Wind speed Extrapolation based on power law correction through a Bayesian heteroskedastic model

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Abstract – **In this study we propose a new methodology for extrapolate (in height) the wind speed given a fixed location. This problem is of great importance given that reals measurements at different heights are very costly, however, many times it is necessary to account for the complete profile of speed. In particular, this problem arises in the evaluation of wind energy projects. Although there several methods for extrapolation, such as the power law, there are no consistent methodologies that use real data, mechanistic models, and at the same time can reproduce the stochastic behavior of wind speed.**
The model we propose translate the wind speed probability distribution to a continuum of heights combining a statistical model (from data taken at selected points) and a theoretical model (power law). We use the Box-Cox transformation and a heteroskedastic Bayesian model that uses the physical model as a prior information. We prove by simulation that the method performs adequately and has a better fit than using just the statistical or the physical model.

KEYWORDS – Bayesian Methods, Extrapolation, Heteroskedasticity, Power Law

1. Introduction

Wind energy, being one of the main sources of renewable energy in the world, is presented as a great attraction nowadays, based on the growing technological development of efficient turbines(Zekai, Tarkan, & Abdusselam, 2012). Hence, the tendency is to develop models of wind turbine with powers between 6-8 MW (megawatts), with heights regularly above 100 m. In a scenario with a vertiginous growth, chasing the wind at a height of 100 m implementing the classic meteorological masts is an increasingly expensive solution. The use of instruments such as LIDAR or SODAR is certainly more appropriate for determining the wind profile, although it greatly increases the costs of the wind energy project, often making it economically unfeasible (Gualtieri, 2015).

In the specific case of Colombia, the country counts with abundant hydro resources, and the historic lack of recognition of the benefits of renewable energy generation, have limited the call for government incentives in sectors like wind, geothermal and solar. There is a widespread perception that additional hydro capacity is the best solution, with fossil fuel capacity laid by for dry years (Norton Rose Fulbright, 2016).

Due to their intermittent nature, in Colombia wind and solar energy are not competitive when compared to other kinds of energy generation. Hydrocarbons, on the other hand, have the advantage of being easily transportable and relatively cheap. These characteristics still make hydrocarbons more attractive to consumers compared to wind and solar sources. It remains true that consumers tend to prefer the cheapest and most effective products (Vergara, Deeb, Toba, Cramton, & Leino, 2010).

In addition, there are very few companies with the financial and technical capabilities to assume the high initial costs of renewables projects, including the lack of public funds for the development of viable projects. Moreover, uncertainty about the generation capacity and reliability of non-dispatchable energy increases the financial sector's perception of risk (Norton Rose Fulbright, 2016). This constitutes an economic barrier to any project and to any potential economies of scale which could lower the price of non-conventional energy. Higher costs create disincentives for companies to replace conventional sources of energy with non-conventional ones.

Therefore, in the Colombian context, it would be ideal to propose a model that allows reducing the initial investment cost in a significant way. The development of a wind energy field requires preliminary studies that guarantee a constant source of wind, so that an adequate energy service can be provided. As will be mentioned, traditional models have high uncertainty and the use of technologies may involve high costs that few existing investors are unwilling to assume. Hence, the proposed model should be simple to implement but robust enough to adequately estimate the desired wind speed profile.

2. Extrapolation models

Over the last decades, various mathematical and modeling approaches have been implemented to estimate the wind resource at the height of the turbines. The foregoing, in order to evaluate the feasibility of carrying out a wind energy project in a determined place. The models used include but are not limited to: numerical models of data scale reduction, CFD (computational fluid dynamics) models and statistical methodologies such as machine learning or artificial neural networks (Gualtieri, 2017). In addition to requiring a large amount of input data, the above methods are usually computationally expensive and are mostly site-specific. On the other hand, knowledge about extrapolation models of wind speed has preferences for a broader spectrum of applications to predict the wind resource at different heights of turbine centers (Gualtieri & Secci, 2012). Usually, based on information that is easily accessed at surface height.

The uncertainty in the extrapolation of wind speed is considered one of the most critical factors when evaluating wind energy projects, specifically considering the increase in the size of modern wind turbines. Generally, in the absence of wind speed measurements at heights relevant to the wind farm, it is necessary to extrapolate the speed profile from data at available heights. Depending on the extrapolation made, critical errors can occur between the estimated energy production and the actual production. The difference between the predicted and observed wind energy production can be up to 40% due to the effects of turbulence, the time interval of measurement of the data and the extrapolation of the data at the height of the wind turbine (Zekai et al., 2012).

