A Receding Horizon Approach to Mean Field Games

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Abstract

Urban drainage systems (UDSs) are complex large-scale systems that carry stormwater and wastewater throughout urban areas. During heavy rain scenarios, UDSs are not able to handle the amount of extra water that enters the network and flooding occurs. Usually, this might happen because the network is not being used efficiently, i.e., some structures remain underused while many others are overused. This paper proposes a control methology based on mean field game theory and model predictive control that aims to efficiently use the existing network elements in order to minimize overflows and properly manage the water resource. The proposed controller is tested on a UDS located in the city of Barcelona, Spain, and is compared with a centralized MPC achieving similar results in terms of flooding minimization and wastewater treatement plant usage, but only using local information on non-centralized controllers and using less computation times.

Contents

Chapter 1

Introduction

Urban drainage systems (UDS) are complex networks of interconnected pipes and nodes that carry stormwater and wasterwater to wastewater treatment plants, which treats it and sends it to the enviroment [4]. In many cases, the design of these networks ends up being underdimensioned, because of rapid urbanization in cities and climate change scenarios, not being taken into account in early stages of the design process [24]. For that reason, heavy flooding may appear in urban areas, and some serious sanitary problems may occur due to the unproper management of wastewater that comes out of the network into street level [4]. Some of the adressed solutions to that problem seek to do a restructuration to the hydraulic design of the network by adding storage elements throughout the system, so that the overflows are totally avoided [20]. Even though these solutions are quite effective, they are extremely expensive in both time and money.

The problem above could be solved, without having many large modifications being performed to the general design of the network, by using real-time control (RTC) techniques [11]. These techniques seek to find a way to properly manage the active elements of the network, e.g., retention and redirection gates, in order to achieve an efficient management of the wastewater, and thus, assuring a minimization of overflows that may appear. Optimization-based control techniques have been the most widely used techniques in the literature to solve the problem of minimization of overflows in UDS. For instance, model predictive control (MPC) has been widely used to solve the problem [6, 23], due to its flexibility in the selection of performance functions, constraints, and its multiple-inputs multiple-outputs capabilities [17]. However, many of the proposed techniques are based upon centralized schemes for the determination of control actions to be perfomed, which could derive into heavy computational burden problems [18] and cyber-security-related problems [5].

In order to deal with some of the computational burden problems, aggregated models of the UDSs, e.g., the so-called virtual-tank (VT) model, are used to reduced the number of states of the system and in term, the size of the optimization problems. This approaches are quite effective for many system, but deliver poor results when the network is heavly interconnected, i.e., when there is a large amount of connections among the pipes and nodes [14]. For that matter, there has been an incresed interest in techniques that do not use aggregated models of the system, such as the one proposed in [14], where each element of the system is taken into consideration, without sacrificing computational time due to its linearity. Nonetheless, it is possible to encounter quite complex networks that required partitioning and decentralization, in order to guarantee suitable computation times for real-time applications.

For that reason, there has been an increased interest in studying distributed control techniques [7]. For instance, [1] proposes a distributed control methodology based on population dynamics, that achieves an efficient use of the network and guarantees a minimization of flooding. However, on that methodology, local controllers are not able to consider proper cost functions, which can be problematic if there are multiple control goals such as, moving wastewater between a wastewater treatment plant (WWTP) and out of the network, while efficiently using the network and minimizing flooding. Moreover, the technique requires an aggregated model, which could derive into poor results, as stated before.

This paper proposes a technique that aims to solve problems related to distributed information on local controllers, as well as problems related to the aggregation of large portions of the netwoks into single variables, by using a game-theoretic approach (i.e., dynamic games) combined with a hybrid linear delayed (HLD)-based MPC. Differential game (DG) theory [2] gives a natural extension of optimal control to scenarios with multiple controllers that are optimizing its own performance criteria [19], and thus its framework is well suited for optimization-based non-centralized control applications. This type of games have been used in the literature to solve problems related to the formation control of mobile robots [12], problems related to demand response in power grids [10], and the control of surge tanks [9]. This is due to the fact that DGs have the ability to consider multiple cost functions as well as non-centralized information on distributed controllers. As for the UDS control problem, it has been reported that these networks can be seen as partitioned systems that are being controlled by multiple local agents that interact with each other [1]. Hence, it is a suitable idea to apply the DGs framework to the control of UDS, where multiple local controllers act as players of a game where they interact with each other, in order to guarantee a proper operation of the network in terms of wastewater management.

It is quite important to point out that, even though DGs are quite useful for many non-centralized control application, they generally fail to succeed when the number of sub-systems, i.e., the number of agents, is large, because in order to compute the solution to a DG, it is required to solve a set coupuled partial differential equations (one per each agent in the game). Nonetheless, a novel tool called the mean field games (MFGs) [15, 13] allow to solve large scale DGs in which the number of agents tend to infinity. Hence, it still is suitable idea to use DG to solve non-centralized control problems.

The main contribution of this paper is the design of a non-centralized control methodology based on a large scale DGs, i.e., a MFG, in the same spirit as in [22, 3], which seeks to determine the optimal behavior of each active element of the UDS by using a consensus-like algorithm, only using local information of the network. The proposed methodology has the advantages of optimization-based techniques used for the control of UDS, e.g., MPC, as well as the ability to have distributed information on the controllers. Moreover, since only local information is used, less data is needed, and thus less computational resources are involved in the computation of the control inputs. The proposed methodology is flexible enough that it allows to combine game-theoretic approaches with more traditional approaches, such as the MPC.

