Optimizing periodic maintenance operations for railroad tracks subjected to deterioration

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Abstract

Railroad operations are inherently complex and source of several problems. Most of the maintenance operations require specialized teams with large and expensive equipment, so it is essential that these resources are used efficiently in order to guarantee adequate safety levels under a restricted budget. Our objective throughout this paper is propose an optimization model that minimizes the cost of periodic maintenance operations for railroad tracks subjected to deterioration. Our main contribution consists in unify the deterioration-based maintenance problem with the maintenance workforce routing by a sequential procedure composed by two stages. Real-world case study on a existing US railroad shows that the proposed methodology yields reliable an economical solutions in which the repair costs and the expected cost related to the absence of maintenance are balanced. Finally, we evidenced that our proposed model can be easily extended to each new inspection performed.

Keywords: Railroad Maintenance, Deterioration, Gamma Process, Workforce routing

1. Introduction

Rail is a crucial mode of transportation in the United States. Railroads account for approximately 40 percent of intercity freight volume - more than any mode of transportation (AAR, 2015). In addition, Amtrak, the National Railroad Passenger Corporation transports an average of 86000 passengers every day. Analyzing track geometry defects is critical for keeping freight and passenger trains moving safely. According to the US Federal Railroad Administration Office of Safety Analysis (FRA, 2014), track defects are one of the leading causes of train accidents in the United States. For instance, among the 1747 train accidents that happened in 2012, 577 (33.03%) were caused by track defects, resulting in a total reportable damage of \$ 102.9 million (Peng et al., 2013).

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Railroad infrastructure maintenance consumes large budgets, it is complicated to organize and has numerous challenging planning problems such the deterioration-based maintenance scheduling and the maintenance workforce routing (Lidén, 2015). Several maintenance activities require specialized teams with large and expensive equipment, so it is essential that these resources are used efficiently in order to guarantee adequate safety levels. Our objective throughout this paper is propose an optimization model that minimizes the cost of periodic maintenance operations for railroad tracks subjected to deterioration. This model considers safety standards of the railroad and a restricted workforce capacity. Our main contribution consists in unify the deterioration-based maintenance problem with the maintenance workforce routing by a sequential procedure composed by two stages. The remainder of this paper is organized as follows: Section 2 reviews the literature related to maintenance and routing problems, also introduces the available data. Section 3 describes the proposed model analyzing its structure and steps for its implementation. Section 4 summarizes the results obtained for a case study showing the utility of the model. Finally, several concluding remarks are given in the last section.

2. Background, data and model preliminaries

2.1. Background

When a railroad network is put into use, physical changes to the system occur over time. These changes may be the result of internal processes; for instance, natural changes in material properties, or external processes, such as environmental conditions. Regardless of the cause, these changes may result in deterioration leading to loss of system capacity to perform its intended function (Sánchez-Silva, 2015). Under this deterioration-based perspective, Markov decision processes have been used to develop maintenance planning systems for deteriorating assets such as road pavements (Ben-Akiva et al, 1993; White, 1993; Feighan et al, 1998). The deterioration-based maintenance problem is also studied in Quiroga et al. (2011), where a set of maintenance operations are scheduled with special focus on the prediction uncertainty in the degradation model. This uncertainty is also studied in Andrade and Teixeira (2012) where a Bayesian approach is used.

On the other hand, the maintenance vehicle routing problem aim to assign and schedule a given set of maintenance jobs on different maintenance teams with varying capabilities, equipment and home locations. This problem has been studied by several authors (Gorman et al. 2010; Nemani et al. 2010; Bog et al. 2011; Peng et al. 2011) using different formulations and solution techniques. Both problems are highly correlated, therefore they must be evaluated simultaneously for obtaining better maintenance policies. Different applications of maintenance operations tightly coupled with vehicle routing problems arise naturally in the oil industry (Lopez-Santana et al, 2016), telecoms (Tang et al, 2007), public utilities (Goel et al, 2013) and the financial sector (Blakeley et al, 2003). Specifically, for the railroad industry He et al. (2014) proposed an analytical framework to reduce the probability of derailments. They integrated a track deterioration model and an optimization model to efficiently plan track rectification activities.

