

1 **A Colombian inter-urban transport model: a tool for planning in developing countries**

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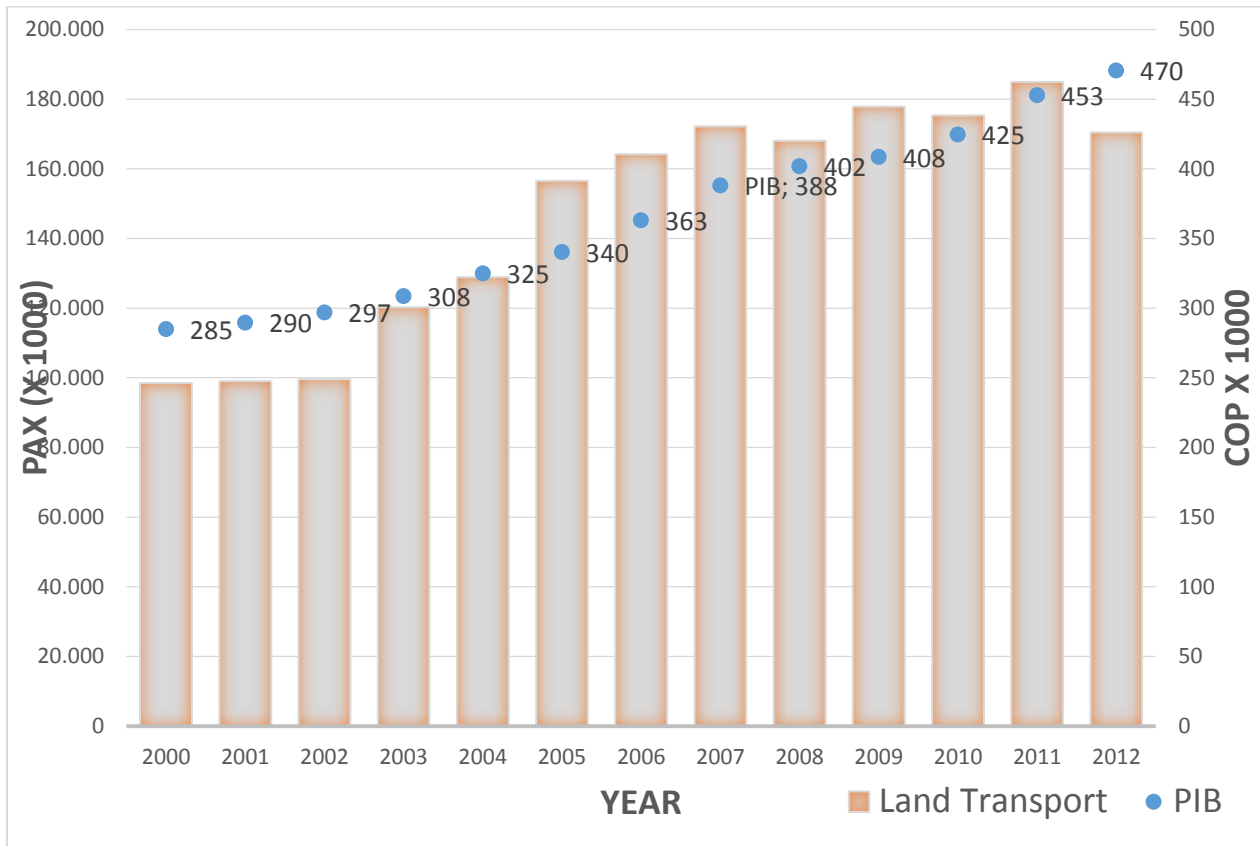
**1 ABSTRACT**

2 Bus transport is the most widely used form of inter-urban passenger transportation in Colombia and in  
3 many other developing countries in which long-distance passenger rail travel is not an option. It is  
4 estimated that Colombia currently transports over half a million passengers per day. In many developing  
5 countries, private operators provide inter-urban bus systems, with limited control being exercised by  
6 governments. In these countries, the contribution that bus travel makes to the volume of inter-urban  
7 transport can usually only be calculated by slow and costly forms of data collection. Thus, governments  
8 often have a limited capacity to determine levels of oversupply or unmet demand. This investigation  
9 develops a gravity transport model for inter-urban trips, the aim of which is to help decision-making in  
10 situations where the available information or resources do not support the use of a traditional four-step  
11 model. Variables such as population, distance and travel time are used to determine levels of travel  
12 between cities. The model is applied to, and balanced and calibrated for, 24 cities in Colombia. It  
13 demonstrates the importance of the relationship between population and infrastructure, which can be  
14 considered to be a key variable in explaining inter-urban travel. The model emphasises that transport  
15 corridors that are likely to increase inter-urban demands as a result of the need for new roads and  
16 identifies the growth of areas in which a possible oversupply could occur. The model could be developed  
17 as an effective planning and management tool for inter-urban travel in Colombia and other countries.