The remainder of this paper is organized as follows. Section II presents the relationship that exists between the dynamic games and the UDSs. Therein, the concepts behind the agents and the enviroment are presented, as well as the different cost functions that the agents might minimize. Section III states the main problem to be solve in this paper, as well as the required tools for a real-time implementation of the solution. Section IV presentes the case study in which the approach is tested. It shows an UDS found in the city of Barcelona, Spain, called the Riera Blanca network. Section V presents the main results obtained using the proposed approach, as well as a comparison between the proposed scheme and a more traditional tool for solving the problem.

Finally, Section VI collets all the conclusions.

Chapter 2

Dynamic Games and UDS

Typically, UDS have a strong convergence topology where many pipes end up into a common outlet node, until the drain node is reached. This causes most of the burden to be suffered downstream and quite little burden to be taken upstream of the network. This means that, usually, upstream pipes remain underused, so they could retain some water (by using retention gates) in order to minimize overflows downstream. The proposed scheme aims to solve that problem, by using a decentralized controller based on dynamic games, so that most of the pipes on the UDS are used efficiently, and the total overflow is minimized. Moreover, the proposed scheme seeks an integration with state-of-the-art techniques, e.g., MPC, to deliver more suitable results when other objectives are required, e.g., the minimization of combined sewer overflow (CSO).

2.1 The Dynamic Game Definition

According to [2], in order to propertly describe a dynamic game theory problem, it is required to state what an agent is and what his actuation mechanisms are for the selected application, in order to propertly described how the strategy can be applied. Moreover, it is required to state how the different agents in the game are reasoning, i.e., what function they are trying to optimize, as well as how the overall game is evolving, i.e., the system dynamics. It is important to point out that this definitions are not unique and depend heavily on the proposed scheme.

The first required definition is the concept of an agent. Agents are local controllers that are able to manipulate the physical actuation mechanisms of the UDS. For instance, an agent might be a local controller capable of changing the inflow to a particular sewer pipe or storage unit. This means that in general, the agents are the responsibles for the selection of the appropiate control signals in order to guarantee a suitable operation of the system. This leads to the second required definition: the reasoning of the agents. As has been state before, the reasoning of the agents are measuring functions that depend on the actions of the agents, and determine their performance in the game. For this paper, the main goal of the agents is to distribute evenly the rainwater that enters into the network during a heavy rain event, so that no part of the system is prone to overflow due to the overuse of the capacity of the pipes and storage units. This reasoning can be capture by the minimization of the following cost function

$$J_i = \int_0^T (x_i(t) - \phi(\mathbf{x}(t)))^2 + r u_i(t)^2 dt, \quad \forall i \in \{1, 2, \ldots, N\}, \tag{2.1}$$

where x_i is the state of the system that the i-th agent is able to manipulate, $\phi(\mathbf{x})$ is a Lipschitz function of all the states of the system, u_i is the action that the i-th agent is taking, N is the total number of agents, T is a time horizon, and r is a weight parameter. It is important to point out that a single agent might be able to change multiple states of the system; if that is case, cost function would have a quadratic form of the states, instead of a simple substraction. Cost function (2.1) express the desire of each agent to change the states that his capable to manipulate, so that they become as close as possible to some function of the states system. Notice that having the function $\phi(\mathbf{x})$ is quite flexible in the sense that it allows to express the desire of a particular agent to modify the volume of a cluster of sewer pipes that are under his control, so that it become as equal as possible to the volume of some other cluster of pipes.

Finally, the third require definition is the dynamics of the game, which ultimately determine how the agents interact with each other and with the system. This definition is quite simple, given that the game dynamics derive from mass balance equations. This causes the state equations of the different sewer pipes to be linear with respect to the inflows and outflows to themself. The general evolution of the game is given by (this equation is going to be explined in more detail in the forthcoming sections)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \sum_{i=1}^{N} \mathbf{B}_i u_i(t), \tag{2.2}$$

where **A** and **B** are matrices of proper dimensions. Notice that (2.2) is only able to give information about the mass balance of the UDS, since is not a hybrid state equation. It does not capture other phenomena such as the switching flow in weirs. Nonetheless, for the proposed control strategy, that is the only information that is going to be needed, because the strategies u_i are based on volumes exclusively, as it is going to be presented in the forthcoming sections.

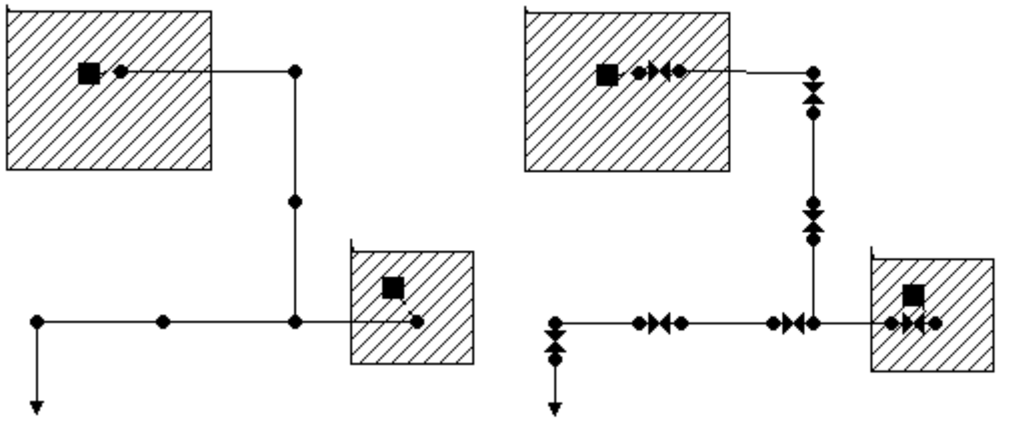

Figure 2.1: After the model extention is performed to the system, each sewer pipe has a retention gate at its entrance. This causes all the inflows to sewer pipes to be controllable.

