ON THE DESIGN OF RAINWATER HARVESTING AND GREYWATER RECYCLING SYSTEMS FOR URBAN AREAS

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Abstract

Rainwater harvesting is the process of collecting, storing, and distributing rainwater for reuse, rather than allowing it to run off. Greywater recycling consists of purifying processes that remove contaminants present in wastewater coming from faucets, showers, baths, clothes washing, and dishwashing, so that it can be reused, instead of becoming inlet wastewater for municipal wastewater treatment plants. In urban areas, rainwater harvesting and greywater recycling systems provide additional water supplies for houses, buildings, and industries, and also help to mitigate flooding events and pollution. To design an integrated system that includes rainwater harvesting and greywater recycling in residential units, this paper proposes: 1) a multi-objective optimization model that minimizes: potable water consumption, maintenance and operational costs plus potable water cost, and construction costs; and 2) a two-stage stochastic optimization model that incorporates the uncertainty associated with rainfall events. To evaluate the applicability of the proposed optimization models a case study in the city of Soacha in Colombia is presented. The results show a reduction of more than 30% of the potable water consumed.
Keywords: multi-objective optimization, two-stage stochastic optimization, rainwater harvesting, greywater recycling, ecofriendly design, sustainable construction.

1. Introduction

Due to warnings of increasing potable water scarcity in the world (UN-Water, 2012), there is a global tendency for making an efficient use of water in urban centers. The efficient use of water ultimately reduces potable water use by substituting fresh water consumption with recycled water, in uses where its potability is not required.

Water is used in houses for multiple purposes such as: shower, bath, faucet, toilet flushing, clothes washing, dishwashing, watering plants, and cleaning. According to a study made by Gato-Trinidad et al. (2011) in Melbure, Australia, 19% of domestic water consumption is used in toilet flushing, 26% in clothes washing, 31% in shower, 24% in other indoor uses such as faucet, bath, and dishwashing, 5% is lost in leaks, and the remaining 29% is used in outdoor uses such as watering plants and cleaning. In urban areas, the general tendency is that the water that supplies all these needs comes from the aqueduct, even if its potability is not required for certain needs. For example, toilet flushing, cleaning, and watering plants are needs that can be supplied with recycled water, and represent 48% reduction in potable water consumption in Melburn. Similar information provided by the American Water Works Association Research Foundation (1999), show that in North America 30% of the potable water can be reduced because of the same reason.

In Colombia, the efficient use of potable water is necessary. During 2005, 94.3% of urban areas had access to potable water (DANE, 2005). During the same year a study of the Colombia Advocacy Office (2005) surveyed 86% of municipalities and cities of Colombia. The study reveal that only 18% of these municipalities and cities had access to potable water that meets the established quality standards. Given the limited availability of potable water, substituting potable water consumption promotes the efficient use of the resource to provide maximum availability (Roldan et al., 2015). In Bogota, for example, there is a potential saving of potable water in homes for uses as irrigation and toilet flushes greater than 27% (Cortés, 2012).

Rainwater harvesting and greywater recycling are two strategies used to achieve a net-zero water use in buildings. Net-zero water use relies on using renewable sources of water, which allows a building to operate independently of the aqueduct (3p contributor, 2015). Rainwater harvesting systems collect rainwater in catchment surfaces, store it in storage tanks and distribute it for reuse. Greywater recycling systems treat wastewater in treatment plants and store it in storage tanks so that it can be reused. Implementing systems that combine rainwater harvesting and greywater recycling reduces potable water consumption, provides additional water supplies for houses, buildings, and industries, and helps to mitigate flooding events and reduces the volume of wastewater that reaches the municipal treatment plants (Rojas, 2004). The design of a *Rainwater Harvesting and Greywater Recycling System* (RHGRS) involves decisions related to the location, size, material, and technology of storage tanks and treatment plants. Additionally, catchment surfaces that will supply the system with rainwater need to be identified. Finally, connections between users and the storage tanks must be determined.

A system that combines rainwater harvesting and greywater recycling for multiple customers can ensure a long-term supply of water with adequate quality. Rainwater harvesting allows the water on rainy seasons to be collected while greywater recycling provides a constant source of recycled water. With the right components, treatment technology, and location, a RHGRS can be installed in new or existing residential units (García-Montoya et al., 2015b; Stec & Kordana, 2015). The executive director of the Colombian Sustainable Building Council (CCCS) stated that buildings that incorporate these systems reduce water consumption in up to 40% (Portfolio, 2015).

However, designing such systems is an engineering challenge because construction costs, operation and maintenance costs, as well as conditions associated with the system configuration must be taken into a count. In this sense, there is a need to develop tools to support design decisions related to these systems.

To design an integrated system that includes rainwater harvesting and greywater recycling in residential units, this paper proposes: 1) a multi-objective optimization model that minimizes: potable water consumption, maintenance and operational costs plus potable water cost, and construction costs; and 2) a two-stage stochastic optimization model that incorporates the uncertainty associated with rainfall events. To evaluate the applicability of the proposed optimization models a case study in the city of Soacha in Colombia is presented. The results show a reduction of more than 30% of the potable water consumed.

2. Rainwater Harvesting and Greywater Recycling System Design

In any RHGRS system, there are several input elements that must be identified. First, a set of possible locations for storage tanks and greywater treatment plants, a set of potential catchment surfaces, a set of customers, and a planning horizon for the system evaluation. Second, physical characteristics of storage tanks and greywater treatment plants such as dimensions, weight, volume, and prices can be defined (Krishna, 2005; Pidou et al., 2007). Third, the cost of all the components and accessories that must be installed to transport rainwater from all catchment surfaces to all storage tanks, pipes that transport greywater from every customer to every treatment plants, and pipes that transport recycled water from any storage tanks to any customer can be identified.

Weather conditions such as the amount and frequency of rainfall are required to calculate rainwater supply. Since rainfall amount and frequency vary along different places, a careful study must be performed on the rain data provided by local authorities. The usual measure of rainfall is in inches per square meter per unit of time. The time units used are hours, days, weeks, months, or years. Average rainfall and median rainfall are commonly estimators of rainfall (Krishna, 2005). For planning purposes, median rainfall can be used to estimate water availability. The median value for rainfall is usually lower than the average value, since large rainfall events tend to overestimate the average value (Krishna, 2001). Thus, the median rainfall provides a more conservative calculation of system sizing than average rainfall. Another consideration is that most rainfall occurs seasonally (Krishna, 2005). Annual rainfall is not evenly distributed throughout the 12 months of the year, therefore the monthly distribution of rainfall is an important factor to consider (Krishna, 2005).

Non-potable water demand is determined by characteristics of the customers in the system, namely, types of customers (houses or buildings), number of users, and individual water consumption of non-potable water. Based on social and economic characteristics of the residential unit, these characteristics might vary.

Aside from rainwater and non-potable water demand estimation, additional information for water losses is incorporated. For instance, not all the rainwater that falls on a catchment surface is collected. Depending on the physical properties of the materials and surfaces, each one will have a runoff coefficient. This coefficient is the percentage of real amount of rainwater collected. In the case of greywater recycling, 80% of the greywater provided by domestic uses will enter to the treatment plant. Extra rainwater losses, caused by first flush diverter, evaporation, splash-out or overshoot from the gutters in hard rains, and possibly leaks, will affect the amount of rainwater that is stored in the storage tanks.