1 **INTRODUCTION AND BACKGROUND**

2 According to data from the Ministry of Transport, in 2012 there were 218 million inter-urban trips  
 3 nationally, of which around 184 involved land based public passenger transport.

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Figure 1. Inter-urban trip evolution VS GDP

9 In Colombia, the Ministry of Transport is responsible for establishing policies and planning inter-urban  
 10 travel. In order to do this, it is necessary to gather data about trips between cities. As in many developing  
 11 countries, the Ministry usually uses intersect surveys, which makes the decisions and evaluation of  
 12 scenarios, such as modification and creating new infrastructure, very inefficient as well as costly.  
 13 Furthermore, in Colombia the majority of the passenger inter-urban service permits were granted more  
 14 than 30 years ago, creating discrepancies between permits (supply) and demand.

15

16 Therefore, in order to use new tools to efficiently (from a resources perspective) various evaluations of  
 17 scenarios of road travel are required. For this investigation, a gravity transport model, based on secondary  
 18 information, was developed. The aim of the model is to support policy decision-making in the areas of  
 19 service planning and modelling future infrastructure scenarios.

20

21 The methodology uses an analogy to a traditional gravitational method in which the size of the bodies (in  
 22 the case of transport, the characteristics of cities such as population, industrialization, etc) determines the  
 23 magnitude of the force between them (number of trips between the cities). After multiple calibration  
 24 methodologies and validation against real secondary data for Colombia, an effective model was

1 developed, which allowed the evaluation of various scenarios. This was the main objective of the  
2 investigation.

3  
4 In the next section a literature review is presented. This provided a theoretical foundation for the model.

### 5 **LITERATURE REVIEW AND THEORETICAL BACKGROUND**

6 The gravity model is usually used as an urban travel distribution model, applying functions of cost  
7 between the areas (4). The case study was implemented in China and was called ‘An urban gravity model  
8 based on cross-correlation function and Fourier analyses of spatio-temporal processes’ (3). It involved  
9 additional spatio-temporal variables in the gravity spectrum between two cities with regard to four  
10 particular cities in the country.

11  
12 The gravity model was based on an analogy with the Universal Law of Gravity which can be presented as  
13 follows: the travel that two towns, A and B, generate between themselves in proportion to the numbers of  
14 their populations and other variables is inversely proportional to the square of the distance (d) between  
15 them. (4).

16

$$V_{AB} = \frac{P_A P_B}{d^2}$$

17

18 Where:

19  $P_A$ : Is the population of A.

20  $P_B$ : Is the population of B.

21  $d^2$ : Is the distance between the cities.

22

23 This interpretation of the model could be referred to as being the simplest since it makes an analogy  
24 between ewton’s law and passenger demand and does not involve additional variables that could  
25 potentially improve the assessment.

26

27 The first rigorous use of the gravity model was conducted by Casey (4), which suggested the following  
28 approximation to synthesize the trips made by sales and its catchments between the cities in a region. In  
29 the simplest formulation the model presents the following formula:

$$T_{ij} = \frac{\alpha P_i P_j}{d_{ij}^2}$$

30

31 Where  $P_i$  and  $P_j$  are the population of the origin and destination cities,  $d_{ij}$  is the distance between i and j,  
32 and  $\alpha$  is a proportionality factor between both towns.

33

34 The model presents variations in the improvement process and thus varies from its initial expression to  
35 include the coefficient n and changes the population from produced (O) and attracted (D).

