.23}$$

subject to

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ x(0) &= x(k), \\ U_{min} \leq & U(k) \leq U_{max}, \\ X_{min} \leq & \Psi x(k) + \Lambda U(k) \leq X_{max}. \end{aligned}$$

Since the cost function J_2 is quadratic on U, this is a quadratic programming

(QP) problem. Additionally, J_2 is a convex function, which means that optimization problem (4.23) is a convex QP problem. This condition ensures that the OP will always have a global minimum, so there should always be an optimal solution, as long as the feasible region of the problem is not empty.

Unlike the unconstrained controller, in this case the control law is not necessarily linear. In this case, the optimal control law may be nonlinear when one or more variables are activating any of the constraints of the system. For this reason, it is necessary to solve the OP (4.23) at every time instant. Standard algorithms already exist for solving QP problems [59].

It should be noted that OP (4.23) takes into account restrictions on both the system state variables and inputs. Generally, restrictions on $U(k)$ are due to the actuators in the system, and must be always satisfied. On the other hand, it may not be feasible to always satisfy the contraints asociated to the system state variables. If this is the case, these constraints must be 'softened' (*e.g.*, through the use of barrier functions [59]). Additionally, other restrictions may be added in the system (*e.g.*, restrictions in the rate of change of the system inputs (ΔU)).

4.1.3 MPC taking disturbances into account

As it was explained in Chapter 3, MPC application to UDS may consider disturbances in the system for a better performance of the controller. This can be achieved by changing the state Equation (3.1) into (3.6), as it was previously stated.

Consider the disturbance vector

$$D(k) = [\hat{d}(k+1|k)^\top, ..., \hat{d}(k+H_p|k)^\top]^\top, \tag{4.24}$$

where $\hat{d}(k+j|k)$ is the estimated or predicted value of $d(k+j)$, from knowing the value of $d(k)$. Using this vector, prediction models 4.8 and 4.19 can be updated to

$$X(k) = \Psi x(k) + \Upsilon u(k-1) + \Theta \Delta U(k) + \Omega D(k) \tag{4.25}$$

$$X(k) = \Psi x(k) + \Lambda U(k) + \Omega D(k), \tag{4.26}$$

respectively. In both cases Ω is the same matrix, defined as

$$\Omega = \begin{bmatrix} B_p & 0 & \dots & 0 \\ AB_p & B_p & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{H_p-1}B_p & A^{H_p-2}B_p & \dots & B_p \end{bmatrix}. \tag{4.27}$$

By describing the system dynamics through a state equation of the form of 3.6, matrix Ω can be obtained, and the prediction model for the system can be updated. Additionally, being able to obtain the vector $D(K)$ for the system at every time instant allows the MPC controllers to take disturbances into account when determining the optimal control law. By considering the corresponding updated model in the solution of the OP, the system disturbances will be taken into account for both constrained and unconstrained MPC controllers that are presented in this Section. For example, in the unconstrained MPC controller the tracking error signal will now be defined as

$$\mathcal{E}(k) = R(k) - \Psi x(k) - \Upsilon u(k-1) - \Omega D(k). \tag{4.28}$$

It is important to note that this formulation implies two important matters. First, the controller is required to be able to know and measure the value of the system disturbances at every time instant. Moreover, the system needs to have a way of predicting the future values of its disturbances with an adequate precision, in order to obtain vector $D(k)$. If it is not possible to measure and predict the system disturbances with an acceptable accuracy, it may not be convenient for the controller to consider them trough this approach.

4.2 Nonlinear approach

Most systems and phenomena in nature have a nonlinear behavior [60]. Hydraulic systems, and particularly UDS, are not the exception. For this reason, it is of great interest being able to apply MPC to nonlinear systems.

As it has been stated in Chapter 3, MPC strategies highly depend on a mathematical model of the system to be controlled. When applying MPC to nonlinear systems, any linear model used to simulate and predict the behavior of a particular system will be only an approximation, and will therefore imply loss of information on the original system dynamics. This loss of information may affect predictions of the future system behavior, and thus worsen the controller performance. Because of this, in the case of nonlinear systems it may be better to work with nonlinear mathematical representations of the systems that include a more accurate information about their dynamics.