It is noteworthy that the offer of greywater and rainwater is limited, and sometimes may not be enough to meet the demand of recycled water. Since user's needs must be satisfied regardless the availability of recycled water, potable water from the aqueduct can be consumed. During all the planning horizon, supply and demand of water will cause storage level changes. The processing time of greywater treatment plants will also affect water availability.

Each possible location for the storage tanks or treatment plants has installation conditions. Storage tanks or treatment plants dimensions cannot exceed the space limitations, and can only be installed on a structure capable of holding the corresponding weight. More than one storage tank can be located in one location, but at most one treatment plant can be placed on treatment plants tentative locations. Consequently, each location is characterized by a set of possible storage tank types and/or treatment plants that can be placed at the former.

Connections among the system components are subject to the installation of storage tanks and treatment plants. Connection costs are computed taking into account the types of pipes and the distance between the components of the system. Depending on the water quality (rainwater, greywater or recycled water) to be transported, different types of pipes (each of them with a specific cost) are available to build the connections. Required pumps in the system also have a installation and operational cost.

The design decisions to make are: which storage tank locations are going to be used and which type of tank is going to be installed, which catchment surfaces are going to collect and send water to the located storage tanks, which treatment plant locations are going to be used and what treatment plant is going to be installed, which source customers are supplying greywater to the selected treatment plants, which treatment plants are supplying recycled water to an installed storage tank, and which treatment tank will supply each customer demand.

Figure shows an example of a RHGRS that collects rainwater in parking lots, building roofs, and green areas and has two storage tanks. With only one treatment plant, greywater is treated and later stored in one of the storage tanks that contains rainwater. Water is kept in storage tanks until needed. Note that RHGRSs are not independently installed, they share the delivery system and storage tanks. The uses that can demand recycled water are toilet flushing, cleaning, and watering plants (e.g., irrigating green areas of residential units).

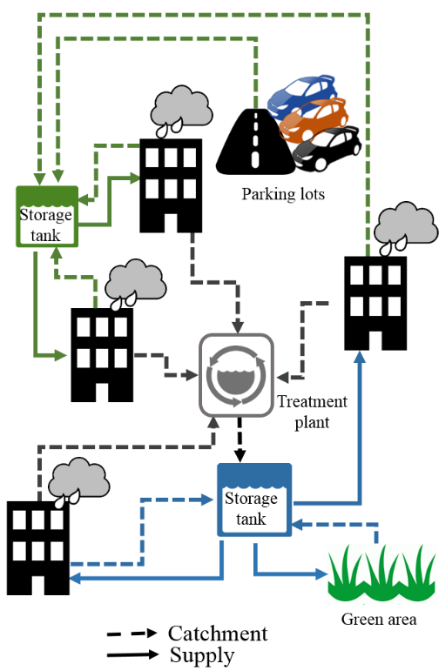

Figure 1. Example of a RHGRS design.

3. Literature Review

Related to this topic several academic articles can be found in the literature. Liaw et al. (2004) proposed a methodology to determine the optimal size of rainwater storage tanks depending on the materials and shape of the roof that are used as rainwater catchment surface. Solyali et al. (2015) made a linear optimization model that seeks to minimize the construction cost of tanks and the costs of potable water consumption, considering different billing intervals for a residential unit in the Northern Cyprus. Sample & Liu (2014) assessed the feasibility of implementing a rainwater harvesting system in buildings with different concentrations of people and different uses (residential and commercial) through a simulation model that determines the capacity of the tanks, catchment areas, irrigation areas and demand for non-potable water that an installed system for each building should hold, calculating an indicator of system reliability. Farreny et al. (2011) made a cost-benefit analysis on the implementation of different construction alternatives under two rainfall scenarios in Granollers (Spain) using the net present value as an indicator of viability. Moralez-Pinzon et al. (2015) proposed a simulation model that integrates structural design elements as the size of tanks, estimated costs, and environmental analysis, based on the YAS (yield-after-spill) and YBS (yield-before-spill) algorithms.

For the design of systems in residential complexes and neighborhoods in urban areas, Bocanegra-Martinez et al. (2014) proposed a mixed integer multi-objective optimization model for the design of a rainwater harvesting system. Garcia-Montoya et al. (2015a) also used a mixed integer multi-objective optimization model to design a greywater recycling system. Garcia-Montoya et al. (2015b) proposed a simultaneous design of water reusing and rainwater harvesting systems, by integrating the two optimization models posed by Bocanegra-Martinez et al. (2014) and Garcia-Montoya et al. (2015b). This new model independently designed two systems that combined with the potable water source supply water to the users. Garcia-Montoya et al. (2015c) used the multi-objective optimization model proposed by Garcia-Montoya et al. (2015b) but incorporate the environmental impact of this type of systems by using the life cycle assessment (LCA) methodology.

To incorporate rainfall events variability, Su et al. (2009) used a simulation model that incorporates historical rainfall information to identify the probability distribution function of the rate of recycled water deficit in the system with specific sizes of storage tanks. Youn et al. (2012) incorporated to the previous simulation model rainfall variability associated with climate change and also incorporated changes on the demand of users depending on the weekdays. Unami et al. (2015) proposed a stochastic control model to describe the dynamic behavior of a rainwater harvesting system. Stochastic optimization has not been use to solve problems related with rainwater harvesting or greywater recycling system design, but a similar application is urban water resource allocation. Wang et al. (2016) proposed a two-stage stochastic optimization model that incorporates uncertainty in the availability of water resources to determine strategic plans of water allocation to meet the needs of the users.

4. Optimization Models

To solve the design problem described in previous sections, two optimization models are presented in this section. First, a multi-objective optimization model that comprises three optimization steps to address the objectives of environmental organizations, system users, and construction companies. And second, a two-stage stochastic model that uses scenarios to describe rainfall events variability. Both models have the structure of a network design mix integer programming model.

Each of the optimization models considers all the design conditions of the system. In brief:

- Every possible location can hold a certain type of tanks or treatment system.
- Supply of rainwater and greywater is limited.
- Demand should be satisfied with recycled or potable water.
- Water processing time in treatment plants must be considered.
- The stock of water (inventory) in each tank varies in time.
- The capacity of the tanks depends on the selected tank type to be installed.

Additionally, three main cost components must be identified to determine the optimization objectives: 1) potable water cost, 2) operation and maintenance costs: which are pumping and treatment plants operation and maintenance costs, and 3) construction cost: which includes installation costs of storage tanks, treatment plants, pipes, pumps, and catchment areas.

4.1 Multi-objective optimization model

Given the nature of the problem, there are three important agents that are involved in the decision making process: environmental organizations, system users, and construction companies, each of them with its own objective to assess the solution quality of the system design. Environmental organizations want to save as much potable water as possible. That is, supply users demand as much as possible with recycled water (either rainwater or treated greywater). System users or project owners —where the recycling system is implemented— are interested in a system that saves potable water, with low maintenance and operational costs, i.e., a system that minimizes maintenance costs of treatment plants and pumping units, operational costs, and potable water cost. Finally, construction companies want to minimize the construction costs of the system, i.e., minimize the cost of installing storage tanks, catchment surfaces, treatment plants, pipes, and pumps. Consequently, the objective of designing a RHGRS is to reduce the consumption of potable water while minimizing the total cost of the system.