In this paper we consider a set of defects that are spatially distributed over tracks. These defects are classified into two severity levels - red tags and yellow tags. Red tag defects violate FRA track safety standards and must be treated as soon as possible. Yellow tag defects satisfy FRA standards; however, they will eventually become red tag defects if they are not fixed. Ability to predict yellow tags which are potentially turning into red tags, before they are actually measured as red tag defects, allows railroads to more efficiently maintain the rail and remain in FRA compliance. Moreover, we consider a fleet of technicians who are missioned to perform rectification operations with a restricted time capacity. Given a planning horizon, this problem consists of determining which defects must be treated on the whole track and assign each rectifying operation to a technician. Additionally, for each technician an specific sequence in which these operations must be carried out must be planned. The novelty of our approach lies in combining maintenance and routing models in order to optimize periodic maintenance operations.

2.2. Data Summary

Every year, North American railroads spend millions of dollars on periodic rail inspection. A fleet of track geometry vehicles travel on the railroad network and examine rail tracks for external and internal rail defects using visual inspection and technologies such as induction and ultrasonic devices (Cannon et al., 2003). Additionally, they are equipped with Global Positioning System (GPS) to accurately identify the location where measurements are taken. These vehicles have the ability to identify around 40 different types of defects; however, only three specific types were analyzed: XLEVEL (XLE) is the difference in elevation between the top surfaces of the rails at a single point in a tangent track segment, SURFACE (SUR) exceptions are determined by depressions or humps in the rail surface and DIP (DIP) is the largest change in elevation of the centreline of the track within a certain moving window distance. DIP may represent either a depression or a hump in the track and approximates the profile of the centreline of the track. In figure 1, it can be observed a diagram of the three types of failure.

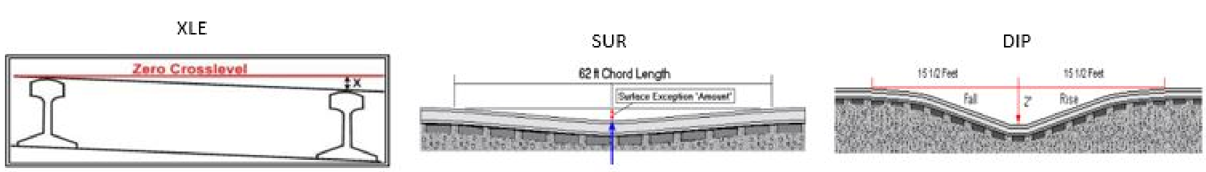

Figure 1: Failure types diagram

Our research is based on a field dataset from 4 tracks including 4 different classes of railroads (II, III, IV, V). The database contains registers of the three types of defects (XLE, SUR, DIP) measured by the track geometry vehicles from 2007 to 2013. In total, there are approximately 6500 red tag defect records and 17500 yellow tag defect records. The limits that separate a yellow tag defect from a red tag defect for a specific railroad class were obtained from the Track and Rail and Infrastructure Integrity Compliance Manual (FRA, 2014). For modeling purposes, we defined ϕ_i as the amplitude in inches of the i^{th} record, t_i as the date in which the i^{th} record was registered and Y_i represents the color of the tag in the i^{th} record ($c_i = 1$ if red tag or $c_i = 0$ if yellow tag).

2.3. Data Tidying

Data tidying consists of giving structure to the dataset to facilitate its analysis. Tidy data sets are easy to manipulate, model and visualize (Wickham, 2014). A considerable effort was spent cleaning data to get it ready for analysis, using a sequential process composed of three stages (Aggregation, Joining, Cleaning).

- **Aggregation**: The dataset was divided into 12 subsets, each of which containing all the records for a specific combination of track and defect type. To generate consistent spatial units and accommodate different modeling purposes, we divided each track in smaller segments called lots, following the strategy presented by He et al. (He et al, 2014) used for track deterioration analysis. Each lot is 0.02 mile (about 100 ft) in length. Then, we aggregate the registers within each lot. Given that there is more than one register per date per lot, we took the register of the maximum amplitude to represent the track segment condition for the inspection run under consideration.