The Power Law and Logarithmic law are the two analytical models most used to extrapolate wind speeds at higher altitudes. In fact, although there is no uniform analytical expression valid for all stability conditions, it has been found that the PL (Power law) provides a reasonably better representation and better wind speed profiles than the LogL (Logarithmic law), at least in unstable and neutral conditions (Gualtieri, 2017). In short, the model is based on finding the magnitude of the exponent (wind shear coefficient), which provides the best adjustment to the wind speed between two heights. Therefore, since surface velocity wind measurements (9-20 m) are fairly easy to achieve, the availability of a reliable estimation method is particularly useful for extrapolating the wind resource at the height of the hub(Gualtieri & Secci, 2014).

2.1 Power Law

The PL equation is a simple but useful model to estimate the vertical wind profile, it was proposed for the first time by Hellman et al. (1916):

$$\left(\frac{Z_1}{Z_2}\right)^n = \left(\frac{V_1}{V_2}\right)$$

Where,

$\boldsymbol{V_1}, \boldsymbol{V_2}$: Wind speed (m/s) at each height level, knowing that $\boldsymbol{V_2} > \boldsymbol{V_1}$

$\boldsymbol{Z_1}, \boldsymbol{Z_2}$: Height (m) at which a certain wind speed is found, such that $\boldsymbol{Z_2} > \boldsymbol{Z_1}$

$\boldsymbol{n}$: It is the exponent of PL, which is a complex function of weather conditions, topography, roughness of the surface, environmental conditions, humidity stability and the rate of meteorological lapse.

It should be noted that the PL equation is an engineer / empirical formula used to represent the degree of stability or turbulence through a number, but this does not present a physical foundation (Gualtieri & Secci, 2014). Obviously, within the model it is necessary to take into account standard deviations and cross correlation coefficients since these parameters are inherent to the instability of the conditions.

2.2 Weibull Distribution Parameter Extrapolation

On the other hand, the extrapolation of the wind speed profile has approximation by time series. The probability distribution function (pdf) Weibull has been used for the empirical distribution of the wind velocity profile. Starting from the definition of the Pdf Weibull:

$$P(V) = \left(\frac{k}{c}\right)\left(\frac{V}{c}\right)^{k-1} \exp\left[-\left(\frac{V}{k}\right)^k\right]$$

Where,

k: Shape a dimensional parameter.
c: It is a scale parameter with the velocity dimension.

The distribution presents its expected value, variance and moment of order m as shown below:

$$E(V) = c\Gamma\left(1+\frac{1}{k}\right)$$

$$Var(V) = c^2\left[r\left(1+\frac{2}{k}\right) - r^2\left(1+\frac{1}{k}\right)\right]$$

$$E(V^m) = c^m\Gamma\left(1+\frac{m}{k}\right)$$

Now, starting from what was proposed by Zekai et al. (2012) and by statistical definition the coefficient of variation is obtained as:

$$C_V = \sqrt{\frac{\Gamma^2\left(1+\frac{2}{k}\right)}{\Gamma\left(1+\frac{1}{k}\right)} - 1}$$

Where do you get,

$$k = \frac{1}{C_V^{1.086}}$$

$$c = \frac{\bar{V}}{\Gamma\left(1+\frac{1}{k}\right)}$$

When replacing in the final extension of the PL it is obtained that:

$$\left(\frac{Z_1}{Z_2}\right)^n = \left(\frac{c_1}{c_2}\right)[1 - r_{12}(k_1k_2)^{-0.921} + k_2^{-1.841}]$$

Thus,

$$n = \frac{\ln\frac{c_1}{c_2} + \ln[1 - r_{12}(k_1k_2)^{-0.921} + k_2^{-1.841}]}{\ln\left[\frac{Z_1}{Z_2}\right]}$$

3. METHODOLOGY

Statistical models are commonly used in studies seeking quality improvement. However, such models tend to have a poor performance if it is necessary to make predictions outside the range of study. On the other hand, engineering models derived from purely physical processes do not always coincide in an accurate way with reality (Roshan & Melkote, 2009). Hence, an approach is considered in which statistical models are developed that are estimated from data generated from engineering or physical models. This type of models seeks to generate predictions more accurate than those granted by theoretical models but at the same time they are less computationally expensive. Next, the example proposed by Roshan et al. (2009):

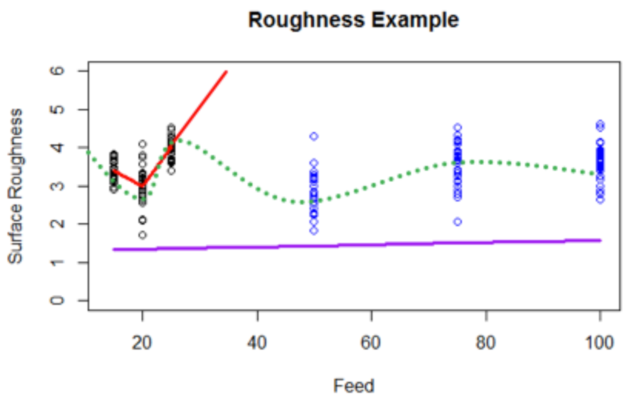

Figure 1 Engineering-Statistical Model vs others(Roshan & Melkote, 2009). Red Line represents the statistical model. Purple line represents the Engineering model. Green dotted line represents the statistical engineering model.