4.1.1 Model formulation

Notation

Sets

T: Set of time periods (Planning horizon)
$\mathcal{A}$: Set of arcs (Connections)
$\mathcal{N}$: Set of nodes
$\mathcal{N} = U \cup B \cup H \cup S \cup D \cup J \cup \{d\}$
Where
U: Set of potential locations for tanks
B: Set of potential locations for tratment plants
H: Set of customer supplying greywater
D: Set of customer demanding water
S: Set of catchement surfaces
J: Set of intervals for water billing
d: Sink node
K: Set of tank sizes
F: Set of treatment plant daily processing capacities (m^3)
C_i: Set of posible sizes for tanks or treatment plants in the location $i \in U \cup B$

Parameters

b_{it} = Supply/demand $i \in \mathcal{N}, t \in T$
b'_{it} = Potable water demand $i \in D, t \in T$
e_i = Supply/demand $i \in J \cup \{d\}$
β_{ij} = Connection cost between $i, j \in \mathcal{N}$
λ_c = Installing costs of a tank or a treatment plant of size $c \in F \cup K$
γ_{ij} = Pumping cost per m^3 from $i \in \mathcal{N}$ a $j \in \mathcal{N}$
φ_f = Operation and maintenance costs of the treatment plant $f \in F$
η_i = Adequacy costs of a catchment surface $i \in S$
ρ_i = Cost of the pump placed on the catchment surface, tank location or treatment plant location $i \in S \cup U \cup B$
ε_k = Charge per m^3 of water from the aqueduct on the billing interval $k \in J$
m_k = Maximum amount of m^3 of water from the aqueduct provided on the billing interval $k \in J$
α_f = Efficiency of treatment plant $f \in F$
r_f = Time periods of water treatment for the plant $f \in F$
a_i = Available area in location $i \in B \cup U$
h_c = Area occupied by the tank or treatment plant of capacity $c \in K \cup F$

$$l_{ij} = \begin{cases} 1, \text{If location } i \in U \text{ and location } j \in B \text{ are in the same place} \\ 0, \text{ Otherwise} \end{cases}$$

g = Time periods per two months
τ = Days per time period
M = Large number

Decision variables

$x_{ijtt'}$ = Water flowing through the arc $(i, j, t, t') \in A$

$$z_{ij} = \begin{cases} 1, \text{If node } i \in \mathcal{N} \text{ is connected with node } j \in \mathcal{N} \\ 0, \text{ Otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{ If the catchment surface } i \in S \text{ is used} \\ 0, \text{Otherwise} \end{cases}$$

$$w_{ic} = \begin{cases} 1, \text{If the tank or treatment plant location } i \in U \cup B \\ \quad \text{hold a tank or treatment plant of capacity } c \in C_i \\ 0, \text{Otherwise} \end{cases}$$

$$q_i = \begin{cases} 1, \text{ If a pump is installed on the catchment surface,} \\ \quad \text{tank or treatment plant location } i \in S \cup U \cup B \\ 0, \text{Otherwise} \end{cases}$$

$$o_{itc} = \text{Amount of water entering the plant location } i \in B \text{ in time } t \in T \text{ through a treatment plant of size } c \in C_i$$

Let $G = (\mathcal{N}, \mathcal{A})$ be a directed graph where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ the set of arcs. Set $\mathcal{A} = \bigcup_{i=1...10} \mathcal{A}_i$ is divided into ten subsets depending on the type of arc. Subsets $A_i, i = 1 \dots 10$, are divided in two categories: arcs of the static network and arcs of the dynamic network. A static network is a set of arcs and nodes that represent the entire system in every time period $t \in T \backslash \{|T|\}$. A dynamic network is the complete set of arcs and nodes (G) that comprise all time periods' static networks, and some additional ones that relate one time period with another.

Figure 2 shows graph general form associated with the static network of each time period $t \in T \backslash \{|T|\}$. The 4-tuple (i, j, t, t'), which indicates that node i of the time period t is connected with the node j of the time period t', represent an arc. Subsets $\mathcal{A}_1$ to $\mathcal{A}_4$ are the arcs of the static network of time period $t \in T \backslash \{|T|\}$. For all $t \in T \backslash \{|T|\}$, $\mathcal{A}_1 \coloneqq \{(i, j, t, t) | i \in S, j \in U\}$ is the set of connection from catchment surfaces to tank locations, $\mathcal{A}_2 \coloneqq \{(i, j, t, t) | i \in U, j \in D\}$ is the set of connection from tank locations to customers that demand water, $\mathcal{A}_3 \coloneqq \{(i, j, t, t) | i \in H, j \in B\}$ is the set of connection from customers that supply greywater to treatment plant locations, and $\mathcal{A}_4 \coloneqq \{(i, j, t, t) | i \in B, j \in U\}$ is the set of connection from tank locations to treatment plant locations.

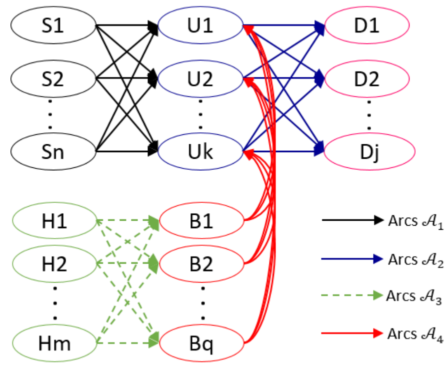

Figure 2. Static network

Subsets $\mathcal{A}_5$ to $\mathcal{A}_{10}$ are arcs only of the dynamic network. $\mathcal{A}_5 \coloneqq \{(i, j, t, |T|) | i \in S \cup H \cup B, j \in \{d\}, t \in T \backslash \{|T|\}\}$ is the set of connection from catchment surfaces, customers that supply greywater and treatment plant locations to the sink node. $\mathcal{A}_6 \coloneqq \{(i, j, |T|, t) | i \in J, j \in D, t \in T \backslash \{|T|\}\}$ is the set of connection from the intervals for water billing, that represent the aqueduct, to customers that demand water. $\mathcal{A}_7 \coloneqq \{(i, i, t, t+1) | i \in U, t \in T \backslash \{|T|\}\}$ is the set of connection from tank location to tank location, this set represent the inventory of water that is stored in each storage tank. $\mathcal{A}_8 \coloneqq \{(i, j, t, t') | i \in H, j \in B, t, t' \in T \backslash \{|T|\}\}$ is the set of connection from customers that supply greywater to treatment plant locations, these arcs are associated with the delay of water treatment in each treatment plant. $\mathcal{A}_9 \coloneqq \{(i, j, |T|-1, |T|) | i \in U, j \in \{d\}\}$ is the set of connection from tank locations of the last time period to the sink node. Finally, $\mathcal{A}_{10} \coloneqq \{(i, j, |T|, |T|) | i \in J, j \in \{d\}\}$ is the set of connection from the aqueduct to the sink node. Figure 3 is a sketch of the graph's general form for the dynamic network.

The nodes in S, H and J, offer rainwater, greywater and potable water, respectively. Nodes in U and B are transit nodes. And nodes in D demand water. The sink node captures surplus of water in the network.