- **Joining**: Given that we are interested in the evolution of the defects over time, a single record of the defect i is not enough to develop our analysis. Hence, we search the next record of the defect in time for each of the defects. Using the two consecutive registers for a specific defect and lot, we computed $\Delta_{\phi_i} = |\phi_{i+1}| - |\phi_i|$ as the difference between the absolute value of the amplitude of the $i+1$ and the i defect. Moreover, we calculate $\Delta t_i = t_{i+1} - t_i$ as the number of days in which the Δ_{ϕ_i} change in amplitude occurred.

- **Cleaning**: From the constructed data set of joint registers, we remove the cases in which the following situations occur:

  - $\Delta_{\phi_i} < 0$: There are cases where the cumulative deterioration decreases over time. Removing these cases from the data set is equivalent to assuming that the defects always get worse along time in absence of a preventive or corrective maintenance.

– $\Delta t_i > 365$: There are long periods of time without any record for a specific defect. Assuming that the track geometry cars inspect the tracks at least once a year, it is almost certain that during a $\Delta t_i > 365$ a preventive or corrective maintenance was done given the lack of records.

In conclusion, for each possible combination of tracks and defect types we developed a dataset containing the following variables: Δ_{ϕ_i} represents the change in amplitude of the defect i in Δt_i days. Moreover, we computed α_i as the missing amplitude of the yellow tag defect i to reach the red tag level.

3. Proposed model

In order to improve current track rectification decisions, this study aims to help existing railroads address the following questions: Which defects must be repaired immediately and how these operations must be carried out by the set of technicians?. This paper presents a two-step sequential procedure in which the solution minimizes the total expected cost of maintenance given a fleet of repairers. Moreover, the solution satisfy constraints associated to a minimum safety standard of the railroad and a restricted capacity of technicians. Figure 2 shows an explanatory diagram of the optimal solution provided by the proposed model. In this case, 12 defects are evaluated for its maintenance by three groups of technicians.

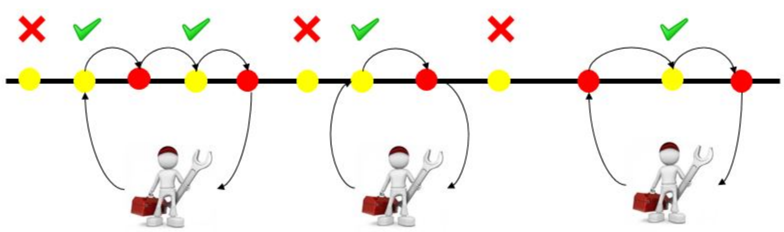

Figure 2: Explanatory diagram of the optimal solution provided by the proposed model

As mentioned before, we have analyzed this problem under a two-step sequential procedure. In the first step, an stochastic deterioration model determines yellow tag defects which are potentially turning into red tags. For each yellow tag defect, we computed the probability of turning into red in a certain interval of time using a gamma process. In the second step, we went through a integer linear programming (ILP) model to decide which defects must be repaired and assign these maintenance operations to each technician over the planning horizon. The ILP proposed receives as an input the probabilities computed by the gamma process to decide more efficiently which yellow tag defects must be repaired given their likelihoods of turning into red tags. In conclusion, our proposed solution follows the structure presented in Figure 3.