Several MPC strategies for nonlinear systems already exist, and are generally known as nonlinear model predictive control (NMPC) [61]. However, MPC application in linear systems presents numerous advantages over the use of NMPC. Some of these advantages are [62]:

- Identifying nonlinear models from experimental data is still an open problem. On the other hand, currently there are numerous identification techniques available for linear systems, making this task much simpler.

- When applying MPC to linear systems, a correct choice of the cost function can ensure that the associated OP will be convex, which guarantees that its solution will be located in a global minimum. However, convexity of the OP is much more difficult to guarantee in the case of nonlinear systems. This makes it difficult to obtain a global minimum, and can potentially cause stability problems in the controller.

- Less computational costs and efforts are required for solving linear, convex optimization problems. This is very valuable, specially in real-time applications.

- The study of aspects such as stability and robustness from an analytical point of view is more complex in the case of nonlinear systems. This field is still open.

Because of this, is of great interest to use the advantages of linear MPC described above for nonlinear systems, without losing much information of the system dynamics due to the use of only one linear approximation of its mathematical model. Two methods for approaching this problem are proposed below.

4.2.1 Gain-scheduled MPC

The first approach to control nonlinear systems by using linear MPC controllers is through the technique of gain scheduling (GS) [63]. The purpose here is to use both GS and MPC techniques together to control nonlinear systems. To achieve this, a strategy denoted as gain scheduled - model predictive control (GS-MPC) is proposed [64].

GS is a control design methodology that consists in decomposing the design of a nonlinear controller through a finite number of linear controllers. These controllers are designed by linearizing the system around several values of a given set of variables, known as scheduling variables. During system operation, the linear controllers are enabled and switched depending on the current value of the scheduling variables [65].

Development of a GS controller for tracking applications in nonlinear systems can be summarized in the following steps [60]:

1. Linearize the nonlinear model about a family of operating (equilibrium) points.

2. Design a parametrized family of linear controllers to achieve the specified performance at each operating point.

3. Build a gain-scheduled controller such that:

    - The closed-loop system under the gain-scheduled controller has the same equilibrium point as the closed-loop system under the fixed-gain controller.

    - The linearization of the closed-loop system under the gain-scheduled controller and under the fixed-gain controller are equivalent.

4. Check the nonlocal performance of the gain-scheduled controller.

For a given nonlinear system, the GS-MPC technique consists in applying GS to choose a model linearized around some predetermined operating points of the established scheduling variables of the system. Once that the most appropriate linear model to the current conditions of the system is chosen, linear MPC can be applied based on this model. Thus, the nonlinearity of the system can be considered, while taking advantage of the benefits of applying linear MPC.

Consider a nonlinear system described by the following equations

$$\dot{x}(t) = f(x(t), u(t)), \tag{4.29}$$

$$y(t) = h(x(t), u(t)), \tag{4.30}$$

where $f(x, u)$ y $h(x, u)$ are nonlinear functions that describe the dynamic behavior of the system. Let σ be a vector containing the scheduling variables of the system. Once that the operation points for σ are defined, the linear models around these points can be obtained. This generates a set of linear models of the form

$$\dot{x}(t) = A_c(\sigma)x(t) + B_c(\sigma)u(t), \tag{4.31}$$

$$y(t) = C_c(\sigma)x(t) + D_c(\sigma)u(t). \tag{4.32}$$

These linear state space representations are defined by the matrices A_c, B_c, C_c, D_c, whose coefficients depend on the value of σ at every time instant. These models can be easily taken to discrete time, obtaining models of the form

$$x(k+1) = A_d(\sigma)x(k) + B_d(\sigma)u(k), \tag{4.33}$$

$$y(k) = C_d(\sigma)x(k) + D_d(\sigma)u(k). \tag{4.34}$$

For each linear model in the form of (4.33), (4.34) a discrete time linear MPC controller can be designed. This produces as many controllers as the number of operating points of σ, creating a bank or database of linear models and controllers. At this point, it is necessary to define a way of assigning the MPC controllers at each instant of the system operation. One way to do this is to measure (or estimate) the system scheduling variables at each time instant σk, and establishing the closest operating point to the current values σ^*. Once that this point is determined, the controller designed around σ^* can be chosen.