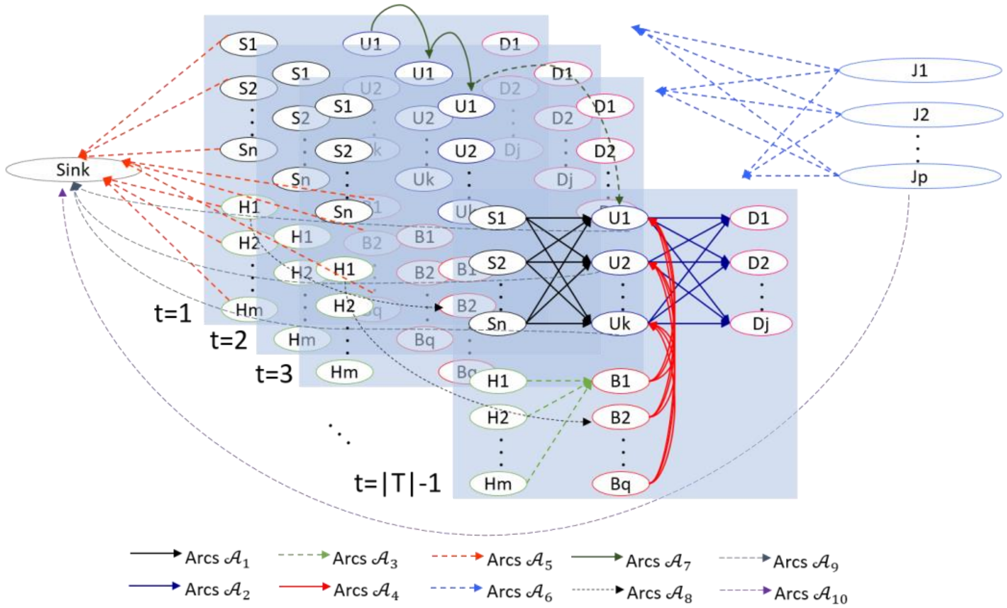

Figure 3. Dynamic network

Constraints

$$\sum_{\{j\in\mathcal{N},t'\in T:\,(i,j,t,t')\in\mathcal{A}\}} x_{ijtt'} - \sum_{\{j\in\mathcal{N},t'\in T:\,(j,i,t',t)\in\mathcal{A}\}} x_{jit't} = b_{it} \quad \forall i\in\mathcal{N}\setminus J\cup\{d\}, t\in T \tag{1}$$

$$-\sum_{\{j\in\mathcal{N},t\in T,t'\in T:\,(j,i,t',t)\in\mathcal{A}\}} x_{jit't} = e_i \quad \forall i\in\{d\} \tag{2}$$

$$\sum_{\{j\in\mathcal{N},t\in T,t'\in T:\,(i,j,t,t')\in\mathcal{A}\}} x_{ijtt'} = e_i \quad \forall i\in J \tag{3}$$

$$\sum_{c\in C_i} w_{ic} \leq 1 \quad \forall i\in U\cup B \tag{4}$$

$$\sum_{\{j\in H,t\in T,t'\in T:(i,j,t,t')\in\mathcal{A}\,\wedge\,(t'-t)\neq r_c\}} x_{jitt'} \leq M(1-w_{ic}) \quad \forall i\in B, c\in C_i \tag{5}$$

$$\sum_{c\in C_i} o_{itc} = \sum_{\{j\in\mathcal{N},t'\in T:\,(j,i,t',t)\in\mathcal{A}\}} x_{jit't} \quad \forall i\in B,\ t\in T \tag{6}$$

$$o_{itc} \leq M\, w_{ic} \quad \forall i\in B,\ t\in T,\ c\in C_i \tag{7}$$

$$x_{ijtt'} \geq \sum_{c\in C_i} \alpha_c o_{itc} \quad \forall i\in B, j\in\{d\}, t\in T, t'\in T\,|\,(i,j,t,t')\in\mathcal{A} \tag{8}$$

$$x_{ijtt'} \leq \tau \sum_{c\in C_i} c\, w_{ic} \quad \forall i\in B, j\in U, t\in T, t'\in T\,|\,(i,j,t,t')\in\mathcal{A} \tag{9}$$

$$x_{iitt+1} \leq \sum_{c\in C_i} c\, w_{ic} \quad \forall i\in U, t\in T\setminus|T|-1\,|\,(i,i,t,t+1)\in\mathcal{A} \tag{10}$$

$$\sum_{\{j\in\mathcal{N},t\in T,t'\in T:\,(j,i,t',t)\in\mathcal{A}\}} x_{jit't} \leq M \sum_{c\in C_i} w_{ic} \quad \forall i\in U \tag{11}$$

$$\sum_{\{t'\in\{1+g\,n,\,\ldots,g+g\,n\}:(i,j,t,t')\in\mathcal{A}\}} x_{ijtt'} \leq m_i \quad \forall i\in J, \in D, t\in T, n\in\left\{0,\ldots,\left\lfloor\frac{|T|-1}{g}\right\rfloor-1\right\} \tag{12}$$

$$\sum_{\left\{t'\in\left\{1+g\left(\left\lfloor\frac{|T|-1}{g}\right\rfloor-1\right),\,\ldots,|T|\right\}:(i,j,t,t')\in\mathcal{A}\right\}} x_{ijtt'} \leq m_i \quad \forall i\in J,\ j\in D, t\in T \tag{13}$$

$$\sum_{\{j\in\mathcal{N}\setminus\{d\},t\in T:(i,j,t,t)\in\mathcal{A}\}} x_{ijtt} \leq M\, y_i \quad \forall i\in S \tag{14}$$

$$\sum_{\{t\in T,t'\in T:(i,j,t,t')\in\mathcal{A}\}} x_{ijtt'} \leq M\, z_{ij} \quad \forall i,j\in\mathcal{N} \tag{15}$$

$$\sum_{\{j\in\mathcal{N}|\tilde{p}_{ij}>0\}} z_{ij} \leq M\, q_i \quad \forall i\in\mathcal{N} \tag{16}$$

$$q_i \leq \sum_{\{j\in\mathcal{N}|\tilde{p}_{ij}>0\}} z_{ij} \quad \forall i\in\mathcal{N} \tag{17}$$

$$\sum_{c\in C_i} h_c w_{ic} + \sum_{c\in C_j} h_c w_{jc} \le a_i \quad \forall i \in U, j \in B | l_{ij} = 1 \quad (18)$$

$$\sum_{\{j\in J, t'\in T:\,(j,i,t',t)\in\mathcal{A}\}} x_{jit't} \ge b'_{it} \quad \forall i \in D,\ t \in T \quad (19)$$

$$x_{ijtt'} \ge 0 \quad \forall (i,j,t,t') \in \mathcal{A} \quad (20)$$

$$z_{ij} \in \{0,1\} \quad \forall i,j \in \mathcal{N}, t \in T, t' \in T | (i,j,t,t') \in \mathcal{A} \quad (21)$$

$$y_i \in \{0,1\} \quad \forall i \in S \quad (22)$$

$$w_{ic} \in \{0,1\} \quad \forall i \in K \cup P,\ c \in C_i \quad (23)$$

$$q_i \in \{0,1\} \quad \forall i \in S \cup U \cup B | \rho_i > 0 \quad (24)$$

$$o_{itc} \ge 0 \quad \forall i \in B,\ t \in T, c \in C_i \quad (25)$$

Equations (1) to (3) are the balance constraints of the network. Equation (4) states that only one treatment plant or tank can be installed per location. Equation (5) guarantees that if a treatment plant capacity is installed in a plant location, greywater supply can only come from a supplier r_c time periods before. Equations (6) and (7) relate variables o_{itc} and $x_{jit't}$ to determine the amount of water that flows to a treatment plant location through a specific treatment plant capacity. Equations (8), (9) and (10) determine the minimum flow of the outgoing arcs from the plant locations nodes to the sink node, the maximum flow of the outgoing arcs from the plant locations nodes to tank location nodes, and the maximum flow of the outgoing arcs of tank location nodes to tank location nodes, respectively. Equation (11) guarantees that the incoming water flow to a tank is allowed in a tank location if any tank size is installed. Equations (12) and (13) bound the amount of water provided by the aqueduct in every billing interval for two months. Equations (14) to (17) activate the decision variables that indicate if a catchment surface is used, if a connection exists, and if a pump is installed. Equation (18) states that tanks or treatment plant installed in any location cannot exceed the available area. Equation (19) guarantees that the potable water required by any customer is satisfied. Finally equations (20) to (25) indicate the variables nature.