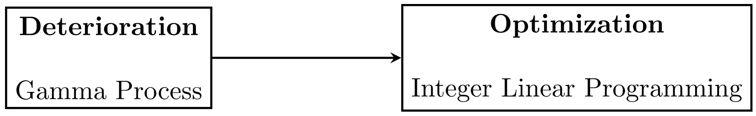

Figure 3: Solution Structure

3.1. Gamma Process

Since the introduction of the gamma process in the area of reliability in 1975, it has been increasingly used to model deterioration in terms of a time-dependent stochastic process for optimizing maintenance (Noortwijk, 2009). The gamma process is suitable to model gradual damage monotonically accumulating over time in a sequence of tiny increments (Riascos-Ochoa et al. 2015), such as corrosion (Kallen et al., 2005), crack growth (Lawless et al., 2004) and concrete creep (Cinlar et al., 1977). In mathematical terms, the gamma process is defined as follows (Noortwijk, 2009). Recall that a random quantity Z has a gamma distribution with shape parameter $v > 0$ and scale parameter $u > 0$ if its probability density function is given by $Ga(z|v, u) = \frac{u^v}{\Gamma(v)} z^{v-1} e^{-uz}$, where $z \geq 0$ and $\Gamma(a) = \int_{x=0}^{\infty} x^{a-1} e^{-x} dx$ is the gamma function for $a > 0$. Furthermore, let $v(t)$ be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $v(0) = 0$. The gamma process with shape function $v(t) > 0$ and scale parameter $u > 0$ is a continuous-time stochastic process $\{X(t), t \geq 0\}$ with the following properties:

- $X(0) = 0$ with probability one
- $X(\tau) - X(t) \sim Ga(v(\tau) - v(t), u)$ for all $\tau > t \geq 0$
- $X(t)$ has independent increments.

Let $X(t)$ denote the cumulative deterioration at time t, $t \geq 0$, and let the probability density function of $X(t)$, in accordance with the definition of the gamma process, be given by $f_{X(t)}(x) = Ga(x|v(t), u)$ with expectation and variance as follows:

$$E(X(t)) = \frac{v(t)}{u} \quad \text{and} \quad Var(X(t)) = \frac{v(t)}{u^2}$$

In order to apply the gamma process model in the rail network problem, for each track and defect type, we aggregated all the records of Δ_{ϕ_i} representing the change in amplitude of the defect in Δt_i days into a data set. This procedure is equivalent to assuming that all the defects in a particular combination of track and defect type follow the same pattern of deterioration. In conclusion, we estimated 12 gamma processes (one for each combination of track and defect type). Each data set consists of inspection times t_i, $i = 1, ..., n$, where

$0 = t_0 < t_1 < t_2 < ... < t_n$, and corresponding observations of the cumulative amounts of deterioration x_i, $i = 1, ..., n$, where $0 = x_0 \leq x_1 \leq x_2 \leq ... \leq x_n$. Considering a gamma process with shape function $v(t) = ct^b$ and scale parameter u, the parameters c and u are estimated by the method of maximum likelihood initially presented by Cinlar et al. (Cinlar et al., 1977). We assume that the value of the power b is initially known and equal to 1, but c and u are unknown. However, c and u can be obtained by maximizing the logarithm of the likelihood function of the increments. The likelihood function of the observed deterioration increments $\delta_i = x_i - x_{i-1}$ where $i = 1, ..., n$ is presented in equation 1.

$$L(\delta_1, ..., \delta_n | c, u) = \prod_{i=1}^{n} f_{X_{(t_i)} - X_{(t_{i-1})}}(\delta_i) = \prod_{i=1}^{n} \frac{u^{c[t_i^b - t^b{}_{i-1}]}}{\Gamma(c[t_i^b - t^b{}_{i-1}])} \delta_i^{c[t_i^b - t^b{}_{i-1}]-1} e^{-u\delta_i} \tag{1}$$

By computing the first partial derivatives of the loglikelihood function of the increments with respect to c and u, the maximum-likelihood estimates $\hat{c}$ and $\hat{u}$ can be solved from equation 2.

$$\hat{u} = \frac{\hat{c}t_n^b}{x_n} \quad and \quad \sum_{i=1}^{n} [t_i^b - t_{i-1}^b]\{\psi(\hat{c}[t_i^b - t_{i-1}^b]) - log\delta_i\} = t_n^b log(\frac{\hat{c}t_n^b}{x_n}) \tag{2}$$

The maximum-likelihood method to estimate the parameters c and u can be extended to estimate the parameter b as well. The parameter b then must be determined by numerically maximizing the likelihood function L (Nicolai et al.,2007). A particular gamma process was estimated for each track and type of defect, the parameters are shown in Table 1. Moreover, Figure 4 shows the fit of the gamma process to the actual data for the specific case Track 1 - XLE.