It is necessary to note that, although all the MPC controllers in this technique are designed around a linear model, the resulting control law is not necessarily linear, as it has been shown previously in this Chapter.

4.2.2 Linear-Parameter Varying MPC

As it was previously presented, the first step in a GS design is to derive a linear parameter-dependent description for the nonlinear system. As a result, a family of linear systems parametrized by the scheduling variable vector σ is obtained. In the case of large systems, or systems with very complex dynamics that involve a high number of operating scenarios, the resulting family of linear models and controllers may become too big. For example, in a system with 10 scheduling variables, each of them with 4 relevant operating points, the GS technique will produce 4^{10} linear models. This number of models is way too big, and a real-time controller will probably not be able to manage it.

This feature is a potential drawback of the GS-MPC strategy, and can make this method not suitable for large systems. Other features that may affect the performance of GS controllers include [66]:

- The performance of linearization scheduled systems typically deteriorates as exogenous reference and scheduling signals vary more rapidly.

- Stability properties of the closed-loop system can be guaranteed only in a neighborhood of the equilibrium points and under an assumption of slow variation of signals.

The first of these problems can be solved trough the use of the GS-MPC technique, since at every iteration the controller selects the most appropriate linear model and solves an MPC control problem based on that model. This allows the controller to start all over again in each iteration, and thus eliminate the negative effects of rapidly varying scheduling signals.

However, the second listed issue puts at risk the stability of the system, if not enough operating points are established for the scheduling signals. This leaves to a difficult situation. On one hand, determining too few scheduling variables and /or operating points may cause stability problems in the GS-MPC controller. On the other hand, defining many of these variables and points will generate an unmanageable number of linear models and controllers.

This situation motivates the second nonlinear approach proposed in this Section, denoted as linear parameter-varying MPC (LPV-MPC). This is a modification of the GS-MPC techniques, and relies on the concept of a linear parameter-varying (LPV) system, *i.e.*, a linear system whose dynamics depend on time-varying exogenous parameters whose trajectories are unknown a priori, but can be measured (or estimated) on-line by the controller [67].

Unlike GS-MPC, the LPV-MPC technique does not create a finite number of linear models of the system off-line. Instead, this method generates a linear model of the nonlinear system at each iteration, approximated around the current values of the scheduling variables. For doing that, the controller is required to be able to generate linear models in real-time. This can be done either by knowing and linearizing the system nonlinear mathematical model, or by being able to obtain a linear approximation from real system data. LPV-MPC technique can be summarized in three steps:

1. Measure (or estimate) the current values of the scheduling variables $\sigma(k)$.

2. Generate a linearized model of the system around $\sigma(k)$.

3. Based on the generated linear model, design a linear MPC controller and apply the optimal control law to the system.

This method may require more computational resources from the controller and may take more time at each iteration, but it solves the issues associated with the GS-MPC approach.

The two nonlinear approaches presented in this Section modify the closed control loop shown in Figure 3.2. In the case of nonlinear systems, a new element of the controller is needed to obtain a linear model of the system at each time instant, according to its current conditions. Figure 4.1 shows how this block, denoted as 'linear model generator' can be included in the control loop.

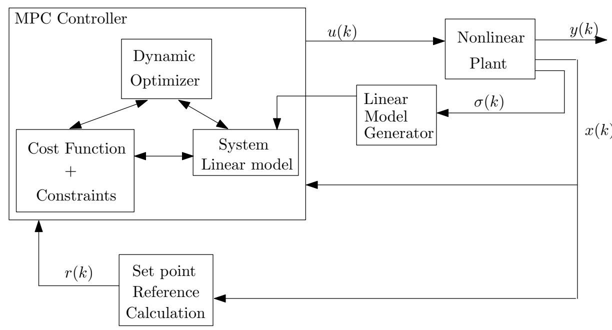
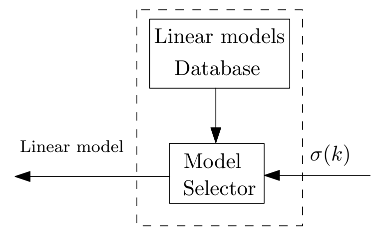
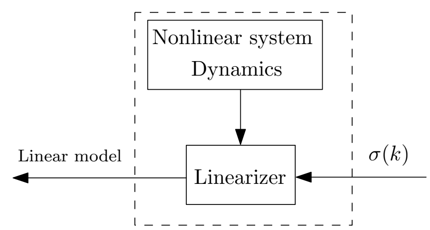