In order to consider the different objectives that serve the interests of the agents described before, the following optimization methodology determines an optimal design of the RHGRS based on a solution strategy that has three optimization steps. Each step uses a mixed integer program (MIP).

The solution methodology is inspired in lexicographic multi-objective optimization (also called ε-constraint method), and consists in solving three sequential MIP models. Each model represents one of the three agents previously described by selecting a single objective function in each step. In order to incorporate the objective of the three agents, the second step model incorporates a constraint on the objective function value of the first step model, allowing the design to deviate a given percentage of the optimal solution obtained in the first step model. Similarly, the third step model incorporates an additional constraint on the objective function value of the second step model. The spirit of the methodology is to reach a compromise solution that serves the interest of all the agents in the system while aiming an optimized design. Figure 4 depicts the solution methodology. Notice that there are six possible sequences in which the models might be executed and that the order will affect the final design.

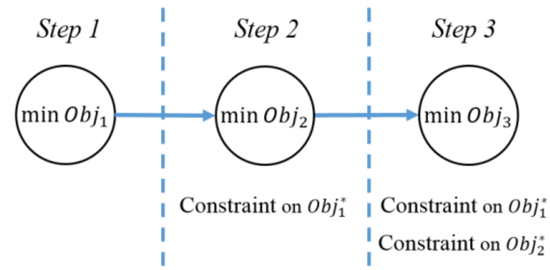

Figure 4. Methodology steps

For example, assume for steps 1, 2, and 3 that the objective function to be used in each model is minimizing the amount of water consumption, minimizing operational and maintenance costs, and minimizing construction costs, respectively. Step 1 encounters a design with a minimum amount of potable water consumption z_1^*. The following model finds a design that minimizes the operation and maintenance costs, but this is subject to a water consumption constraint imposed by the model in step 1. If z_1 is the amount of potable water consumed in the second design, it might deteriorate as much as β_1 percent of z_1^*, i.e., $z_1 \le z_1^*(1+\beta_1)$ in the second model. The solution of the second model has an operation and maintenance cost of z_2^*, which is plugged in the third model in a similar way, i.e., $z_2 \le z_2^*(1+\beta_2)$. Notice that the third model carries the constraints from both step 1 and step 2 models. The final design minimizes the construction cost, but also considers the other two objectives. Figure5 shows a diagram of the complete methodology using the previous example.

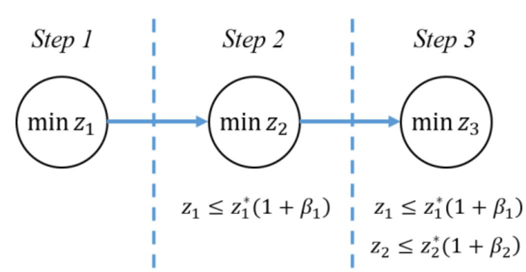

Figure 5. Example of the solution methodology.

With this methodology, there are two ways to prioritize the different objectives. First, the order in which the objective functions are sequenced prioritizes the objectives, that is, the most important goal must go first and then the other two according to their relative importance. Secondly, the percentage of slack β, that is inversely proportional to the importance that wants to be given to each objective. Indeed, the less slack granted to the optimum value of an objective function the more important it is, while if it is not important the percentage of slack increases and thus the objective value may worsen further from the optimal.

Note that whenever the first objective placed in the methodology is to minimize construction costs (construction companies' objective, z_3), the optimization model will decide not to build a RHGRS. For this reason, it is required that the most important objectives are the ones associated with the system user (z_2) and environmental organizations (z_1).

The mathematical expressions for z_1, z_2 and z_3 are:

$$z_1 = \sum_{\{i \in J, j \in D, t \in T, t' \in T : (i,j,t,t') \in \mathcal{A}\}} x_{ijtt'} \qquad (26)$$

$$z_2 = \left(\begin{array}{c} \sum_{i \in B} \sum_{c \in C_i} \varphi_c w_{ic} \\ + \sum_{\{i \in \mathcal{N}, j \in \mathcal{N}, t \in T, t' \in T : (i,j,t,t') \in \mathcal{A} \wedge \gamma_{ij} > 0\}} \gamma_{ij}\, x_{ijtt'} \\ + \sum_{\{i \in J, j \in D, t \in T, t' \in T : (i,j,t,t') \in \mathcal{A}\}} \varepsilon_i\, x_{ijtt'} \end{array} \right) \qquad (27)$$

$$z_3 = \begin{array}{c} \sum_{i \in S} \eta_i\, y_i + \sum_{i \in U \cup B} \sum_{c \in C_i} \lambda_c w_{ic} \\ + \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \beta_{ij}\, z_{ij} + \sum_{i \in S \cup U \cup B} \rho_i\, q_i \end{array} \qquad (28)$$

The expression for operation and maintenance cost (equation (27)) is the sum of three terms: maintenance and operational cost of treatment plants, pumping cost and potable water cost. Construction costs (equation (28)) is the sum of four terms: adequacy costs of catchment surfaces, tanks and treatment plants costs, connection pipes costs and pumps costs.

4.2 Two-stage stochastic optimization model

To incorporate the variability associated with rainfall events a two-stage stochastic optimization model is defined. Essentially, this model has the same network structure of the multi-objective optimization model. The first stage variables are the ones associated with the RHGRS construction, and the second stage variables are those that represent water flow through the network. By using scenario representation, the proposed model incorporate the uncertainty of rainfall. In this case, scenarios correspond to historical data of rainfall events. A probability of occurrence is linked to each scenario. This probability is used to calculate the expected value of the measures that vary depending on the amount of rainfall in each scenario.

Rainfall variability impact in two ways the performance of the system: tank usage and rainwater overflow. It is desirable to construct a system that stores as much rainwater as possible and has the adequate size of storage tanks. On one hand, if the installed tanks are huge all rainwater is stored, and the installation cost is high. Probably during rainy seasons, all rainwater is stored but during dry periods the tank will be empty. In this case the utilization rate of the tank is low and there is no overflow. In the other hand, if the installed tanks are small rainwater is not completely stored, and the installation cost is low. This causes a high utilization rate of tanks, and rainwater overflow during rainy seasons.

The objective function of the two-stage stochastic optimization model is to minimize all the total annual costs of the system. These costs are divided in primary and secondary costs. The primary costs are: potable water cost, operation and maintenance costs, and construction costs. And the secondary costs are three economic penalties that are going to be charge if the system has low tank usage, rainwater overflow, and reusable water wastage.

This problem is known in the literature as a stochastic recourse problem (RP). The behavior of the climate is the recourse, in this case rainfall, and it affects the decisions that are taken on am initial stage.