Track 1 - XLE		**Track 2 - XLE**		**Track 3 - XLE**		**Track 4 - XLE**	
$\hat{b}$	1.2008	$\hat{b}$	1.1076	$\hat{b}$	0.9367	$\hat{b}$	0.7861
$\hat{u}$	1.377	$\hat{u}$	1.5022	$\hat{u}$	1.5488	$\hat{u}$	0.9397
$\hat{c}$	0.0010	$\hat{c}$	0.0022	$\hat{c}$	0.0176	$\hat{c}$	0.0292
Track 1 - SUR		**Track 2 - SUR**		**Track 3 - SUR**		**Track 4 - SUR**	
$\hat{b}$	0.8743	$\hat{b}$	0.9921	$\hat{b}$	1.0047	$\hat{b}$	0.9570
$\hat{u}$	0.5146	$\hat{u}$	0.6948	$\hat{u}$	0.6843	$\hat{u}$	0.6312
$\hat{c}$	0.0246	$\hat{c}$	0.0118	$\hat{c}$	0.0094	$\hat{c}$	0.0153
Track 1 - DIP		**Track 2 - DIP**		**Track 3 - DIP**		**Track 4 - DIP**	
$\hat{b}$	0.9658	$\hat{b}$	1.1449	$\hat{b}$	1.0309	$\hat{b}$	1.1705
$\hat{u}$	0.5909	$\hat{u}$	0.7216	$\hat{u}$	0.7120	$\hat{u}$	0.8320
$\hat{c}$	0.0128	$\hat{c}$	0.0024	$\hat{c}$	0.0069	$\hat{c}$	0.0017

Table 1: Gamma Process parameters

In this way, we calculated the probability that a yellow tag defect remains tagged as yellow after t days using the cumulative distribution function of the gamma process estimated. Basically, recall that we defined $X(t)$ as the deterioration in the amplitude of the defect after t days. Assuming that $X(t) \sim Ga(v(t) = ct^b, u)$,

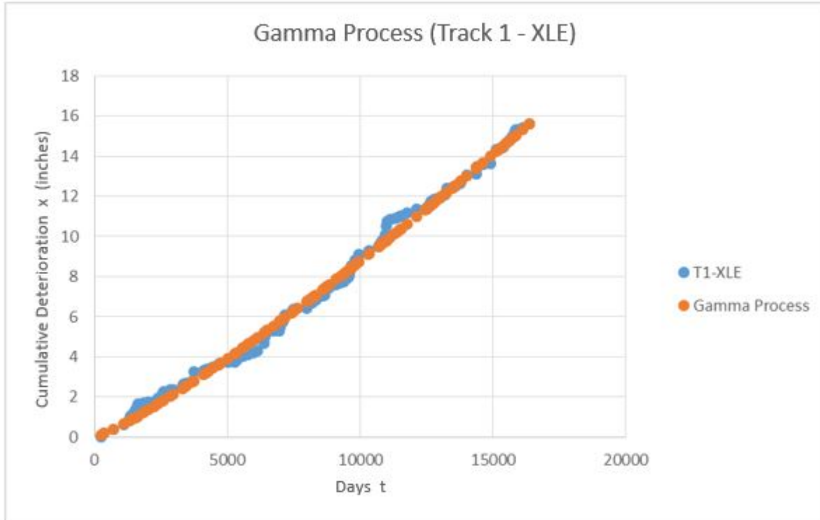

Figure 4: Fit of the Gamma Process to the actual data

we computed

$$P(Yellow) = P(c = 0) = F_{X(t)} = (P(X(t) \leq \alpha).$$

Thereby, we are computing the probability that the deterioration of the defect in these t days does not exceed the missing amplitude to reach red tag level. On the other hand, the probability that a yellow tag defect reaches the red level after t days is estimated using the complement, so

$$P(Red) = P(c = 1) = 1 - P(c = 0).$$

In conclusion, we computed ${p_i}^R$ as the probability that the yellow tag defect i reaches the red level after t days. This measure will be used in the next step of our solution to decide more efficiently which yellow tag defects must be repaired.