2.2 Model Extention

Typically, a UDS has a small amount of active elements, i.e., retention gates and redirection gates, which derives in a very limited controllability of the flows that run through each sewer pipe. This may cause problems if a control strategy requires full controllability of all the states, and may lead to poor performances. For that matter, it is first assumed that there is a retention gate at the entrance of each sewer pipe in the network. This means that the inflow to each sewer pipe is completely controlable, and thus, there is going to be an agent of the game associated with that retention gate. This may seem like an unreasonable assumtion, so it will be dropped latter on, as is only required for design purposes. Figure 2.1 illustrates how a simple network with six sewer pipes and two catchments is extended after the assumption has taken place. Notice that it would now be possible to control all flows running through the system.

Given this assumption, it is now know that the evolution on the game can be express in terms of the evolution of the volume of water inside each sewer pipe, because it is possible to relate one agent with each sewer pipe. Moreover, it is now know that each agent is not only associated with a fictional retention gate, but with a volume of a pipe as well. Hence, it can be said that the dynamics of each agent of the game are given by

$$\dot{v}_i(t) = u_i - q_i^{out}, \tag{2.3}$$

where v_i is the state of the i-th agent, i.e., the volume stored in the i-th pipe, u_i is the action of the i-th agent, i.e., the controlled inflow to the i-th pipe, and q_i^{out} is the total outflow of the i-th pipe. It should be noted that there are some constraints in u_i, since the maximum inflow to a particular pipe cannot be greater that the total outflow from pipes whose outputs are directly connected to the i-th pipe, plus the rainwater entering to the network via i-th link. As been stated by [14], the outflow of a given sewer pipe can be expressed as a function of its inflow in a delayed time period, and thus, the state equation (2.3) can be simplified into an equation that only depends on the action of the agent itself.

It is interesting to point out that this model extention is a direct opossite of a model aggregation such as the virtual tank model [23], where many states of the system are associated to a single control variable, instead of adding a control variable for each state in the system. This relationship is useful, because it allows to use the proposed scheme for some aggregated representations of a UDS.

2.3 Differential Game Problem Statement

Consider a set of N agents playing a DG, where the evolution of each state of the system is given by (2.3), and each agent is trying to minimize

$$J_i = \int_0^T (v_i(t) - \gamma(\frac{1}{N}\sum_{j=1}^{N} v_j(t) + \eta))^2 + r u_i(t)^2 dt, \qquad \forall i \in \{1, 2, \ldots, N\}, \tag{2.4}$$

where γ and η are known parammeters. The main problem is to find a set of actions $\{u_1^*, u_2^*, \ldots, u_i^*, \ldots, u_N^*\}$, i.e., inflows to sewer pipes, such that the following inequalitites

$$J_i(u_1^*, u_2^*, \ldots, u_i^*, \ldots, u_N^*) \leq J_i(u_1^*, u_2^*, \ldots, u_i, \ldots, u_N^*), \tag{2.5}$$

are simultaneously satisfied $\forall i \in \{1, 2, \ldots, N\}$. In other words, the problem is to find a Nash equilibrium (NE) for the proposed DG [21, 2, 8].

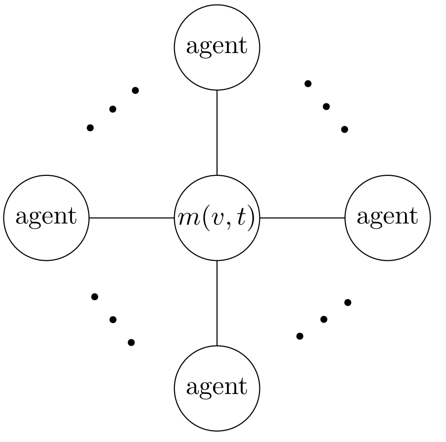

Figure 2.2: Non-centralized model of the MFG where agents have available the information of the distribution.

2.4 MFG Problem Statement

A typical UDS may have hundreds or even thousands of interconnected sewer pipes [14, 4]. This causes the DG presented in the previous section to be large scale in nature, since there is a large amount of both states and actions. For that matter, the solution to the game becomes untractable, and novel tools are required in order to find the NE. Thus, a MFG is proposed as a relief to large scale problem in the DG. Following the basic MFG descriptions from [15] and [13], it is assumed that the volume of water stored in the sewer pipes, i.e., the states of the game, behave as a random variable with probability distribution $m(v,t)$ at time t. This simply means that all the different volumes are condensed in a single variable. Finally, it is assumed that agents have available the probability distribution of the volumes, i.e., $m(v,t)$. It can be said that a MFG has a non-centralized information patter, in which all agents interact inderectly using the distribution of all the system. This information patter is illustrated in Figure 2.2.

For the MFG, the actions of the agents are based upon the information of the probability distribution of the volumes instead of the information from the actual volumes. This mean that a particular agent is more interested in the proportion of volumes that are in a particular level, rather than the real volumes. Hence, the agents in the MFG game are trying to minimize

$$J_i^{MFG} = \int_0^T (v_i(t) - \gamma(\bar{m}(t) + \eta))^2 + r u_i^2 dt, \qquad \forall i \in \{1, 2, \ldots, N\}, \tag{2.6}$$

where $\bar{m}(t) = \int_{\mathbf{R}} v m(v,t) dv$ is the mean of the probability distribution of the volumes. Notice that Equations (2.1) and (2.6) are identical if $N \to \infty$ in the differential game [15].