36

$$V_{ij} = \frac{K O_i D_j}{d_{ij}^n}$$

37 (4), The n value must not be an integer, and according to the author many studies have found that its value  
38 is between 0.6 and 3.5 and that the model was later generalized by considering that the impact of the  
39 distance could be modelled using a digressive function related to the distance or the cost of a trip, as the  
40 following equation shows:

$$T_{ij} = \alpha O_i D_j f(C_{ij})$$

$f(C_{ij})$  is the general function for the cost of the trip in which the calibrating parameters and represents the resistance to travel when the distance or cost is increased, among other factors. The following are the general forms of the equations:

$$f(C_{ij}) = e^{-\beta C_{ij}} : \text{Exponential function}$$

$$f(C_{ij}) = C_{ij}^{-n} : \text{Potential function}$$

$$f(C_{ij}) = C_{ij}^n e^{-\beta C_{ij}} : \text{Combined function}$$

Furthermore, to use the matrix O (origin) – D (destination) the following condition must be met in the following table.

Origin	Destination					$\sum_j T_{ij}$
	1	2	3	...j	...z	
1	$T_{11}$	$T_{12}$	$T_{13}$	... $T_{1j}$	... $T_{1z}$	$O_1$
2	$T_{21}$	$T_{22}$	$T_{23}$	... $T_{2j}$	... $T_{2z}$	$O_2$
3	$T_{31}$	$T_{32}$	$T_{33}$	... $T_{3j}$	... $T_{3z}$	$O_3$
...						
i	$T_{i1}$	$T_{i2}$	$T_{i3}$	... $T_{ij}$	... $T_{iz}$	$O_i$
...						
z	$T_{z1}$	$T_{z2}$	$T_{z3}$	... $T_{zj}$	$T_{zz}$	$O_z$
$\sum_i T_{ij}$	$D_1$	$D_2$	$D_3$	... $D_j$	... $D_z$	$\sum_i T_{ij} = T$

Table 1: Mathematical condition of the gravity model matrix (Source: Modelling Transport (4))

$$\sum_j T_{ij} = O_i$$

$$\sum_i T_{ij} = D_j$$

In order to have a balanced gravity model, the sum of the trips in one row must be the same as the sum of all trips originating in that area and the sum of trips in one column must be equal to the sum of all the trips attracted by the entire universe of cities considered. If both conditions are met, the model is bound by two restrictions. In order for this to happen, factor  $\alpha$  is replaced by two balancing factors  $A_i$  and  $B_j$ , which leads to the following equation:

$$T_{ij} = A_i O_i B_j D_j f(D_{ij})$$

These are the values for the balancing factors mentioned above:

$$A_i = \frac{1}{\sum_j B_j D_j f(C_{ij})}$$

$$B_i = \frac{1}{\sum_i A_i O_i f(C_{ij})}$$

According to this information, the balancing factors depend on one another. The process begins by doing all the  $B_j=1$  and computing the values for  $A_i$  until the process converges.

Because of the use of the application to maximize the entropy, the gravity model assumes that all the micro-conditions can be added as meso or macro conditions. Thus, after mathematical processes have been conducted, the following equation is achieved:

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta c_{ij})$$

The balancing factors for the equation are:

$$A_i = \frac{1}{\sum_j B_j D_j \exp(-\beta c_{ij})} \quad y \quad B_j = \frac{1}{\sum_i A_i O_i \exp(-\beta c_{ij})}$$

The factor  $\beta$  is given to satisfy the equation and gauge the model in relation to the cost of the trip or its generalized cost:

$$C - \sum_{ij} T_{ij} c_{ij} = 0$$

$C_{ij}$ : is the calibrating factor times the cost of trips.

The calibration and validation for the model is conducted in many stages:

- The calibration of parameters  $A_i$  and  $B_j$  so that the model can be bound by two restrictions.
- The calibration of  $\beta$ , which is independent from the other parameters such as population.
- The validation, which is conducted by comparing the results gathered from the previous study or with a defined matrix.

One of the simplest calibration techniques to determine  $\beta$  is to assume an initial value, run the model and iterate until the results obtained in the model are as close as possible to the ones observed (4).

A study in Korea, called Intercity Express Bus Flow in Korea and its Network Analysis (1) applied a gravity model to inter-urban trips related to the behavior of the inter-urban movements of the population. The study was performed in a network of express buses that spreads through 74 cities with 170 routes and is serviced by 6,692 vehicles. After the application of the model it was proven that the inter-urban bus flow was proportional to the square root of the population of town A multiplied by the population of town B. Furthermore, the total flow of the buses in one city is only proportional to its population, and additionally, a considerable influence of infrastructure on inter-urban travel was shown.