3.2. Integer Linear Programming (ILP)

Integer Linear Programming (ILP) is the name given to LP problems which have the additional constraint that all the variables have to be integer. Several types of LP problems have been extensively used for planning maintenance operations (Levi et al. 2014; Bajestani et al. 2014; Tantardini et al. 2014; Manzini et al. 2015) and routing (Kallehauge et al. 2005; Solomon, 1987; Kohl et al. 1997; Larsen, 2004). In this section, the ILP model is formulated to decide which defects must be repaired and assign these maintenance operations to each technician over the planning horizon while minimizing the total expected cost of maintenance operations.

Our problem is defined by a set of defects $\mathcal{D}$, a fleet of repairers $\mathcal{R}$ and a directed graph $\mathcal{G}$. The fleet is considered to be homogeneous meaning that all repairers are identical. The graph consists of $|\mathcal{D}|+2$ vertices, where the defects are denoted $1, 2, ..., n$. Also, the starting point and ending point is represented by the vertices 0 and $n+1$ respectively. The set of all vertices, that is, $0, 1, ..., n+1$ is denoted $\mathcal{N}$. The set of arcs $\mathcal{A}$, represents direct connection between vertices. There are no arcs ending at vertex 0 or originating from vertex $n+1$. With each arc (i, j), where $i \neq j$ we associate a distance in miles d_{ij}. For each defect $i \in \mathcal{D}$ we define c_i as the tag color of defect i, u_i as the type of defect i, t_i as the time to repair defect i and ${c_i}^o$ as the cost to repair defect i. Moreover, we defined q_k of the time capacity of the repairer $k \in \mathcal{K}$ and v as the speed of the repairers.

The model contains two set of decision variables $\mathcal{X}$ and $\mathcal{Y}$. For each defect $i \in \mathcal{D}$ we define x_i as

$$x_i = \begin{cases} 1 \text{ if the defect } i \text{ is repaired; } i \in \mathcal{D} \\ 0 \text{ otherwise} \end{cases} \tag{3}$$

In addition, for each arc (i, j), where $i \neq j$, $i \neq n+1$, $j \neq 0$, and each vehicle k we define y_{ijk} as

$$y_{ijk} = \begin{cases} 1 \text{ if the defects } i \text{ and } j \text{ are fixed by the repair } k; \ \forall\ i, j \in \mathcal{N}, \quad \forall\ k \in \mathcal{K} \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

The goal is to design a set of routes to assign the repairers in which the total expected cost is minimized. This objective function is compound by three elements:

- Cost of repairs computed as $\sum_{i \in \mathcal{D}} {c_i}^o x_i$.
- Cost of transportation computed as $\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{K}} c^m d_{ij} Y_{ijk}$ where c^m represents the cost per mile traversed.
- Expected cost related to the absence of maintenance computed as $\sum_{i \in \mathcal{D}} [{p_i}^R (1-x_i)](\Delta {c^o}_i + p^d c^d)$ where ${p_i}^R$ denotes the probability that the yellow tag defect i reaches the red level after t days (estimated with the gamma process), $\Delta {c^o}_i$ represents the change in the repair cost between red tag and yellow tag for an specific defect i, p^d denotes the probability of a train derailment caused by a geometric defect and c^d represent the expected cost associated to a train derailment. In this way, we are assuming that only the non-repaired yellow tag defects that reach the red level tag would cause a derailment in the railroad track.

The ILP model minimizes the objective function subjected to three fundamental constraints. All red tag defects must be repaired (FRA Conditions), the fleet of repairs have a restricted time capacity and the railroad must accomplish a minimum safety standard α computed as the proportion of defects repaired over the total inspected.