From the image above, it is observed that the statistical engineering model is much better than the engineering model and the statistical model. It should be noted that, the statistical model presents the best performance of all, however, when evaluated outside the training range this is completely unfortunate. Likewise, the engineering model adequately represents the behavior of the data, but it is completely remote from the real measurements.

Based on the above, right away the methodology described by Roshan & Melkote et al. (2009) to adjust a statistical-engineering model. This is the methodology that is intended to adjust to the model of the power law for the extrapolation of the wind velocity profile, taking into account as adjustment parameters those referring to the Weibull distribution.

Let Y be the response variable of the theoretical model and $x = (x_1, \dots, x_p)^t$ the factors. The response is a random variable and has an inherent error due to the factors and their measurement. Then you have to,

$$Y = \mu(x) + \epsilon$$

Where, $\mu(x)$ is the mean of Y for a given value of x and $\epsilon \sim N(0, \Sigma_y)$. The objective is to find the unknown function $\mu(x)$. What we have then is an engineering model $f(\boldsymbol{x}; \boldsymbol{\eta})$ and the output of an experiment $(x_1, y_1), \dots, (x_n, y_n)$ where $\boldsymbol{\eta} = (\eta_1, \dots, \eta_q)^t$ corresponds to the unknown calibration parameters. This is a value that is omitted from the equation because it is adjusted with fixed values of $\boldsymbol{\eta}$ with its real value of $\boldsymbol{\eta}^*$. Obviously, the parameters $\boldsymbol{\eta}^*$are unknown and it is necessary to estimate them from the physical properties.

A Bayesian approach is useful in formulating this problem. Because the engineering model is available before obtaining the data, from this a prior distribution is obtained for (x). Specifically, the result of the engineering model is taken as the average of the previous distribution. Then, based on the data, we can obtain the posterior distribution of (x). The subsequent distribution incorporates information on the engineering model, as well as information on the data. This is exactly what we need. The statistical-engineering model is simply the mean of the subsequent distribution (Roshan & Melkote, 2009).

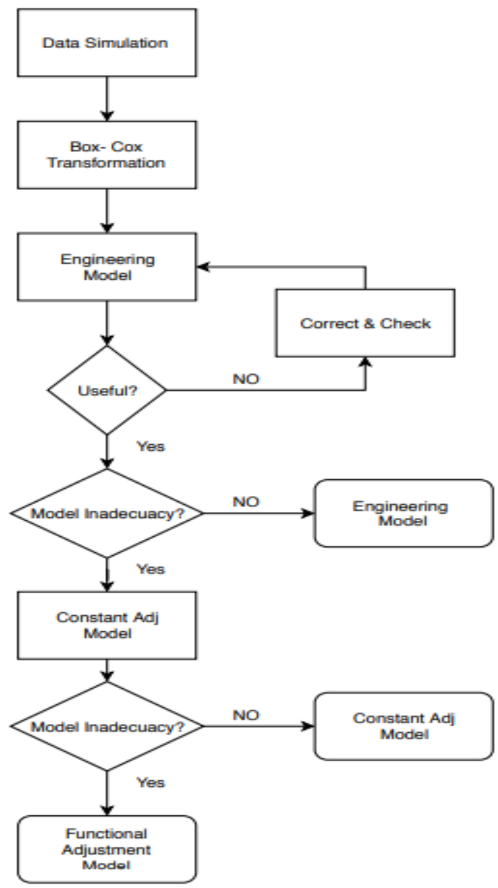

Figure 2 Estrategia para construcción de modelo secuencial(Roshan & Melkote, 2009)

The sequence is as illustrated in the previous figure. In the first place, it is evaluated if the theoretical or engineering model is useful for the forecasting of the data. This can be evidenced by correlation graphs, if there is evidence the model is used, otherwise the model to be used must be re-evaluated. Next, the quality of the engineering model is evaluated. The author in this step uses the "inadequacy measure" index:

$$MI = \frac{1}{n} \sum_{i=1 \in n} \left(y_i - \widehat{\mu_i}^E\right)^2$$

Where $\hat{\mu}^E = f(x_i)$. If the engineering model is good, then $MI = \sigma^2$. Therefore, if the MI is small enough and approaches the variance σ^2 then we can use it for prediction. Otherwise, the necessary statistical adjustments are made. First, a "Location-scale adjustment" is carried out:

$$\mu(x) - f(x) = \beta_0 + \beta_1\big(f(x) - \bar{f}\big)$$

Where $\bar{f} = \sum_{i=1}^{n} \frac{f(x_i)}{n}$. The previous distribution for $\mu(x)$ is calculated from the parameters: $\beta_0 \sim \mathcal{N}(0, \tau_0^2)$ and $\beta_1 \sim \mathcal{N}(0, \tau_1^2)$, where β_0 and β_1 are independent. Note that $\mu\,(x)$ is $f(x)$. This is called the constant-adjusted model because two constants are used for the adjustment. In this stage the predictor is denominated as:

$$\mu^C(x) = f(x) + \beta_0 + \beta_1\big(f(x) - \bar{f}\big)$$

The MI is computed again:

$$MI = \frac{1}{n}\sum_{i=1 \in n} \left(y_i - \widehat{\mu_i}^{C}\right)^2$$

Again, evaluate if the MI is close enough to σ^2, if so, the sequence is stopped, otherwise proceed to the next step. On this occasion, we seek to adjust a more sophisticated model:

$$\mu(x) - \mu^C(x) = \delta(\boldsymbol{x}; \boldsymbol{\alpha})$$

Where $\delta\ (x;\ \alpha)$ is used to capture the "model inadequacy" of the model adjusted by constants. This is called the functional adjusted model, since the functional form of the predictor is different from that established by the engineering model. The form of the function δ is obtained by residuals analysis of the model adjusted by constants. We use the methodology of $\delta(\boldsymbol{x}; \boldsymbol{\alpha}) = \sum_{i=0}^{m} \alpha_i u_i(x)$, where u_i are known functions in x. The previous distribution chosen for α is carried out so that $E\ (\delta\ (x;\ \alpha))\ =\ 0$. In this way, $\mu^C(x)$ and α of the data are estimated; the author recommends doing this step by step, avoiding that the statistical model replaces the engineering model. Now, the functional adjustment is given by:

$$\hat{\mu}^F(x) = \hat{\mu}^C(x) + \delta(\boldsymbol{x}; \boldsymbol{\alpha})$$

The unknown calibration parameters (η) can easily be incorporated into the construction sequence by treating them as hyperparameters. This is correct because (η) enters the Bayesian model only through $f\ (x;\ \eta)$ which is taken as the mean of the previous distribution of $\mu\ (x)$. We start the procedure with a least squares estimation of the next stage and we obtain a Bayesian empirical estimate from the model adjusted by constants. Since (η) does not appear in $\delta\ (x;\ \alpha)$, the functional adjustment model remains the same as before.

3.1 Box Cox Normalization

It has been found that the observed wind speed distribution is adequately described by a Weibull distribution. However, the proposed Bayesian model requires that the wind distribution profile fits a normal distribution, since $\boldsymbol{\varepsilon} \sim \boldsymbol{N}\,(\boldsymbol{0}, \Sigma_y)$ is expected. Therefore, it is necessary to perform a parametric and robust transformation on the velocity profile for each height $\boldsymbol{z_i}$. The Box-Cox transformation was proposed by Box and Cox in 1964 and was used to transform non-normal data (Yang, See, & Xie, 2003). This transformation uses a parameter $\boldsymbol{\lambda}$, which must be selected in an appropriate way to transform the data as close as possible to normality. To obtain the optimum $\boldsymbol{\lambda}$ value, the Box-Cox transformation methodology requires the maximization of a logarithmic likelihood function. Next the general formulation:

$$X^{(\lambda)} = \begin{cases} \dfrac{X^{\lambda} - 1}{\lambda}, & for\ \lambda \neq 0 \\ \ln X, & for\ \lambda = 0 \end{cases}$$

In this case, a grid of λs is proposed that maximizes the logarithmic likelihood joint function for a normal distribution,

$$l(\mu, \sigma^2; x_1, \dots, x_n) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{n}\left(x_j^{(\lambda)} - \mu\right)^2$$

We would like to found which λ in a maximizes the next joint likelihood distribution,

$$\max \prod_{i=1}^{n} l\left(\mu, \sigma^2; x_1^{(\lambda)}, \dots, x_n^{(\lambda)}\right) \mid \lambda \in \{-2, 2\}$$

Performing normal distributions transformation of the data for each z_i that is presented where $x_i = v_{z_i}$.

3.2 Constant Adjusted Model

Given the characteristics of the input data, a Bayesian heteroskedastic regression model is proposed. Modeling heteroskedasticity should improve the efficiency of regression coefficient estimates, the most important effect being prediction, allowing predictive inferences to be more precise for some units and less accurate for others. Hence, the following regression model is used, for n independent units in k covariates,

$$y_i | \beta, \sigma^2, \theta \sim N\big((X\beta)_i, \sigma^2 f(z_i, \theta)\big)$$

For this case, the variances are considered to be governed by known weights z_i, directly related to the height at which the wind velocity profile is measured, and a weighted least squares regression is modeled where the variance is proportional to $1/z_i$. The following functional form is considered

$$f(z_i, \theta) = \frac{z_i^{-\theta}}{\left(\prod_{i=1}^{n} z_i^{-\theta}\right)^{\frac{1}{n}}}$$

Additionally, non-informative prior distributions for the components of the variance are considered. For a heteroskedastic model there seems to be no clear choice for a non-informative density of the prior over θ, but using

$p(\theta) = 1$ over a unit interval seems reasonable. Therefore, a prior of uniform density in θ will be used.