Thus, the main problem is as follows: consider a MFG with N agents, where the dynamics of a single agent are given by (2.3), and each agent is trying to minimize a cost functional such as (2.6); the the problem is to find a set of actions

$\{u_1^*, u_2^*, \dots, u_i^*, \dots, u_N^*\}$ such that the following inequalitites

$$J_i^{MFG}(u_1^*, u_2^*, \dots, u_i^*, \dots, u_N^*) \leq \\ J_i^{MFG}(u_1^*, u_2^*, \dots, u_i, \dots, u_N^*), \tag{2.7}$$

are simultaneously satisfied $\forall i \in \{1, 2, \dots, N\}$. In other words, the problem is to find a mean field NE of the game [13, 22].

In general, the MFG problem formulation is more suitable for typical UDS because they are large scale in nature. However, the DG problem formulation is useful when there is some kind of model aggregation in the representation and the number of states is small.

2.5 The Multi-population MFG definition

Up to now, the control problem have been focused around the minimization of overflows in the UDS. Although that objective is arguably the most important one, the controller of an UDS might have some other objectives, such as the transport of sewage to a WWTP. If one wanted to considere multiple objective in the cost functional (2.6), the solution would become hard to compute, due to the extra complexity that the multiobjective optimization problem brings. A more convenient idea is to solve the multiobjective MFG by means of a multipopulation MFG (MP-MFG) [3]. For this approach, each population in the game has a distinct goal, e.g., one population might focus on the even water distribution, while other might focus on the transport of the sewage to a WWTP. Even though it is possible to considere as many populations (and in term, as many objectives) as needed, for this thesis only two goals are considered: the minimization of overflows and the maximization of WWTP usage; therefore, only two populations are required.

In a MP-MFG, the decisions of an agent from a particular population are based upon the shared information from its neighbor populations according to a known information graph. This scheme is quite flexible since a given population is not interacting directly with any agent from other populations, but rather with the populations as groups. This means that for this scheme, a single population does not care whether or not a MFG is occuring in the neighbor populations, because agents in the population only require specific information from the neighbors regardless of how it was determined. Thus, it is not required to have a MFG system on each node on the graph, which is convinient if one wants to use different control strategies combined. Figure 2.3 shows a possible configuration of 4 different MFG taking place simultaneously. In this configuration, each game shares information with its neighbors in order to guarantee a certain operation, as in the case of the UDS.

Consider a MFG as the one previously presented. In this approach, all agents are naturally seeking an even volume on all the pipes of the network. This water is stored, and in order to safetly evacuate it, it should be transported into a WWTP so that no recieving enviroment is damage. The task of transporting the water to a WWTP is performed by a MPC based on the hyrbid linear delayed (HLD) model, whose objecive is the maximization of WWTP inflow and minimization of combined sewer overflow (CSO) [14]. Given that the MP-MFG approach does not require the interconnection of multiple MFG schemes, this MPC approach can be used comined with the MFG to minimize the overflows, so that both control objectives are achieved.

Nonetheless, a coupuling term is required to guarantee a suitable interaction between the two strategies. This coupuling interaction is described as follows:

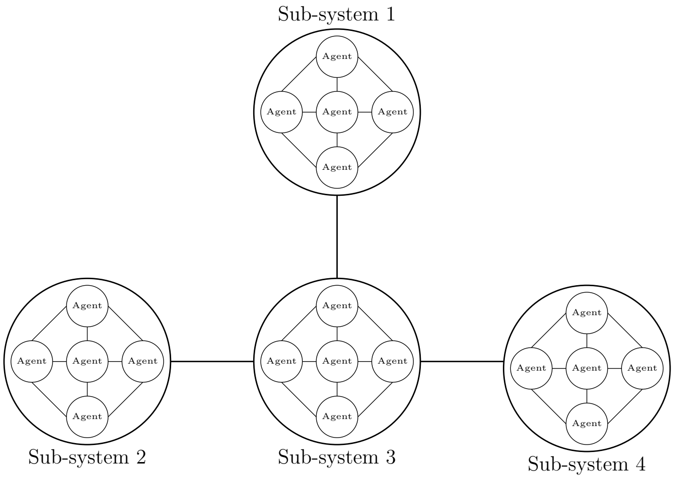

Figure 2.3: Example of a possible configuration of multiple networks interconnected and sharing information.

- The agents in the MFG seek an agreement in the stored volume of water inside the sewer pipes.
- Once the best possible agreement has been reached, all agents as a group follow a certain storage element state of the MPC part, e.g., a collector volume or a tank volume.
- This will naturally decrease the volume stored in the pipes and increse the volume stored in the MPC part.
- While the is sewage stored in the MPC portion, the MPC controller transports the water safetly to the WWTP.

Note that the second step in the interaction above sends water to the MPC section only when it is able to process it.

This interactions can be capture by the following cost fuction

$$J_i^{MP-MFG} = \int_0^T (v_i - \frac{(w_1\bar{m} + w_2 v_{MPC})}{2})^2 + r(q_{in}^i)^2 \, dt, \tag{2.8}$$

where $v_{MPC} \in \mathbb{R}$ is the storage variable from the MPC portion, and $w_1, w_2 \in \mathbb{R}$ are tuning parameters. Notice that this function explicitly states that the agents from the MFG are tracking two variables, one asociated with local interactions, and one with global interactions. Notice that since v_{MPC} is a given value to the MFG, cost functions (2.6) and (2.8) have the same structure.