The ILP model can be stated mathematically as follows:

$$\min \quad \sum_{i\in\mathcal{D}} c_i{}^o x_i + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{K}} c^m d_{ij} Y_{ijk} + \sum_{i\in\mathcal{D}} [p_i{}^R(1-x_i)](\Delta c^o{}_i + p^d c^d) \qquad s.t., \tag{5}$$

$$x_i = 1 \quad \forall\ i \in \mathcal{D} \mid \ c_i = 1 \tag{6}$$

$$\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} (t_i + \frac{d_{ij}}{v}) y_{ijk} \leq q_k \quad \forall\ k \in \mathcal{K} \tag{7}$$

$$\sum_{i\in\mathcal{D}} x_i \geq \alpha\ n \tag{8}$$

$$\sum_{j\in\mathcal{N}} y_{0jk} = 1 \quad \forall\ k \in \mathcal{K} \tag{9}$$

$$\sum_{i\in\mathcal{N}} y_{ihk} = \sum_{j\in\mathcal{N}} y_{hjk} \quad \forall\ k \in \mathcal{K},\ \ \forall\ h \in \mathcal{D} \tag{10}$$

$$\sum_{i\in\mathcal{N}} y_{i,n+1,k} = 1 \quad \forall\ k \in \mathcal{K} \tag{11}$$

$$x_i = \sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{K}} y_{ijk} \quad \forall\ i \in \mathcal{D} \tag{12}$$

$$x_i \in \{0,1\} \quad \forall\ i \in \mathcal{D} \tag{13}$$

$$y_{ijk} \in \{0,1\} \quad \forall\ i,j \in \mathcal{N},\ \ \forall\ k \in \mathcal{K} \tag{14}$$

The objective function (5) minimizes the total expected cost of the maintenance operation. The constraints (6) ensure that each red tag defect must be repaired, and (7) state that a repairer can accomplish the labor assigned in terms of time. Then, constraint (8) ensure that the railroad must accomplish a minimum safety standard α. Next, equation (9), (10) and (11) indicate yhat each repairer must leave the starting point 0; after a repairer arrives at a defect it has to leave for another destination; and finally, all repairers must arrive at ending point $n+1$. Equation (12) establish the relationship between the two sets of variables. Finally, equation (13) and (14) are the integrality constraints.

4. Illustrative case study and results

We illustrate the performance of our proposed approach with an inspection from January 2013 in Track III. This track belongs to the US nationwide line segments, with a considerable length of 200 miles. During the inspection, 47 defects were recorded ($n = 47$). Ten of them were classified as red tags while the remaining as yellow tags. Regarding the types of defects, along the track were found 9 XLE, 24 SUR and 14 DIP distributed throughout all the segment. The next inspection performed in this track was registered three months later; generally, the average time between FRA inspections is 90 days (He et al, 2015). Therefore, considering a 90-day inspection interval Δ_t, we obtained ${p_i}^R(t)$ as the probability that the yellow tag defect i reaches the red level after t days based on the proposed gamma process.

We considered 4 homogeneous repairers with a restricted time capacity of 80 hours related to two labor weeks (Peng et al, 2011), and a velocity of 40 miles per hour to move between defects. In this way, we are planning the maintenance operations for a time horizon of two weeks. The time of rectification for a single defect is distributed uniform between 4 and 8 hours (Peng et al, 2011). Accurately estimating the exact repair costs is very difficult, due to variations in equipment, fleet and personnel. For simplicity, we only take into account the three types of defects and assume that costs for both yellow and red tags stay constant across differents maintenance scenarios as He et al, 2015. In Table 2, we present the cost of the repair operations (c^o) in which implicity can be computed the change in cost according to the color of the tag (Δc^o). Moreover, we assumed a cost per millage traversed (c^m) of \$10 USD.