First of all, consider the engineering model, our power law, without parameter calibration. Therefore, the model adjusted by constant is given by

$$Y - f(\boldsymbol{v_1}, \boldsymbol{z_1}, \boldsymbol{z_2}) = \beta_0 + \beta_1\left(f(\boldsymbol{v_{1i}}, \boldsymbol{z_{1i}}, \boldsymbol{z_{2i}}) - \bar{f}\right) + \epsilon$$

Where

$$f(\boldsymbol{v_{1i}}, \boldsymbol{z_{1i}}, \boldsymbol{z_{2i}}) = v_1\left(\frac{z_2}{z_1}\right)^{\alpha}$$

For $\bar{f} = \sum_{i=1}^{n} f(\boldsymbol{v_{1i}}, \boldsymbol{z_{1i}}, \boldsymbol{z_{2i}})/\boldsymbol{n}$ and $\epsilon \sim N(0, \Sigma_y)$, $\boldsymbol{\beta}|\boldsymbol{\tau^2} \sim N(0, \Sigma_\beta)$ which are independent. Now, re expressed in matrix form, consider that $\boldsymbol{F}$ is a matrix of $n \times 2$, where the first column is $1's$ and the second is $(f_1 - \bar{f}, \ldots, f_n - \bar{f})'$ and $f = (f_1, \ldots, f_n)'$, where $f(v_{1i}, z_{1i}, z_{2i}) = f_i$. Therefore, the model adjusted by constant can be rewritten as

$$\boldsymbol{y} - \boldsymbol{f} = \boldsymbol{F\beta} + \boldsymbol{\epsilon}, \boldsymbol{\epsilon} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma_y}), \boldsymbol{\beta} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$$

Considering the heteroskedastic model conditional on θ, we can use the standard model for a Bayesian linear regression with (Boscardin & Gelman, 1994):

$$(\sigma^2|\theta, y - f) = \frac{S^2}{\chi^2_{n-k}}$$

$$(\beta|\sigma^2, \theta, y) \sim N(\hat{\beta}, \sigma^2 V_\beta)$$

Where

$$W^{-1} = diag\left(f(z_1, \theta), \ldots, f(z_n, \theta)\right)$$

$$\hat{\beta} = (F^T W F)^{-1} F^T W (y - f)$$

$$V_\beta = (F^T W F)^{-1}$$

And

$$S^2 = \left((y - f) - X\hat{\beta}\right)^T W((y - f) - X\hat{\beta})$$

is the sum of the squared weighted residuals. The marginal posterior density remains as

$$p(\beta, \sigma^2, \theta|y) = p(\beta, \sigma^2|\theta, y) p(\theta|y)$$

Hence

$$p(\theta|y) = \frac{p(\beta, \sigma^2, \theta|y)}{p(\beta, \sigma^2|\theta, y)}$$

$$p(\theta|y) \propto \frac{p(y|\beta, \sigma^2, \theta) p(\beta, \sigma^2, \theta)}{p(\beta|\sigma^2, \theta, y) p(\sigma^2|\theta, y)}$$

Substituting the likelihood, and the equations above, we find that

$$p(\theta|y) \propto \frac{\sigma^{-n}\backslash \prod_i f(z_i, \theta)^{-\frac{1}{2}} \exp\left(-\frac{S^2}{2\sigma^2}\right) \sigma^{-2} p(\theta)}{\sigma^{-k}|V_\beta|^{-\frac{1}{2}} \exp\left(\sigma^{-2}(\beta - \hat{\beta})^T V_\beta^{-1}(\beta - \hat{\beta})\right) (S^2)^{\frac{n-k}{2}} \sigma^{-(-n-k+2)} \exp\left(-\frac{S^2}{2\sigma^2}\right)}$$

That based on Boscardin & Gelman et al. (1994) can be reduced to

$$p(\theta|y) \propto |V_{beta}|^{\frac{1}{2}} (S^2)^{-\frac{n-k}{2}} p(\theta)$$

The previous equation does not represent a known distribution. However, it can be used to estimate the non-normalized posterior density for any value θ. Here is a brief algorithm to describe the general procedure to find the posterior value of our parameters

1. Over a grid of θ values $Secuence\ \{-2,2\}\ \ by\ 0.5$ generate $P(\theta|y)$. Normalize data trough step function
2. By the inverse-CDF method simulate 10,000 draws from $P(\theta \mid y)$
3. For each simulation generate: $W, \hat{\beta}, S^2, V_\beta, \sigma^2$
4. Once we have a set of posterior simulations $(\beta, \sigma^2, \theta)$. We calculate the parameters β through $P(\beta|\sigma, \theta, y, \hat{\beta}) \sim N(\hat{\beta}, \sigma^2 V_\beta)$.