Figure 4.1: Modified closed loop MPC configuration.

However, the linear model generator element has different functions and needs different information, depending on the type of control that will be applied. Figure 4.2 shows the composition of the this element, for the two nonlinear control approaches discussed in this Section.



(a) GS-MPC.



(b) LPV-MPC.

Figure 4.2: Linear model generator for both nonlinear control approaches.

Chapter 5

Case Study: Barcelona sewer system

In order to evaluate their performance, the control strategies presented in Chapter 4 were designed and implemented in the simulation model of a real urban drainage network. This Chapter shows the application of MPC strategies to the model of a part of Barcelona's sewer system. The test system is composed of 1 storage tank, 12 pluviometers located at different places that provide a measurement of rain intensity throughout the network, and 5 control actuators that allow to retain and/or redirect drainage flows at some points of the network (Figure 5.1). Additionally, 11 main channels of the system are modeled as virtual tanks, due to their capacity to retain big amounts of drainage. Further information about the system parameters and the concept of virtual tanks can be found in [5].

In this case, the control objective is to minimize the volume released to the city and to the receiving water body (the Mediterranean Sea) due to overflows. The control actions are, as it has already been said, manipulation of flows through the drainage system. The system disturbances are the rain precipitations over the network, and are measured by the system pluviometers. The state variables to be controlled in this case study are the volumes at each tank (virtual or real), expressed in m^3. Taking all of this into account, the system to be controlled has 5 control inputs, 12 state variables, and 12 measured disturbances.

All the control strategies presented so far need a mathematical model that represents the system dynamics in the best possible way. In this study case such model is not available, but it is possible to obtain a linear mathematical approximation of the system dynamics from its simulation oriented model. For this purpose, the system was stimulated with time-varying inputs with adequate values, and data from both inputs and the produced outputs of the system were recorded. Using this data, a least squares optimization problem was formulated and solved in order to

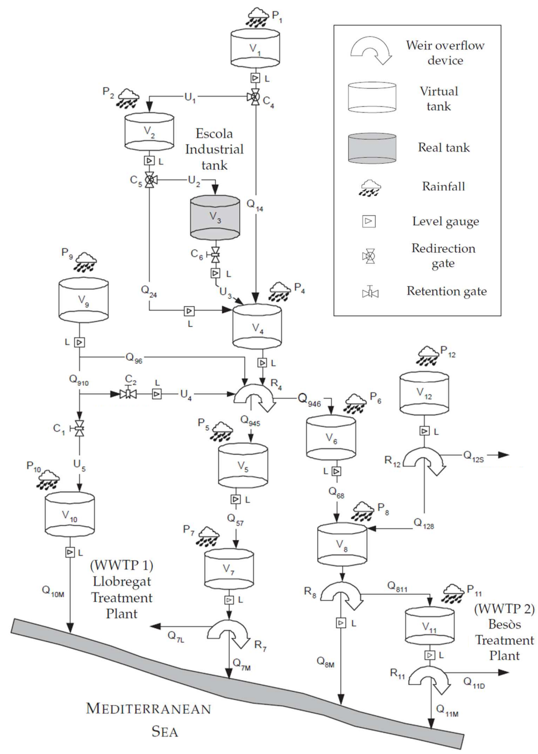
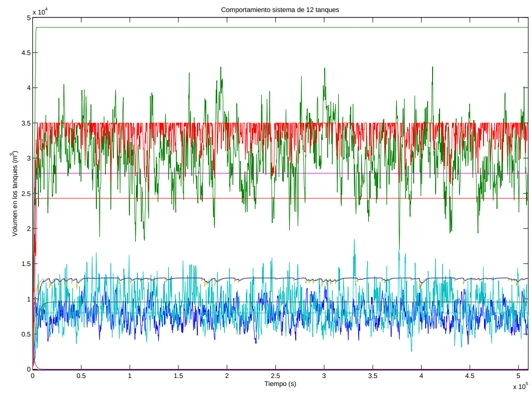


Figure 5.1: Barcelona case study [5].