In Mexico, an investigation was developed that revealed that the distribution of air travel in the country is appropriately related to the gravity model. The empirical data used to develop the model corresponds to the movements of passengers registered by the Dirección General de Aeronáutica Civil of SCT, in 2007. (2). This may be problematical as it uses Euclidian distance as one of the explanatory variables for the

1 gravitational model and its key findings on air transport were heavily influenced by the intensity of short  
 2 distance traffic, due to the prevalence of short inter-urban trips, as are very common in Colombia.

### 4 METHODOLOGY

6 The methodology used to develop the model is shown in figure 2.

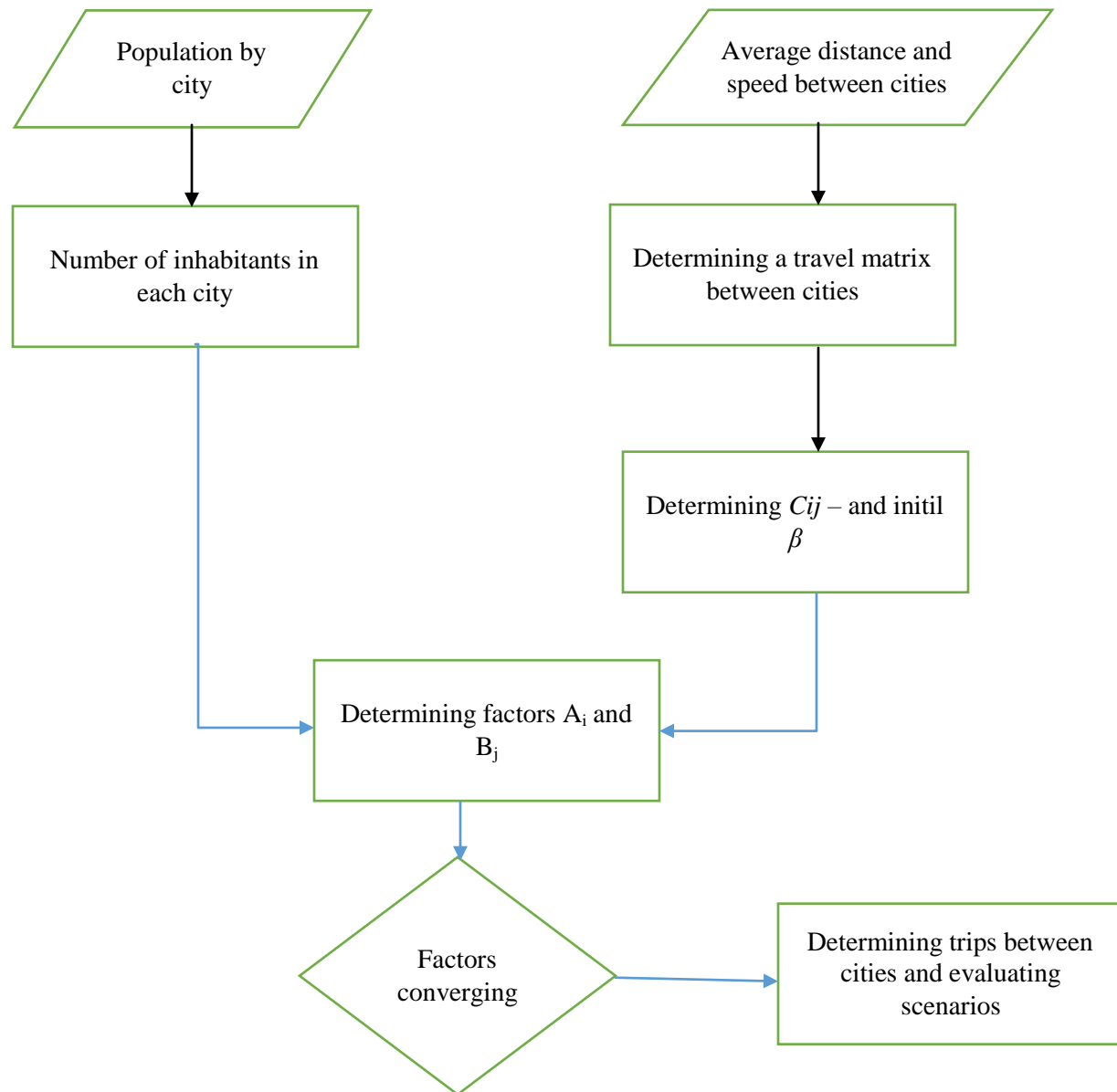


Figure 2. Flowchart

1 In the case of Colombia, data on trips between the 24 main cities is obtained based on the number  
 2 of inhabitants, the distance and time spent travelling. With these population variables the number  
 3 of trips which are generated in each city is determined and this figure is adjusted so that the  
 4 model is bound by two restrictions, as explained in the previous section.