Once we have a set of posterior simulation of $(\hat{\beta}, \sigma^2, \theta)$, we select the mean of each parameter and solve the constant adjustment predictor given by

$$\hat{\mu}^c(x) = f(v_{1i}, z_{1i}, z_{2i}) + \hat{\beta}_0 + \hat{\beta}_1(f(v_{1i}, z_{1i}, z_{21}) - \bar{f})$$

The inadequacy of the model is recalculated as

$$MI = \sum_{i=1}^{n} (y_i - \hat{\mu}_i^c)^2$$

The inadequacy of the model is still significant; therefore, we proceed to fit a functional adjusted model.

3.3 Functional Adjusted Model

As described by Roshan et al. (2009) the functional adjusted model is given by:

$$\boldsymbol{Y} - \boldsymbol{\mu^c}(\boldsymbol{x}) = \boldsymbol{\delta}(\boldsymbol{z}; \boldsymbol{\alpha}) + \boldsymbol{\epsilon}, \qquad \boldsymbol{\epsilon} \sim \boldsymbol{N}(\boldsymbol{0}, \Sigma_\delta)$$

In this case the variable $\boldsymbol{\delta}(\boldsymbol{z}; \boldsymbol{\alpha})$ captures the difference in average between the observations and the constant adjusted model. For our case, we will use a non-linear model such as:

$$\boldsymbol{\delta}(\boldsymbol{z}; \boldsymbol{\alpha}) = \boldsymbol{\alpha_0} + \sum_{i=1}^{m} \boldsymbol{\alpha_i u_i}(\boldsymbol{z})$$

Where $\boldsymbol{u_i}(\boldsymbol{z})$ are known function of $\boldsymbol{z}$ and $\boldsymbol{\alpha} = (\boldsymbol{\alpha_0}, \ldots, \boldsymbol{\alpha_m})^{\boldsymbol{t}}$ are unknown parameters. There will be a linear basis expansion, where our design matrix will be formed by functional basis of $\boldsymbol{x}$ as shown below:

$$u_1(z) = 1, u_2(z) = z, u_3(z) = z^2, u_4(z) = z^3,$$
$$u_5(z) = (z - \xi_i)_+^3$$

As in the constant adjusted model, a Bayesian heteroskedastic regression model is proposed. We maintain the functional form for $f(z_i, \theta)$ and consider:

$$Y - \mu^C(x) = \alpha_0 + \sum_{i=1}^{m} \alpha_i u_i(z) + \epsilon$$

Now, re expressed in matrix form, consider that U is a matrix of $n \times 5$, where the first column is $1's$ and the other ones are de functional basis, where $\delta = Y - \mu^C(x)$. Therefore, the model adjusted by function can be rewritten as

$$\delta = U\alpha + \epsilon, \epsilon \sim N(0, \Sigma_\delta), \alpha \sim N(0, \Sigma)$$

Repeating the process for the constant adjusted model, the functional adjustment predictor is given by

$$\hat{\mu}^F(x) = \hat{\mu}^C(x) + \sum_{i=0}^{m} \hat{\alpha}_i u_i(z)$$

The inadequacy of the model is recalculated as

$$MI = \sum_{i=1}^{n} (y_i - \hat{\mu}_i^F)^2$$

4. Results

4.1 Data Simulation & BoxCox Transformation

The real data required to test the model is not easy to acquire and generally you must pay a certain amount of money or have a wind project in progress. Therefore, wind velocity profile data were simulated based on literature and using the power law.

A speed of $7\frac{m}{s}$ and a height of $10m$ were taken as initial data. The average speed for $h = (30m, 50m, 70m, 90m, 115m)$ was calculated by the power law, with a shear parameter of $\alpha = 0.2$ (Hadi, 2015). Then, using the statistical package R, a Weibull distribution with constant shape and variable scale was adjusted. It should be remembered that the variance in the velocity profile increases proportionally with the height. This behavior is captured by maintaining a constant shape and a variable scale. On the other hand, to not have a simulation completely faithful to the power law, an error in functional form $\gamma * (x)$ is added to each distribution.