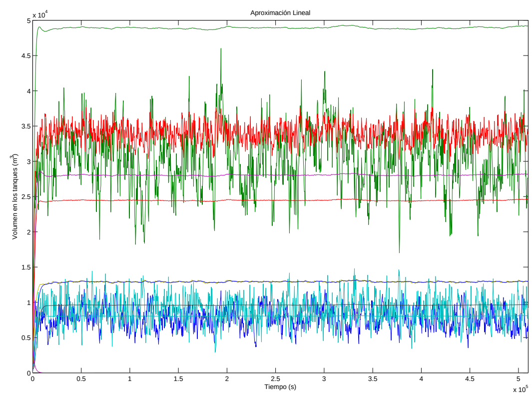
obtain a linear state Equation of the form of (3.1). Figure 5.2 shows the comparison between the system real outputs and the outputs of the approximated linear model, in response to the same set of inputs.

In order to have a better insight of the controllers performance, 9 different scenarios were tested through simulations. In the first place, three different rain events were considered. Two of these events consider the occurrence of a high precipitation peak in the system, while the third case considers two peaks slightly less intense. Figure 5.3 shows the three rain scenarios considered.

Secondly, three different sets of initial conditions (IC) of the system were tested. The first of these initial conditions are the ones that come for default in the simulation model of the drainage system, where the initial volume is very low in the tanks, so the system begins almost empty. The other two IC studied consider



(a) System dynamics.



(b) Approximated linear dynamics.

Figure 5.2: System real and approximated behavior.

the cases in which the initial volumes in each tank constitute the 25% and 50% of the tanks maximum capacity, respectively. Combinations between the different rain events and IC produce the 9 scenarios that were simulated.

The proposed MPC strategies were simulated in all the defined scenarios for the test system. In this case, the control actions produced by the MPC controllers were in turn the set-points for local controllers, located at the actuators.

Figure 5.4 shows the behavior of the sewer network under rain scenario 1. The Figure shows the overflows in the system after a determined amount of time, under different control configurations. The results for MPC and LPV-MPC controllers,

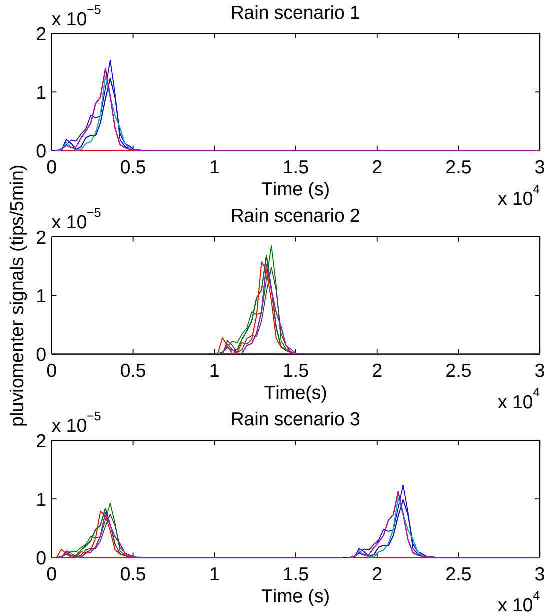


Figure 5.3: Simulated rain scenarios.

both constrained (C) and unconstrained (U), are shown and compared to the case in which there is no control in the system. In this case the GS-MPC technique was not implemented, since the number of linear models required was too big.