5

6 With information about distances and road speeds, travel times were determined which  
 7 corresponded to the impedance of the trips. Therefore, longer travel times are likely to produce  
 8 less trips between the cities. Once the variables were adjusted, the next step is the calibration and  
 9 application of the gravity model.

10

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## 12 **COLOMBIA: A CASE STUDY**

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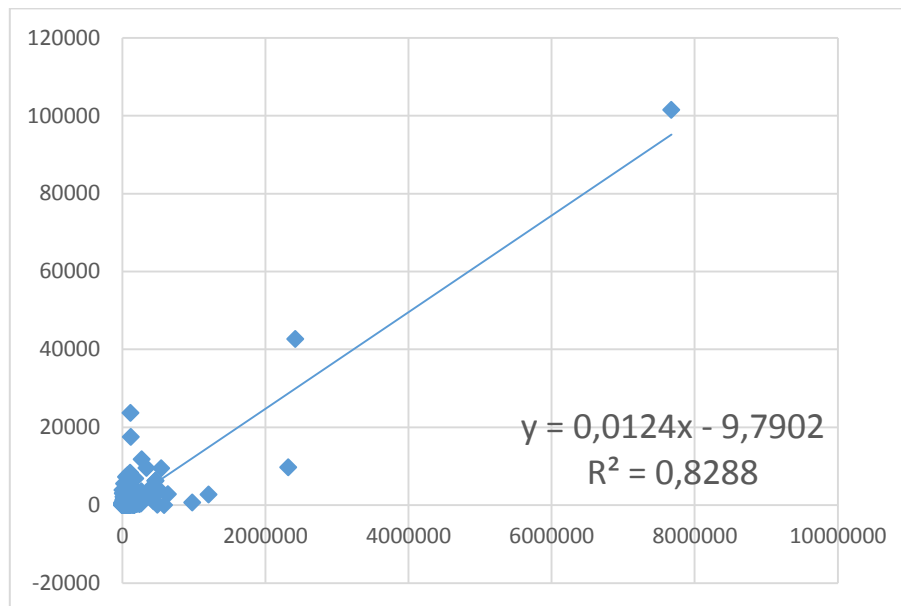
14 The first step in the development of this investigation was the collection of data on population,  
 15 distances and times for travel between cities. This data is now presented, along with its  
 16 corresponding sources:

- 17 • Population database organized by towns and their projections - (5)
- 18 • Distances and speeds between the main cities in Colombia – (6)
- 19 • Origin Destination Matrix developed through observation – (7)

20

21 Initially, standard linear regressions were made for the observed matrix and were set against the  
 22 amount of trips generated and attracted, establishing a correlation with 0.82 and 0.77, which  
 23 validates the model as presented in the next figure.

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Figure 3. Trips Generated vs. population (source: research analysis)

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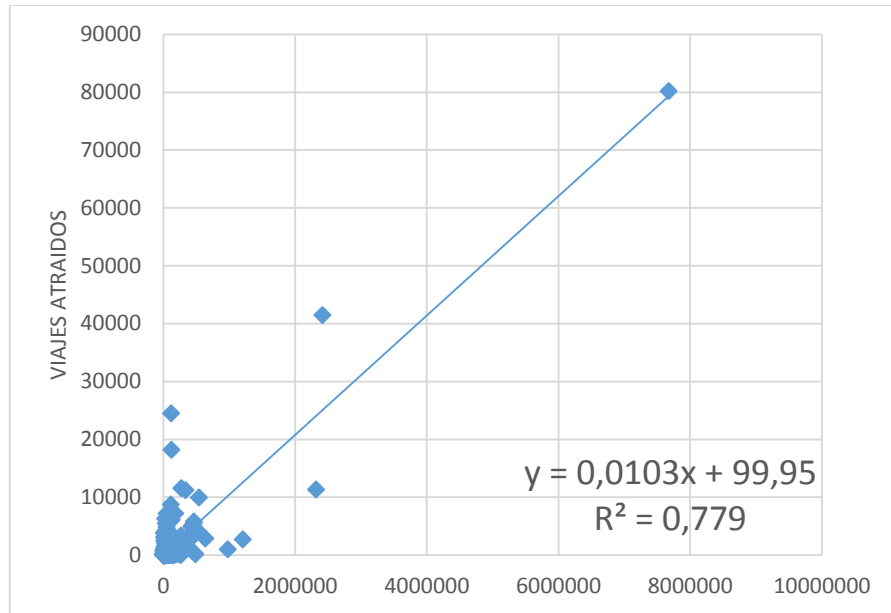