Chapter 3

Problem Statement

Consider a MP-MFG with two populations, where the agents of the first population evolve according to (2.3) and each agent is minimizing the cost functional (2.8). The second population is a MPC-based controller where the system evolves according to a HLD as

$$\sum_{i=0}^{T} M_i X(t-i) = m(t), \tag{3.1}$$

$$\sum_{i=0}^{T} N_i X(t-i) \leq n(t), \tag{3.2}$$

where the definitions of all the parameters are given in [14], and the controller is trying to maximize the usage of a WWTP (which is a state represented in the vector X) over a known prediction horizon. The problem is to find a set of inflows to the pipes of the MFG population $\{u_1^*, u_2^*, \ldots, u_N^*\}$ such that the following inequalities

$$\begin{aligned} J_i^{MP-MFG}&(u_1^*, u_2^*, \ldots, u_i^*, \ldots, u_N^*) \leq \\ &J_i^{MP-MFG}(u_1^*, u_2^*, \ldots, u_i, \ldots, u_N^*), \end{aligned} \tag{3.3}$$

are simultaneously satisfied, while at the same time, the flow through retention and redirection gates in the MPC-based population are maximazing the usage of a WWTP.

3.1 Controller Implementation

The MP-MFG problem formulation presented previously is stated as a continuous time problem. However, the HLD-based MPC used for the management of the WWTP usage (as presented in [14]) is formulated in discrete time. This means that some modification must be performed to either of the approaches to have a succesful coupuling between the two. Since the MPC approach requires an online optimization on each sampling time, a continuous time approach is not convenient. Hence, a discrete time approach to the MFG is proposed, so that a proper coupuling can be achieved. The discretization of the approach is based on the formulation presented in [22], where the MFG is presented as a set of coupuled differential equations, instead of the canonical coupuled non-linear partial differential equations from [15], i.e., the Hamilton-Jacobi-Bellman Fokker-Planck-Kolmogorov system. This simplification is possible due to the linear-quadractic (LQ) nature of the game, where the state equation of the agents is linear and the cost function is quadratic.

For the sake of simplicity, the explicit solution of the game is first enunciated. Following the definitions from [22], it is possible to write an explicit solution to the MP-MFG using traditional tools. This solutions require the implementation of a Riccati equation coupuled with an auxilary equation, to compensate for the mean field effect. Notice that this approach if no different from a traditional LQ tracker [16]. It is convenient to rewrite the optimization problem for each agent with some auxilary variables.

$$\min_{q_{in}^i} \int_0^\infty e^{-\rho t}\left[(v_i - g(\bar{m}))^2 + r(q_{in}^i)^2\right] dt, \tag{3.4}$$

$$\textit{subject to}$$

$$\dot{v}_i(t) = q_{in}^i, \tag{3.5}$$

where $g(\bar{m}) = \frac{(w_1\bar{m} + w_2 v_{MPC})}{2}$, and the exponential is to accomodate for the infinite time horizon. As has been state in [22], the previous optimal control problem can be solved by means of the following equations

$$q_{in}^i = -\frac{1}{r}(pv_i + s) \tag{3.6}$$

$$p^2 + r\rho p - r = 0 \tag{3.7}$$

$$\dot{s}(t) = (\rho + \frac{p}{r})s(t) + g(\bar{m}) \tag{3.8}$$

$$\dot{\bar{m}}(t) = -\frac{1}{r}(p\,\bar{m}(t) + s) \tag{3.9}$$

where Equation (3.6) is the control law, Equation (3.7) and (3.8) are the Riccati equation and the auxilary equation, and Equation (3.9) is the equation that determines the evolution of the mean field. Equation (3.9) is determined by averaging the state equation of each agent, after applying the control law (3.6). It is important to point out that this solution is consistent with the scheme presented in [15], since Equations (3.7) and (3.8) represent the HJB equation, and Equation (3.9) represent the FPK equation.

Given that the previous system of equations is in fact a set of ordinary differential equations, it can be discretizied by means of any discretization tool. For this paper, the system is discretizied by means of an Euler forward approximation, using a Δt equal to the sampling time from the HLD-based MPC. Therefore, the implementation of the control scheme requires the solution to the following set of equations

$$v_i^+ = v_i + \Delta t q_{in}^i \tag{3.10}$$

$$s^+ = s + \Delta t\big[(\rho + \frac{p}{r})s + g(\bar{m})\big] \tag{3.11}$$

$$\bar{m}^+ = \bar{m} + \Delta t\big[-\frac{1}{r}(p\,\bar{m} + s)\big] \tag{3.12}$$

where now every variable is in discrete time, and $v_i^+ = v_i(k+1)\ \forall k \in \mathbb{Z}$. Now the MFG has been discretizied, and it is possible to implement it side-by-side with the MPC approach, where v_{MPC} is a simple constant to the MFG.

Chapter 4

Case Study

The proposed scheme is applied into the Riera Blanca network in the city of Barcelona, Spain. The network is shown in Figure 4.1. This is a typical UDS that drains into the Mediterranean sea and a WWTP located downstream of the network. As many UDSs, this network is a collection of several elements such as pipes, tanks, and weirs, that carry the sewage throughout the city. Table 4.1 shows a summary of all the major elements found in this system. As with most UDSs, this system has a quite strong convergence topology, in which the whole system ultimately converges to a single big sewer pipe. This sewer pipes is a large controllable collector that spans over 1.5km and has a quite little slope, causing it to be a suitable storage element. Following the controllable collector downstream, there are the two outlets of the network: the WWTP and the Mediterranean sea. This WWTP has a maximum capacity of 2 m^3/s, causing any inflow greater than this value to become CSO automatically.