Figure 5.5 shows the results for the three rain scenarios considered, when the sewer system starts with 50% of its full capacity. Once again, the overflows in the system under the different controllers used are compared to the case where there is not any control in the network. Complete results of the simulations are presented in Appendix A. Results show the volume overflows in each case without any control, and the reduction achieved by each controller.

Results show that all the implemented MPC controllers achieve significant reductions in the drainage system, compared to the uncontrolled case. This validates the presented strategies as a way of improving the performance of UDS without requiring changes in their infrastructure. It can be seen that in general the constrained controllers present better results than the unconstrained ones. This shows that the ability of considering the system restrictions into account allows to achieve

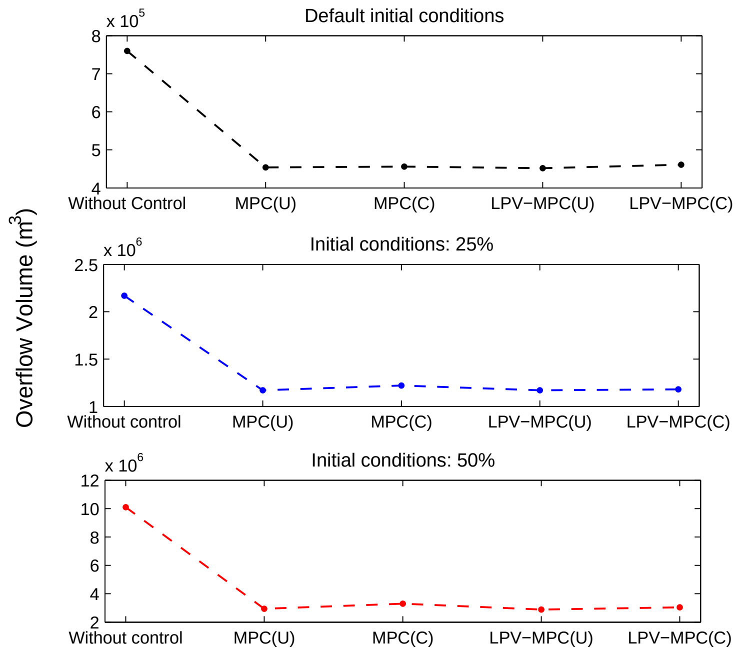


Figure 5.4: Results under rain scenario 1.

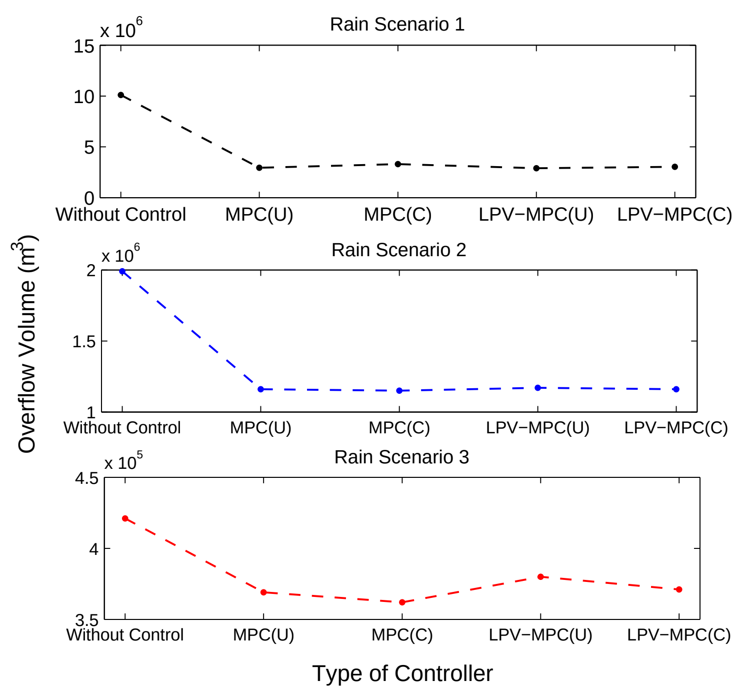


Figure 5.5: Results for all scenarios when initial conditions are 50%.

the control objectives in a better manner.