4.2.1 Model formulation

Consider the same sets, parameters, decision variables, and constrains, defined for the multi-objective optimization model. The following are additional sets, parameters and decision variables:

Sets

L: Set of rainfall scenarios

Parameters

p_l = Probability that scenario $l \in L$ occurs
$\imath$ = Interest rate per period
n = Periods of system life

Decision variables

v_{il} = Water overflow of surface $i \in S$ $f$or scenario $l \in L$

Modify the parameters b_{it} and b'_{it}, and the decision variables $x_{ijtt'}$ and o_{itc} as follows:

Parameters

b_{itl} = Supply/demand $i \in \mathcal{N}, t \in T, l \in L$
b'_{itl} = Potable water demand $i \in D, t \in T, l \in L$

Decision variables

$x_{ijtt'l}$ = Water flowing through the arc $(i,j,t,t',l) \in A$

o_{itcl} = Amount of water entering the plant location $i \in B_l$ in time $t \in T$ through a treatment plant of size $c \in C_i$ for scenario $l \in L$

Constraints (4), (16) to (18), and (21) to (24) will remain the same, the rest will be replaced by the following equations.

Constraints

$$\sum_{\{j\in\mathcal{N},t'\in T:\,(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} - \sum_{\{j\in\mathcal{N},t'\in T,:\,(j,i,t',t,l)\in\mathcal{A}\}} x_{jit'tl} = b_{itl} \quad \forall i \in \mathcal{N}\backslash J \cup \{d\}, t \in T,\ l \in L \quad (29)$$

$$-\sum_{\{j\in\mathcal{N},t\in T,t'\in T,l\in L:\,(j,i,t',t,l)\in\mathcal{A}\}} x_{jit'tl} = e_i \quad \forall i \in \{d\} \quad (30)$$

$$\sum_{\{j\in\mathcal{N},t\in T,t'\in T,l\in L:\,(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} = e_i \quad \forall i \in J \quad (31)$$

$$\sum_{\{j\in H,t\in T,t'\in T,l\in L:(i,j,t,t',l)\in\mathcal{A}\,\wedge\,(t'-t)\neq r_c\}} x_{jitt'l} \leq M(1-w_{ic}) \quad \forall i \in B, c \in C_i \quad (32)$$

$$\sum_{c\in C_i} o_{itcl} = \sum_{\{j\in\mathcal{N},t'\in T:\,(j,i,t',t,l)\in\mathcal{A}\}} x_{jit'tl} \quad \forall i \in B,\ t \in T, l \in L \quad (33)$$

$$o_{itcl} \leq M\, w_{ic} \quad \forall i \in B,\ t \in T,\ c \in C_i, l \in L \quad (34)$$

$$x_{ijtt'l} = \sum_{\{c\in C_i\}} \alpha_c o_{itcl} \quad \forall i \in B, j \in \{d\}, t \in T, t' \in T, l \in L|(i,j,t,t',l) \in \mathcal{A} \quad (35)$$

$$x_{ijtt'l} \leq \tau \sum_{c\in C_i} c\, w_{ic} \quad \forall i \in B, j \in U, t \in T, t' \in T, l \in L|(i,j,t,t',l) \in \mathcal{A} \quad (36)$$

$$x_{iitt+1l} \leq \sum_{c\in C_i} c\, w_{ic} \quad \forall i \in U, t \in T\backslash|T|, l \in L|(i,i,t,t+1,l) \in \mathcal{A} \quad (37)$$

$$\sum_{\{j\in\mathcal{N},t\in T,t'\in T:\,(j,i,t',t,l)\in\mathcal{A}\}} x_{jit'tl} \leq M \sum_{c\in C_i} w_{ic} \quad \forall i \in U, l \in L \quad (38)$$

$$\sum_{\{j\in D,t'\in\{1+gn,\dots,g+gn\}:(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} \leq m_i \quad \forall i \in J, \in D, t \in T, l \in L,\ n \in \left\{0,\dots,\ \left\lfloor\frac{|T|-1}{g}\right\rfloor - 1\right\} \quad (39)$$

$$\sum_{\left\{j\in D,t'\in\left\{1+g\left(\left\lfloor\frac{|T|}{g}\right\rfloor-1\right),\dots,|T|\right\}:(i,j,t,t',l)\in\mathcal{A}\right\}} x_{ijtt'l} \leq m_i \quad \forall i \in J,\ j \in D, t \in T, l \in L \quad (40)$$

$$\sum_{\{j\in\mathcal{N}\backslash\{d\},t\in T,l\in L:(i,j,t,t,l)\in\mathcal{A}\}} x_{ijttl} \leq M\, y_i \quad \forall i \in S \quad (41)$$

$$\sum_{\{t\in T,t'\in T,l\in L:(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} \leq M\, z_{ij} \quad \forall i,j \in \mathcal{N} \quad (42)$$

$$\sum_{\{j\in J,t'\in T:\,(j,i,t',t,l)\in\mathcal{A}\}} x_{jit'tl} \geq b'_{itl} \quad \forall i \in D,\ t \in T, l \in L \quad (43)$$

$$v_{il} \leq M\, y_i \quad \forall i \in S,\ l \in L \quad (44)$$

$$v_{il} \leq \sum_{\{i\in S,j\in\{d\},t\in T,t'\in T:\,(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} \quad \forall i \in S,\ l \in L \quad (45)$$

$$v_{il} \geq \sum_{\{i\in S,j\in\{d\},t\in T,t'\in T:\,(i,j,t,t',l)\in\mathcal{A}\}} x_{ijtt'l} - M(1-y_i) \quad \forall i \in S,\ l \in L \quad (46)$$

$$x_{ijtt'l} \geq 0 \quad \forall (i,j,t,t',l) \in \mathcal{A} \quad (47)$$

$$z_{ij} \in \{0,1\}\ \forall i,j \in \mathcal{N}, t \in T, t' \in T, l \in L|(i,j,t,t',l) \in \mathcal{A} \quad (48)$$

$$o_{itcl} \geq 0 \quad \forall i \in B,\ t \in T, c \in C_i, l \in L \quad (49)$$

$$v_{il} \geq 0 \quad \forall i \in U, l \in L \quad (50)$$

Equations (29) to (31) are the balance constraints of the network. Equation (32) guarantees that if a treatment plant capacity is installed in a plant location, greywater supply can only come from a supplier r_c time periods before. Equations (33) and (34) relate variables o_{itcl} and $x_{jit'tl}$ to determine the amount of water that flows to a treatment plant location through a specific treatment plant capacity. Equations (35), (36) and (37) determine the minimum flow of the outgoing arcs from the plant locations nodes to the sink node, the maximum flow of the outgoing arcs from the plant locations nodes to tank location nodes, and the maximum flow of the outgoing arcs of tank location nodes to tank location nodes, respectively. Equation (38) guarantees that the incoming water flow to a tank is allowed in a tank location if any tank size is installed. Equations (39) and (40) bound the amount of water provided by the aqueduct in every billing interval for two months. Equations (41) and (42) activate the decision variables that indicate if a catchment surface is used, and if a connection exists. Equation (43) guarantees that the potable water required by any customer is satisfied. Equations (44) to (46) model the rainfall overflow. Finally equations (47) to (50) indicate the variables nature.