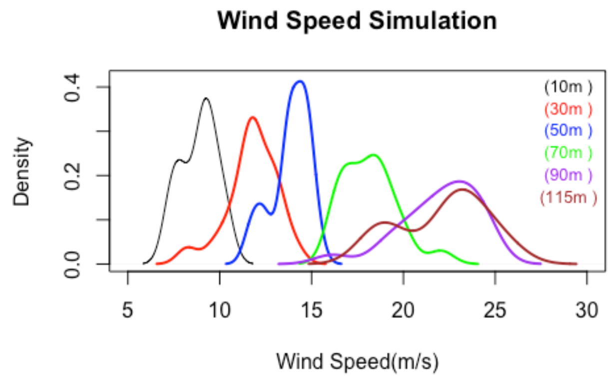

Figure 3. Weibull Wind Speed profile simulation

Finally, 20 random samples of velocity were taken for each of the 5 heights, obtaining a total of 100 data.

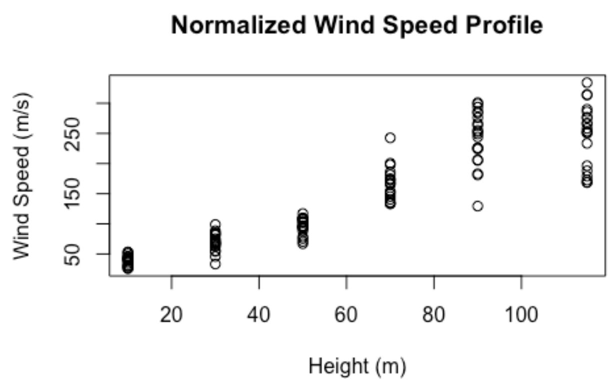

Figure 4 Normalized Wind speed profile after Box Cox Transformation.

The resulting lambda parameter for the Box-Cox transformation was $\lambda = 2$, obtained by the joint function of maximum likelihood.

4.2 Constant Adjustment Model

In the first place, the engineering model of the power law is shown. The model presents an inadequacy measure of $MI = 73,458.97$ which is very high. However, there is a correlation between the existing data and the power law and the constant adjustment model is carried out.

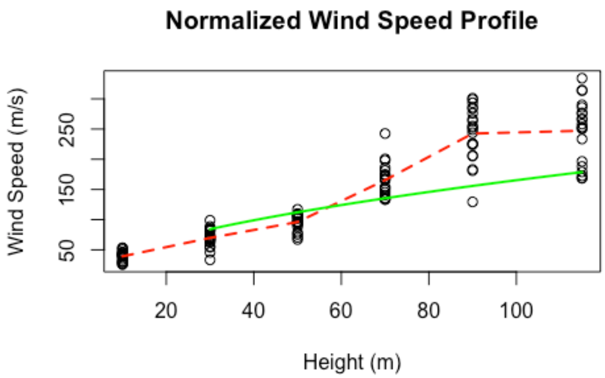

Figure 5 In Green: Power law engineering model. In Red: Real mean of wind speed profile.

From the figure it can be seen that the profile estimated by the power law coincides in some way with the real data. However, the estimate is quite far from the real mean of the data. We proceed to execute the algorithm:

4.2.1 Theta Posterior Distribution $P(\theta|y)$

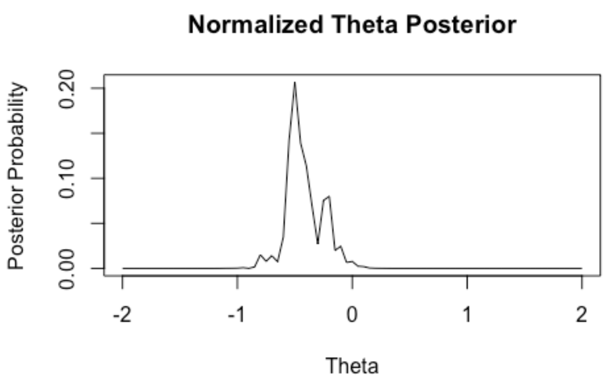

Figure 6 Normalized θ posterior distribution

The estimation of the posterior normalized distribution of beta is congruent with the initial thought, where the increase in height implies a greater variance in the velocity distribution.