Finally, results of the LPV-MPC technique do not show significant improvement with respect to the linear MPC controller in this case study. This may be caused by the fact that the simulation model used for the case study already contains linearizations of many of its elements. This reduces considerably the nonlinearity of the system, and thus does not allow to observe many differences in performance between the linear and nonlinear control approaches proposed in Chapter 4. Application of these two approaches in systems with more nonlinear dynamics may enable a better comparison among them.

Chapter 6

Conclusions

A series of MPC techniques have been presented and formulated for its application to UDS. The application of these techniques in the study case has shown positive results, reducing considerably the amount of drainage released to the city streets and to the environment. The results obtained display MPC, and real-time control in general, as valid and helpful alternatives for the efficient and optimal management of UDS.

Regarding the control strategies presented in this dissertation, two approaches have been proposed for controlling nonlinear systems trough linear MPC controllers, showing great potential. Further work in this direction involves the study of conditions that guarantee stability and robustness of the GS-MPC and LPV-MPC techniques. Application of these strategies in distributed and decentralized systems is also a field of great interest.

Finally, a method for obtaining approximated linear models of a nonlinear system using only real and/or simulated data of the system has been used in this work. Research on this field may get to reduce the model-dependence of MPC techniques, making them suitable for a much wider range of applications.

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Appendix A
Study case results

Tables A.1, A.2, and A.3 show the results of the study case under the rain scenarios 1, 2, and 3, respectively. Notation (U) stands for unconstrained controllers, while (C) stands for constrained controllers.

Table A.1: Results under rain scenario 1.

IC	Controller	Overflow volume (m^3)	Reduction (%)
Default	No Control	7,60E+05	-
	MPC (U)	4,54E+05	40,31
	MPC (C)	4,56E+05	40,06
	LPV-MPC (U)	4,52E+05	40,57
	LPV-MPC (C)	4,61E+05	39,33
25%	No Control	2,17E+06	-
	MPC (U)	1,17E+06	46,08
	MPC (C)	1,22E+06	43,67
	LPV-MPC (U)	1,17E+06	46,03
	LPV-MPC (C)	1,18E+06	45,66
50%	No Control	1,01E+07	-
	MPC (U)	2,94E+06	71,00
	MPC (C)	3,30E+06	67,40
	LPV-MPC (U)	2,89E+06	71,45
	LPV-MPC (C)	3,04E+06	69,96

Table A.2: Results under rain scenario 2.

IC	Controller	Overflow volume (m^3)	Reduction (%)
Default	No Control	1,32E+06	-
	MPC (U)	9,87E+05	25,14
	MPC (C)	9,73E+05	26,15
	LPV-MPC (U)	9,85E+05	25,24
	LPV-MPC (C)	9,78E+05	25,80
25%	No Control	1,57E+06	-
	MPC (U)	1,03E+06	34,52
	MPC (C)	1,03E+06	34,59
	LPV-MPC (U)	1,04E+06	33,82
	LPV-MPC(C)	1,03E+06	34,14
50%	No Control	1,99E+06	-
	MPC (U)	1,16E+06	41,65
	MPC (C)	1,15E+06	41,95
	LPV-MPC (U)	1,17E+06	41,15
	LPV-MPC (C)	1,16E+06	41,60

Table A.3: Results under rain scenario 3.

IC	**Controller**	**Overflow volume (m^3)**	**Reduction (%)**
Default	No Control	1,66E+05	-
	MPC (U)	1,50E+05	9,62
	MPC (C)	1,49E+05	10,64
	LPV-MPC (U)	1,61E+05	3,00
	LPV-MPC (C)	1,59E+05	4,75
25%	No Control	2,56E+05	-
	MPC (U)	2,19E+05	14,43
	MPC (C)	2,17E+05	15,25
	LPV-MPC (U)	2,32E+05	9,67
	LPV-MPC (C)	2,28E+05	11,00
50%	No Control	4,21E+05	-
	MPC (U)	3,69E+05	12,43
	MPC (C)	3,62E+05	13,95
	LPV-MPC (U)	3,80E+05	9,65
	LPV-MPC (C)	3,71E+05	11,79