Objective function

$$
\min \begin{aligned}
&\left[\sum_{i\in S}\eta_i\, y_i + \sum_{i\in U\cup B}\sum_{c\in C_i}\lambda_c w_{ic} + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\beta_{ij}\, z_{ij} + \sum_{i\in S\cup U\cup B}\rho_i\, q_i\right]\left(\frac{i(1+i)^n}{(1+i)^n-1}\right) \\
&+ \sum_{i\in B}\sum_{c\in C_i}\varphi_c w_{ic} \\
&+ \sum_{l\in L} p_l \Bigg[\sum_{\{i\in\mathcal{N},j\in\mathcal{N},t\in T,t'\in T:(i,j,t,t')\in\mathcal{A}\wedge\gamma_{ij}>0\}}\gamma_{ij}\, x_{ijtt'l} + \sum_{\{i\in J,j\in D,t\in T,t'\in T,l\in L:(i,j,t,t')\in\mathcal{A}\}}\varepsilon_i\, x_{ijtt'l} \\
&+ \frac{\sum_{k\in J}\varepsilon_k}{|J|}\left(\sum_{t\in T}\sum_{i\in S\cup H} b_{itl} - \sum_{\{i\in S\cup H,j\in U\cup B,\, t\in T,t'\in T:(j,i,t,t,l)\in\mathcal{A}\}} x_{jitt'l}\right) \\
&+ \frac{\sum_{c\in K}\frac{\lambda_c}{c}}{|K|}\sum_{i\epsilon U}\sum_{t\in T\setminus\{|T|\}}\left(\sum_{c\in C_i} c\, w_{ic} - x_{iitt+1l}\right) \\
&+ \frac{\sum_{k\in J}\varepsilon_k}{|J|}\sum_{i\in S} v_{il}\Bigg]
\end{aligned} \qquad (51)
$$

This objective function is the sum of three terms. The first one represent the annuity that must be pay for the construction of the system, and the second term is the annual operation and maintenance costs of treatment plants. The third term is the expected value of the sum of the pumping costs, potable water costs, penalty for cubic meter of reusable water wasted, penalty for cubic meter that is empty in a storage tank, and penalty for cubic meter of rainwater not used (overflow). The coefficient (or economic penalty) for reusable water wastage and for tank overflow, is the potable water cubic meter average cost of all billing intervals; and the coefficient for low tank usage is the average of the cost of a cubic meter of storage per storage tank.

5. Results

5.1 Case study

A case study of social interest housing residential unit in the city of Soacha in Colombia is used to show the applicability of the proposed optimization models. Composed of 29 buildings, the residential unit has 696 apartments (users), and 2,784 inhabitants. The overall consumption of potable water for all uses is 96,228.5 m^3/year. Shower, bath, faucet, clothes washing, and dishwashing require 65,679.4 m^3/year of potable water, which corresponds to 68.3% of the total potable water consumption. The latter means that 30,549.1 m^3/year, 31.7% of the total potable water consumption, used for toilet flushing, watering plants, and cleaning can be substituted with recycled water.

The potable water billing intervals are three: subsidy, which correspond to the potable water consumption from 0 to 12 m^3 every two months; basic, from 12 to 40 m^3; and no basic, for more than 40 m^3. The cost of a cubic meter of potable water in each interval is $0 USD, $0.87 USD, and $1.44 USD, respectively. Therefore, the cost of the 138.2 m^3 consumed per apartment during one year will be $62 USD.

Figure 6 shows a satellite view of the residential unit and the possible location of tanks and treatment plants. The cost of storage tanks vary from $33 to $115 USD depending on the market size and capacity. The treatment plants costs, capacity and efficiency where the ones specified by Oh et al. (2015) and Do Couto et al. (2015). The available surface area that can be conditioned for rainwater harvesting is 7,750 m^2. The installing cost a catchment surface varies between $35 and $70 USD.

The rainfall data used was from the station P-92 Las Huertas, located in Soacha near the residential unit. The daily rainfall, in millimeters (mm), for the years 1991 to 2010 was available.

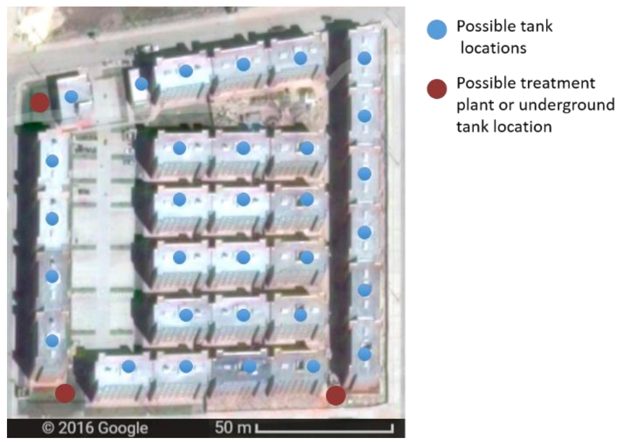

Figure 6. Residential unit satellite view

The optimization models were solved using the optimization software Xpress MP, in a computer with a computer processing unit Intel® CPU E5-2695 v2 @ 2.40Hz with 2 processing units, 32 GB of RAM, and a 64 bits system.

5.2 Multi-objective optimization model results

Table 1 shows the percentage of potable water consumed and saved, the maintenance and operational costs plus the potable water costs, the construction cost, the computational time and the optimality gap, for each step of the proposed optimization methodology. The instance has a planning horizon of one year and a time unit of weeks. There are 138,469 variables and 18,493 constraints in step 1, 18,494 in step 2, and 18,495 in step 3. In this case, the values of β_1 and β_2 are set to cero since it is desirable to have a system that uses the minimum amount of potable water, with the minimum the maintenance and operational costs plus the potable water costs. The rainfall data used is the corresponding to the year 2000, which has the mean value of total millimeters of rainfall, between the years 1991 and 2000. The results show that only 0.4% of the total potable water consumed is used in no potable uses. If the proposed design for the system is construct, the charge for water supply that one user must pay during one year increased 30.6%, respect the charge that the user must pay if no system is construct. An extra charge of $79 USD must be pay by all user for the construction of the system. The three optimization problems were solved to optimality in a total time of 1,131.4s.

Table 1. Multi-objective optimization model results with time unit of weeks

Step	Potable water consumed	Potable water saved	Maintenance and operational costs +potable water cost		Construction cost		CT	GAP
			Total cost	Per user	Total cost	Per user		
1	68.7%	31.3%	$89,854 USD	$129 USD	$388,220 USD	$557 USD	32.6 s	0%
2			$56,201 USD	$81 USD	$281,776 USD	$282 USD	28.9 s	0%
3					$54,833 USD	$79 USD	1,069.9 s	0%

Table 2 shows the same results for the same problem, but using days as the time unit. The results show that 0.5% of the total potable water consumed is used in no potable uses, 0.1% more than the one used with the time unit as weeks. This can be explained due the potential accuracy in the inventory levels in the storage tanks when using a daily time unit. The constraint that guarantees that the storage capacity is not exceeded lowers the amount of recycled water that can be used. If the proposed design for the system is construct, the charge for water supply that one user must pay during one year increased 29%, respect the charge that the user must pay if no system is construct. An extra charge of $80 USD must be pay by all user for the construction of the system. The first two optimization problems were solved to optimality in a total time of 667.8s, and the third problem after 137,341s has an optimality gap of 7%.

Table 2. Multi-objective optimization model results with time unit of days

Step	Potable water consumed	Potable water saved	Maintenance and operational costs +potable water cost		Construction cost		CT	GAP
			Total cost	Per user	Total cost	Per user		
1	68.8%	31.2%	$89,854 USD	$129 USD	$302,282 USD	$434 USD	310.3 s	0%
2			$56,153 USD	$80 USD	$1,058,817 USD	$1,521 USD	357.5 s	0%
3					$55,515 USD	$80 USD	137,341.0 s	7%

Table 3 shows the results for the last step of the optimization methodology for the same problem changing the rainfall input data, using weeks as the time unit. Three scenarios were selected: the year 1997, the year 2000 and the year 2010, which had the minimum, medium, and maximum amount of total rainfall in millimeters, respectively. The RHGRS design is the same for the three scenarios, but differences in the results arise in the amount of potable water consumed and saved, and in the maintenance and operational costs plus the potable water costs. As the amount rainwater increase, the pumping costs and potable water costs decrease.