Figure 4. Number of trips vs. population

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Using a population database, we calculated the number of inhabitants in each of the 24 cities involved in the model, as well as the distance and speed needed to travel between them. This was presented in a matrix to simplify its handling.

The model allows for predictions about inter-urban bus travel between the 24 main cities in Colombia and can easily be adjusted to evaluate changing scenarios in which the number of inhabitants in a city vary, along with changes in infrastructure. Additionally, the methodology can be repeated to create adjusted models in regional cases, for instance in departments, or to evaluate city-region behaviour.

Colombia has a total population of 42 million and has over 24 cities with more than a hundred thousand people (5). Considering the need to validate multiple parameters in the model and limitations in the availability of information, only 24 cities were selected for the case study. The next map shows the cities involved.

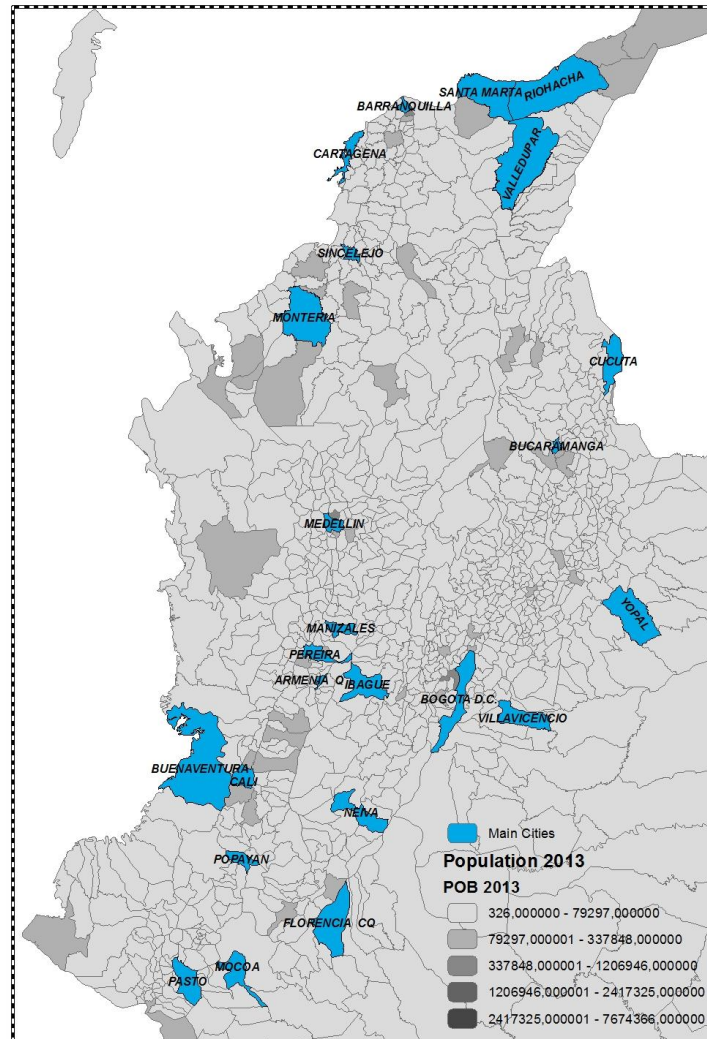


Figure 5. Cities involved

The equation for the gravity model used in this investigation is as follows:

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta c_{ij})$$

Where:

$T_{ij}$ : is the trips between cities

$A_i$  and  $B_j$ : are the balancing factors for the model, which are found for each point of origin  $A_i$  and for each destination  $B_j$ .

$O_i$  y  $D_j$ : is the number of inhabitants of the point of origin and destination town, respectively.

$\beta$ : is the travel time adjusting factor.

$c_{ij}$ : is the travel time between each combination.

The calibration of the model begins by selecting parameters or variables that allow optimisation. As explained in the literature review section, this is done in order to adjust the model in such a way that the results are as close as possible to the observation.