Table 4.1: Parameters of the simplified model

Element	**Quantity**
Tank	2
Pipe	145
Weir	3
Gate	10
Overflow	11
Collector	1
Rain Inflow	68

The proposed approach utilizes a partition of the system in which one portion is performing the flooding minimization task and the other one is managing the WWTP uses. Given that the outlets of the network are located after the collector and it is able to act as a storage element, it is a suitable idea to divide the network at that point, having both outlets and the collector in the same partition. The selected partitioning allows to decentralize the two objective, as has been proposed before in this paper. Figure 4.2 shows the two partitions where the MFG portion is performing the flooding minimization task, while the MPC portion is performing the WWTP usage task. Notice that for this approach, the selected v_{MPC} from Equation (2.8) is the volume from the big sewer pipe. This means that the MFG portion only sends water as long as there is available space inside the big sewer pipe. As for the controller implementation, the

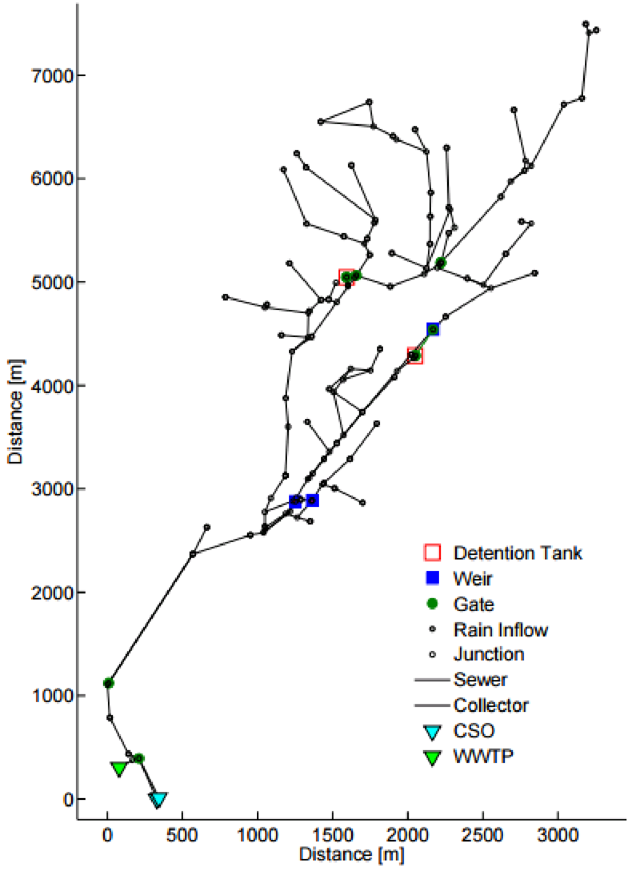

Figure 4.1: Riera Blanca network, Barcelona, Spain.

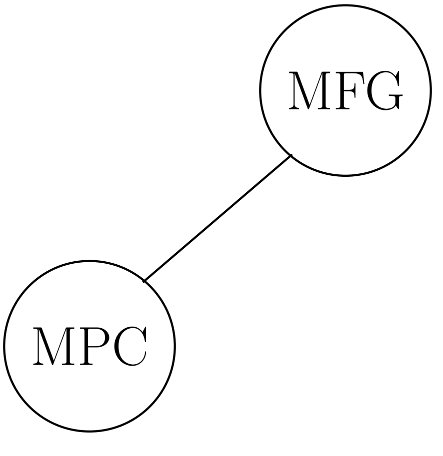

Figure 4.2: Proposed partitioning of the Riera Blanca network for the MP-MFG.

system has a $\Delta t = 1\text{min}$, which is use for both the MP-MFG approach as well as for the MPC approach.

All the information regarding the network is provided by CLABSA (Clavegueram de Barcelona S.A), which includes three-dimensional coordinates of sewer pipes and junctions, crosssectional geometries and materials of sewer pipes, tank geometries and gate characteristics. In order order to test the controllers, a virtual reality of the network is programmed using a HLD callibrated with four different rain scenarios as proposed in [14]. This is a quite accurate model that is useful for simulation purposes, since it is capable to model each element of the system, as well as all the switching phenomena from hybrid elements such as weis and flooding-runoff.

This network has 10 gates that function as active elements for the system. However the proposed scheme requires 1 gate for each sewer pipe in the system, thus it would require 145 gates to propertly function. Having that amount of gates is not possible and an additional tool is required to deal with that problem. The main output of the MFG portion of the approach are all the inflows to each sewer pipe, knowing that there is a constraint on the maximum inflow. Hence, the MFG portion returns a target value for the flows of the network, which can then be pursued by local controller on the gates. This task is performed by a very simple constraint satistaction problem (CSP) as proposed in [14]. This CSP is a follow

$$\begin{gathered} \min_{gates} 0 \\ \textstyle\sum_{i=0}^{T} M_i X(t-i) = m(t), \\ \textstyle\sum_{i=0}^{T} N_i X(t-i) \leq n(t), \end{gathered}$$

where all the flows in vector X are already given. Notice that solving this CSP also regulates the gates that run into the tanks of the network, which ultimately regulates their volumes. Notices that the only desired information from the CSP are the gate flows, which are then pluged into the programmed virtual reality of the network.