Table 3. Multi-objective optimization model results with time unit of weeks for different rainfall input data

Year	Potable water consumed	Potable water saved	Maintenance and operational costs +potable water cost		Construction cost	
			Total cost	Per user	Total cost	Per user
1997	69.0%	31.0%	$57,445 USD	$82.5 USD		
2000	68.7%	31.3%	$56,201 USD	$81.0 USD	$54,833 USD	$79.0 USD
2010	68.6%	31.40	$54,812 USD	$78.7 USD		

5.3 Two-stage stochastic optimization model results

For the two-stage stochastic program, the model used three equally-likely rainfall scenarios corresponding to the years 1997, 2000, and 2010. The value of the resulting objective function is $ 62,518 USD. Note that the objective function of this problem is an artificial object used to guide the search of an optimal design in the feasibility region of the problem. Table 4 shows the results of this optimization problem for each scenario (percentage of potable water consumed and saved, the maintenance and operational costs plus the potable water costs and the construction cost). The construction cost is the same for the three scenarios because only one system is constructed.

Table 4. Two-stage stochastic optimization model results with time unit of weeks

Year	Potable water consumed	Potable water saved	Maintenance and operational costs +potable water cost		Construction cost		CT	GAP
			Total cost	Per user	Total cost	Per user		
1997	69.80%	30.20%	$57,175 USD	$82.1 USD				
2000	69.39%	30.61%	$55,842 USD	$80.2 USD	$53,986 USD	$77 USD	439,573 s	1.2%
2010	69.37%	30.63%	$54,460 USD	$78.2 USD				

To evaluate how much is gained by using the two-stage recourse model, we calculate the *Expected Value of Perfect Information* (EVPI) and the *Value of the Stochastic Solution* (VSS). This values are an estimation of the benefits of using the recourse model, which includes rainfall uncertainty, instead of using the deterministic model, which considers the expected value of rainfall. To calculate EVPI we must solve the perfect information models in which the rainfall parameter is deterministic and corresponds to the rainfall of each scenario (1997, 2000, and 2010). And to find the VSS we must solve the expected value models. First we solve the optimization problem using the expected value of rainfall (EV) of the three scenarios as the deterministic parameter; and second, fixing the values found for the first stage variables we solve the optimization problem for each scenario. Table 5 shows the values for the: objective functions, the best bound, computational time, and the optimality gap found for all the models.

Table 5. Results of the recourse problem, perfect information models and average static models

Model	Objective function (USD)	Best bound (USD)	CT	Gap
RP	$62,518	$61,711	439,573.0 s	1.2%
1997	$62,533	$62,329	439,672.0 s	0.3%
2000	$62,221	$62,020	439,682.0 s	0.3%
2010	$62,298	$61,910	439,732.0 s	0.6%
EV	$68,901	$68,901	3907.8 s	0.0%
EV- 1997	$69,236	$69,236	9.3 s	0.0%
EV- 2000	$69,050	$69,050	23.2 s	0.0%
EV- 2010	$69,058	$69,058	23.0 s	0.0%

Since the optimal solutions for the recourse problem and the perfect information models were not found the EVPI and the VSS, cannot be calculated exactly. Therefore using the objective function (OF) and the best bound a lower bound and upper bound for this values were found (Table 6).

Table 6. Bounds for EVPI and VSS

	Lower bound	Upper bound
EVPI	$ 167 USD	$ 431 USD
% respect the OF	0.27%	0.69%
VSS	$ 6,597 USD	$7,404 USD
% respect the OF	10.55%	12.00%

EVPI can be interpreted as the price one would be willing to pay for complete information on future events if this were possible. Since this value is between $167 USD and $431 USD, which represent 0.27% and 0.69% of the value of the objective function, we can conclude that the role of randomness is in our case study has a low importance. VSS indicates the usefulness of using the stochastic model for this problem. Given that this value is between $6,597 USD and $7,404 USD, which represent 10.55% and 12% of the value of the objective function, we can say that indeed it is useful to use the recourse model.

5.4 Optimization models results comparison

Both optimization models provide a system design that is useful for the three rainfall scenarios selected. Table 7 shows the average of the percentage of potable water consumed and saved, the maintenance and operational costs plus the potable water costs, and the construction cost of the scenarios for both optimization models.

The results show that the designs found using the two optimization models are very similar in terms of potable water saved and system costs. The potable water consumed and save differs in 0.75%, the cost of water supply for a user varies in $0.4 USD from one design to another, and the construction cost of the system found using the multi-objective optimization model is $847 USD more expensive than the one found with the two-stage approach.

Table 7. Average measures for years 1997, 2000, and 2010

Model	Potable water consumed	Potable water saved	Maintenance and operational costs +potable water cost		Construction cost	
			Total cost	**Per user**	**Total cost**	**Per user**
Multi-objective	68.75%	31.23%	$56,152 USD	$80.6 USD	$54,833 USD	$79.0 USD
Two-stage	69.52%	30.48%	$55,825 USD	$80.2 USD	$53,986 USD	$77.0 USD

6. Final remarks and future work

The global need of reducing the potable water consumption requires novel solutions such as water recycle systems. Rainwater harvesting and greywater recycling are two strategies that can be used to reduce potable water consumption, mitigating flooding events and pollution. To solve the problem of designing RHGRSs in residential units, two optimization models are proposed. Using any of the two approaches, multi-objective or two-stage stochastic optimization, more than 30% of the total potable water consumed by the case study residential unit is saved.

Given the results of this two approaches we can conclude that to determine the system design, both approaches involve the different personal interests of the three agents (namely, environmental organizations, system users, and construction companies). This can be supported by the fact that the difference between the average measures for the years 1997, 2000, and 2010, is small for all the measure. This means that the potable water consumed and saved, the maintenance and operational costs plus the potable water costs, and the construction cost are similar with both optimization models.

The two-stage stochastic optimization approach showed that considering the average rainfall of the three selected scenarios for the case study in Soacha is not enough to determine the optimal design of the RHGRS. Also, that it is not necessary to know the exact rainfall per time period, to determine which system can be constructed.

With both optimization models it is shown that the randomness of rainfall has low importance for the selected case study. In one hand, the multi-objective model gave the same system design for the three rainfall scenarios selected, and in the other, the EVPI of the two-stage optimization model was low.

Future work includes decomposition methods to find the optimal solutions for the two-stage stochastic model. Also, use specialized network algorithms to reduce the computational time.

7. Discussion

In the future, it will be necessary to build net-zero water use buildings to mitigate the increasing potable water scarcity in the world. Since net-zero water use relies on using renewable sources of water, installing RHGRSs is one step toward this objective. The question that we must answer is: how to encourage the construction of RHGRSs? For construction companies RHGRS construction often implies cost overrun in residential construction projects, and for users RHGRS operation and maintenance increases the monthly payment for water.

The proposed optimization models are analytic tools for decision makers, who can use them to evaluate different strategies. This models are not only useful for construction companies who's interest is to design RHGRSs, but they can also contribute to create public policies toward the development of sustainable cities. To make this we can use the proposed optimization model to design systems in many study cases and determine general characteristics of residential units that favor the construction of RHGRSs. Local governments can incorporate this information to develop public policies for cities that aim the reduction of potable water consumption.

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