XLE	**(\$ USD)**	**SUR**	**(\$ USD)**	**DIP**	**(\$ USD)**
Yellow Tag	Red Tag	Yellow Tag	Red Tag	Yellow Tag	Red Tag
\$1539	\$1710	\$1125	\$1250	\$1125	\$1250

Table 2: Average rectification cost for individual geo-defects

The non-repaired defects are exposed to be the main cause of a rail derailment. Although rail derailment is infrequent ($p^d = 1.5 * 10^-3$ estimated in Anderson et al, 2005) its consequences can be severe and may result in different forms of costs, including infrastructure; rolling stock; traffic disruptions; injuries and fatalities. Zahurul et al, (2015) estimated this total expected cost as \$ 1.5 Million USD (c^d). Finally, for the base case the desired proportion of defects over the total inspected is set to be $\alpha = 80\%$. Under these conditions, Figure 5 presents the overall results in which total expected cost and routes are shown.

In order to evaluate the sensitivity of the model according to the minimum safety standard α computed as the proportion of defects repaired over the total inspected. We developed an experiment in which the value of α changes between 0% (problem non-restricted by reliability and safety) and 100% (all defects must be repaired). In figure 6 can be observed that regardless of the α value the fleet of repairers must repair at least a 48.93% of the defects if possible. This fact is caused by the balance and compensation between repair costs and the expected cost related to the absence of maintenance. Moreover, in Figure 6 can be observed that the ratio between both costs (non-repair cost over repair cost) is fixed to 0.68 when the value of α is lower than 48.93%. Thereby, it is evidenced the importance of considering the possibility of derailments in

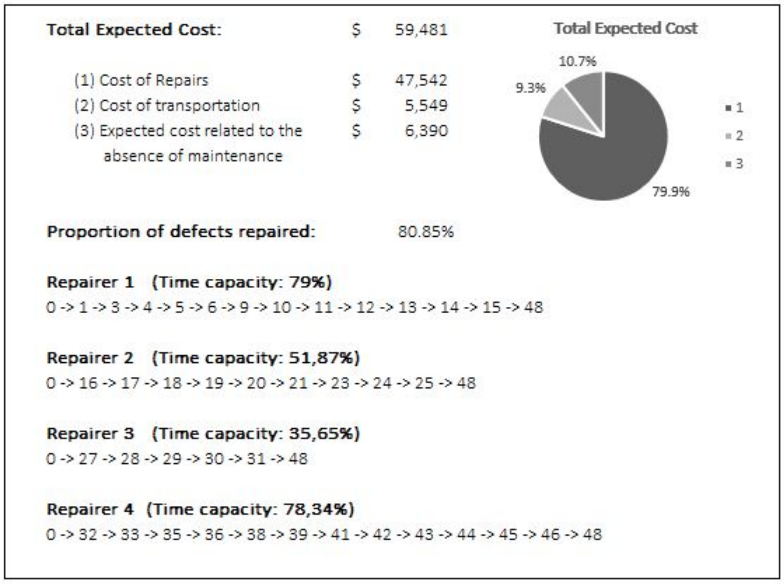

Figure 5: Overall results

the maintenance planning given the high expected costs that an accident would carry. Finally, it is worth to highlight that this model can be implemented for each new inspection performed to plan the maintenance operations.

Figure 6: Reliability and safety standard

5. Concluding remarks

This paper presents an analytical framework to address the combined maintenance and routing problem for a set of defects geographically distributed along a railway. We proposed a solution approach which consists in the integration of deterioration and maintenance models in a two-step sequential procedure. In the first step, an stochastic deterioration model determines yellow tag defects which are potentially turning into red tags. For each yellow tag defect, we computed the probability of turning into red in a certain interval of time using a gamma process. In the second step, we went through a integer linear programming (ILP) model to decide which defects must be repaired and assign these maintenance operations to each technician over the planning horizon. The ILP proposed receives as an input the probabilities computed by the gamma process to decide more efficiently which yellow tag defects must be repaired given their likelihoods of turning into red tags.

Real-world case study on a existing US railroad shows that the proposed methodology yields reliable an economical solutions in which the repair costs and the expected cost related to the absence of maintenance are balanced. Moreover, results suggest that our proposed method could outperform a procedure based on the intuition of maintenance planners that does not include the interaction between maintenance and routing models. In future work, we should focus in considering repair job routing based on existing train timetables to minimize delays and logistical problems. As a final remark, we would like to note that our track rectification model can be easily extended to each new inspection performed.

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