Chapter 5

Results and Discussion

The network is tested using three different scenarios: no controller on the loop, full HLD-based MPC, and the proposed MP-MFG approach. This scenarios allow to show the main problems found in the network, as well as the performance and effectiveness of the proposed scheme compared to a more traditional technique. Each scenarios is tested using four different real-rainfall events provided by CLABSA from years 2002, 2006, and 2011. Figure 5.1 shows the total inflow entering the network during the four rain events. Notice that each rainfall event has a very distinctive charactistic, which makes them suitable as an impartial benchmarks for simulation.

As has been stated before, the two main problems from this network are the heavy flooding and the poor WWTP usage. Thus, all the plots and results are based upon that data, and no other information is shown unless is required.

Figure 5.2 shows the total overflows coming out of the network into street level for all the different control scenarios, and for all the different rain events. When no controller is used in the system, the system presents a serious flooding problem due to the poor management in the active elements. It can be seen that both control strategies, i.e., the MPC and the MP-MFG, are able to reduce the total overflow that the network originally had. It is interesting to see that for the 09-10-2002 scenario, the MP-MFG is not able to do such a good job (compared to the MPC). This is due to the fact that this rain event in not uniform as the others (see Figure 5.1), which causes the mean of the volumes to change quite rapidly, which ultimately misleads the controller. The values of the total volume of overflow are presented in Table 5.1.

Table 5.1: Total overflow for each scenario

Rain Event	**OL [m^3]**	**MPC [m^3]**	**MP-MFG [m^3]**
17-09-2002	3.7094×10^3	11.5870	2.7496
09-10-2002	2.5752×10^4	176.5420	8.8559×10^3
15-08-2006	6.9475×10^3	22.3741	14.4736
30-07-2011	1.8442×10^4	166.3861	748.9536

Figure 5.3 shows the total inflow entering the WWTP for all the different control scenarios, and for all the different rain events. When no controller is used in the system, the total inflow to the WWTP completely surpasses the maximum capacity of the plant, and thus it instantly becomes CSO. However, when any of the controller schemes are applied, the total flow running into the plant stays within its maximum capacity and no

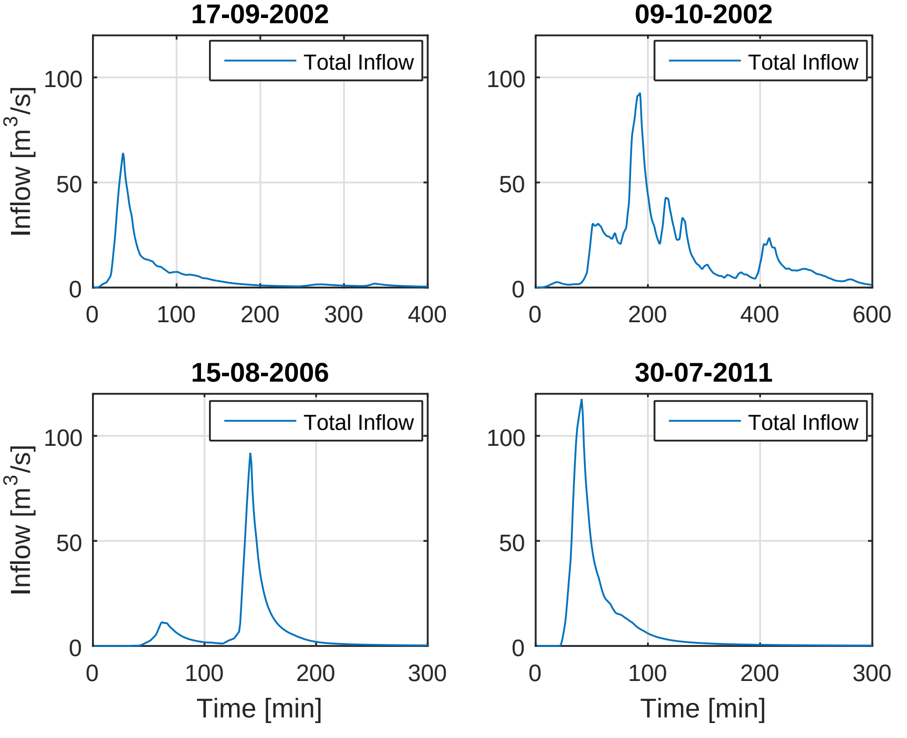

Figure 5.1: Rain-rain scenarios provided by CLABSA used for testing the proposed scheme in the Riera Blanca network.