1  
 2 In order to present this, in a model with two restrictions, the values for the balancing parameters are  
 3 determined:  
 4

$$A_i = \frac{1}{\sum_j B_j D_j \exp(-\beta c_{ij})} \quad y \quad B_j = \frac{1}{\sum_i A_i O_i \exp(-\beta c_{ij})}$$

5  
 6  
 7 The **calibration** of the factor  $\beta$  must be independent of the other parameters  $A_i$  and  $B_j$ , using the inverse  
 8 average of the total cost of trips as an initial value in the matrix.  
 9

10 In the case of Colombia the values for each of the parameters of the model, after multiple interactions, are:

- 11  
 12  $A_i$ : from  $2,16 \times 10^{-5}$  to  $4,01 \times 10^{-5}$   
 13  $B_j$ : from  $8,35 \times 10^{-5}$  to  $1,63$   
 14  $\beta$ : 0.0388.  
 15

16 The **validation** was used by comparing the results acquired with the existing ones in the Ministry of  
 17 Transport (7), in that the model contributes to trips in a similar manner to what was discovered in the base  
 18 line. In this way, it was found that for the total number of trips, the difference was significant for specific  
 19 cities such as Cartagena, where the difference distribution of trips had risen by 4%. For the total  
 20 proportion of trips, the model, compared to the validation data, over estimated demand by 1%  
 21  
 22

23 **Scenarios evaluation**

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 25 Once the model was established, a scenario based on modelling the new group or road concession (called  
 26 the fourth generation) for the country (valued at around 23 billion dollars over the next 20 years (reference  
 27 XXXX) was constructed. In this scenario, population growth rates were applied to the cities investigated  
 28 based on data from the National Statistics Department (8).  
 29

30 Specifically, fourth generation concessions that reduce travel time by 30% were found in the following  
 31 routes:  
 32

Origin	Destination
CALI	CARTAGENA
MEDELLIN	CARTAGENA
MADELLIN	CALI
BOGOTA	CARTAGENA
BOGOTA	BUENAVENTURA
BOGOTA	CALI
BOGOTA	MEDELLIN

33 Table No. 2: Pairs O-D involved in 4G  
 34

35 Finally, after determining the trips, a comparison of supply routes for inter-urban travel was made and it  
 36 was discovered that, because of population growth and changes in infrastructure, it is essential to make  
 37 important readjustments to the main supply due to the fact that these cities will be more attractive for

1 travel and the gravitation between them will increase. The following matrix shows differences found for  
 2 each origin-destination pair.

ORIGEN	DESTINATION	4G	Sin 4g	Increasing
CALI	CARTAGENA	181	137	32%
MEDELLIN	CARTAGENA	263	241	9%
MEDELLIN	CALI	593	549	8%
BOGOTA	CARTAGENA	633	516	23%
BOGOTA	BUENAVENTURA	469	395	19%
BOGOTA	CALI	1502	1347	12%
BOGOTA	MEDELLIN	1589	1451	10%

6 Table No. 3: Increasing trips in 4g roads matrix of differences scenarios

## 8 CONCLUSIONS

9 The model was applied, balanced and calibrated for 24 cities in Colombia in order to estimate the impact  
 10 on demand of a new generation of large highway concessions released by the Colombian government.  
 11 The results show the relationship between population and infrastructure as being the key variable in  
 12 explaining the nature of inter-urban travel.

13  
 14 In particular, the model highlighted the fact that transport corridors are likely to increase inter-urban  
 15 demand, due to the existence of new roads as well as areas in where the government would need to  
 16 manage a possible oversupply.

17  
 18 The development of the model revealed that, in the case of Colombia, there is a strong correlation  
 19 between the number of inhabitants in a city and the trips that it generates or attracts. Also, it is possible to  
 20 establish an easy to adjust gravity model to evaluate different changes in the population and the  
 21 infrastructure.

22  
 23 The important influence that improvements in infrastructure can have in trips is also focused on in this  
 24 study. These improvements should be complemented with enhancements to other transport related  
 25 services such as bus terminals and intermediate stops. In relation to transport planning, the model appears  
 26 to be a useful tool that is inexpensive, fast and easy to use, particularly for governments in developing  
 27 countries. The model could be developed further in order for it to become a very useful planning and  
 28 management tool for inter-urban travel in Colombia and other countries

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