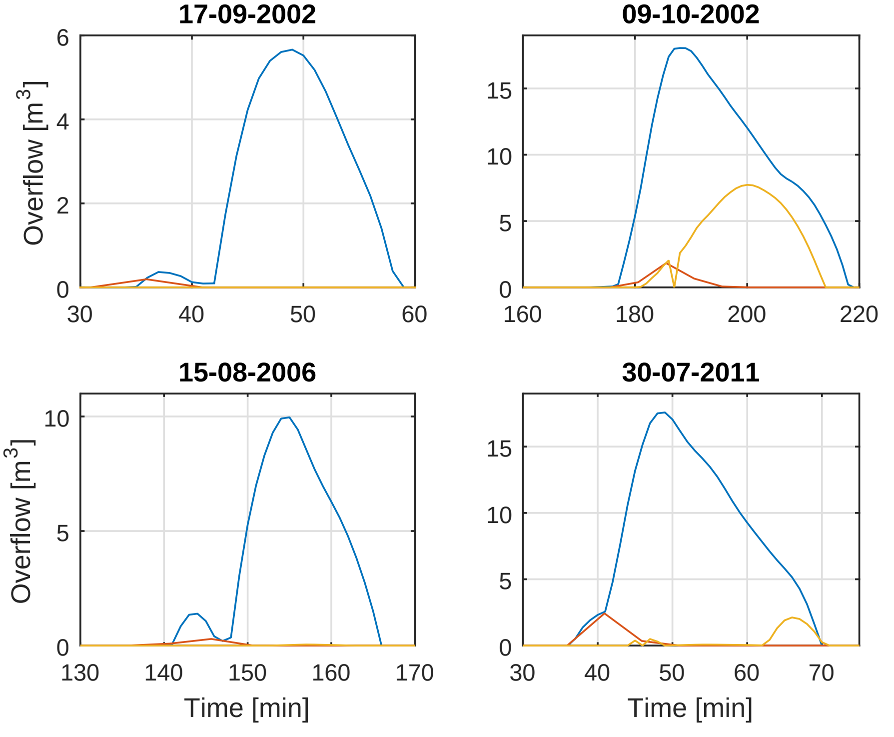

Figure 5.2: Total overflow coming out of the network for all the proposed scenarios, and for the different rain events. The open-loop (OL) overflows are in blue in all graphs, MPC overflows are in red in all graphs, and MP-MFG overflows are in yellow in all graphs.

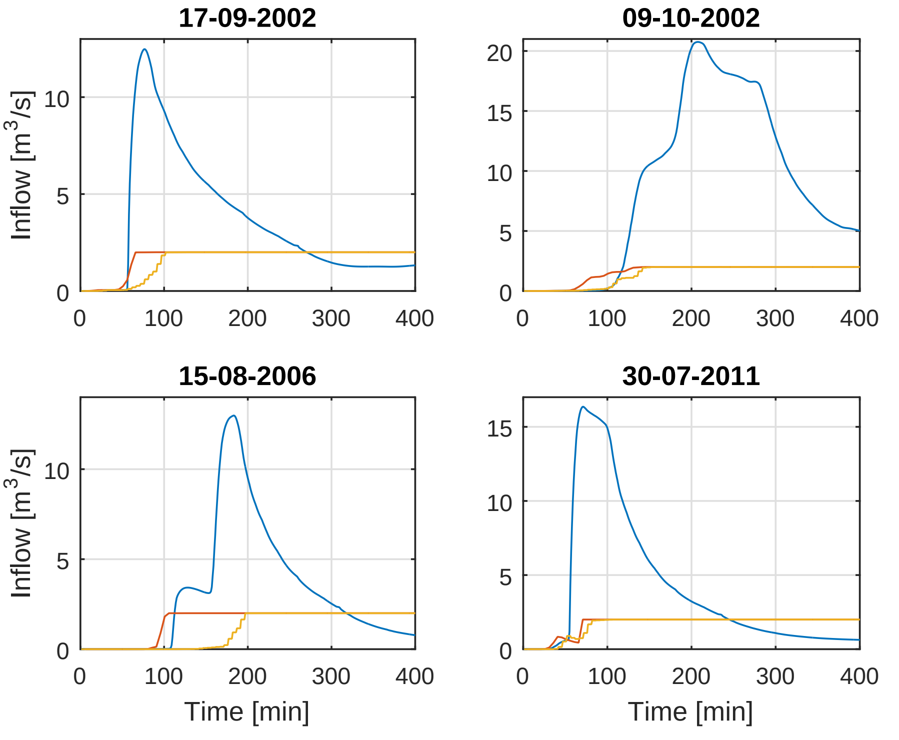

Figure 5.3: Total inflow entering the WWTP for all the proposed scenarios, and for the different rain events. The open-loop (OL) inflows are in blue in all graphs, MPC inflows are in red in all graphs, and MP-MFG inflows are in yellow in all graphs.

sewage is directly sent into the mediterranean sea. It is interesting to notice that for all cases, the MP-MFG takes longer to reach the maximum capacity of the WWTP. This is due to retention property from the microscopic interaction inside the MFG portion of the scheme. Since pipes seek an agreement on their volumes, it is less important to send water downstream.

Both approaches are able to fullfil the requirements of the system, and guaranteeing a suitable operation. Nonetheless, the MPC approach performes slightly better than the MP-MFG approach. However, this improvement in performance brings causes the MPC approach to take longer computation times, compared to the MP-MFG approach. From all the rain events presented, the most complex, computationalwise is the 15-08-2006, due to the double peak found in the rain gauge. For this rain event, the MPC approach takes an average of 2.1 time units to compute the solution, while the MP-MFG approach takes 1 time unit, making it faster. This is particulary useful in real-time appliacations where the computational times are an important decision factor.

Chapter 6

Conclusion

This paper has proposed a non-centralized control scheme based on a MP-MFG that allows to sussesfully combine two control strategies, in order to guarantee a high quality operation of a UDS. The two combined strategies are a controller based on a MFG that uses a consensus-like algorithm in order to achieve a even use of the system, and thus, minimizing the flooding during heavy rain scenarios, and a HLD-based MPC that is capable of managing the usage of WWTP, which minimizes the impact of wastewater to the enviroment. The proposed controller allows to have non-centralized information in the two strategies, only requiring the share of a single coupuling variable, that ultimately determines the macroscopic behavior of the controller. The proposed controller has been tested in a model of real UDS located in the city of Barcelona, Spain, achieving quite suitable performance compared to state-of-the-art solutions, such as a MPC. Moreover, computational time analysis has determined that the proposed scheme is able to achieve similar results to other solutions, while using quite fewer computational resources, and thus, requiring less time.

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