4.2.2 Prior Parameter distribution

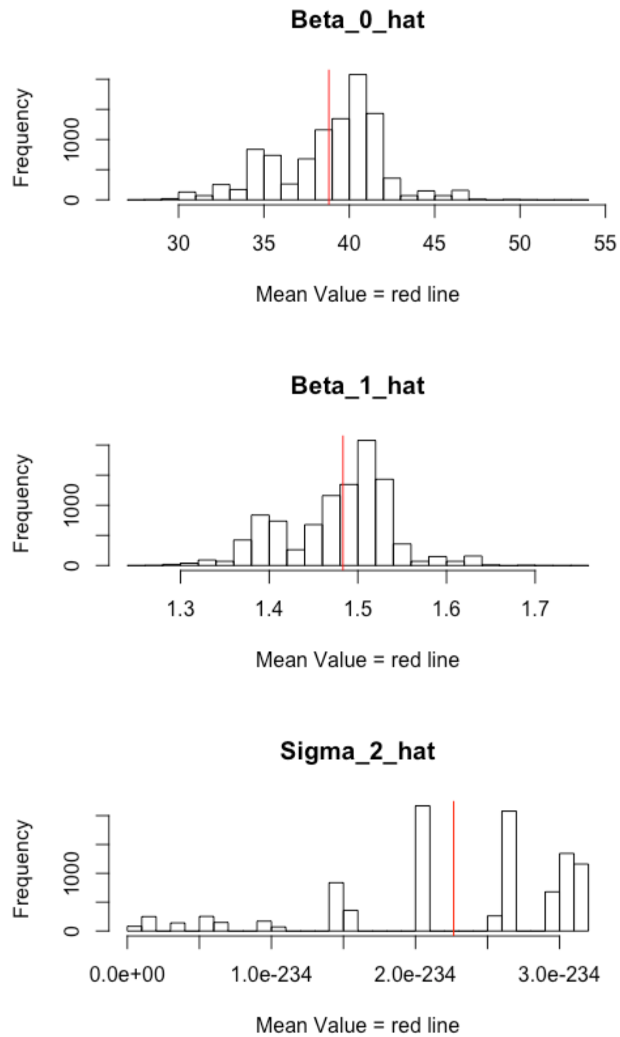

Figure 7 Prior distribution for parameters $\beta_0, \beta_1, \sigma^2$. Red line indicates de mean of the parameter.

After 10,000 simulations we obtain the prior distribution for the regression coefficients.

4.2.3 Posterior Parameter distribution

Once we have a set of posterior simulations $(\beta, \sigma^2, \theta)$. We calculate the parameters β through
$P(\beta|\sigma, \theta, y, \hat{\beta}) \sim N(\hat{\beta}, \sigma^2 V_\beta)$

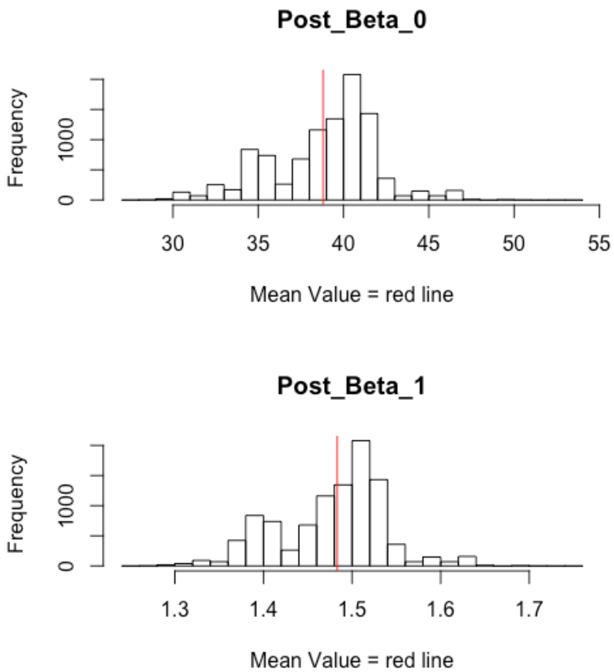

Figure 8 Posterior distribution $P(\beta|\sigma,\theta,y,\hat{\beta}) \sim N(\hat{\beta},\sigma^2\ V_\beta\ )$ for the estimated parameters.

With the data estimated for the subsequent betas, we proceed to estimate the constant adjusted model and evaluate its performance.

4.2.4 Final Constant Adjustment Model

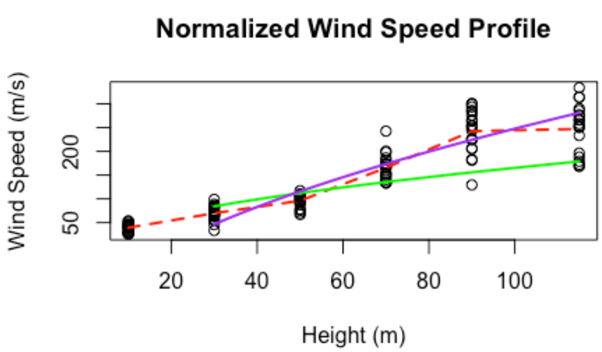

Figure 9 In Green: Power law engineering model. In Red: Real mean of wind speed profile. In Purple: Constant Adjusted Model

As a result, we obtain the constant adjustment model with

$$\mu(x)^C = f(v_{1i}, z_{1i}, z_{2i}) + 38.86 + 1.48 * \left(f(v_{1i}, z_{1i}, z_{2i}) - \bar{f}\right)$$

With

$$MI = \frac{1}{100}\sum_{i=1}^{100}(y_i - \hat{\mu}^C)^2 = 1627.441$$

From the results, you can see that the MI of the model decreases drastically. Additionally, when observing the calculated coefficients, it is evident that the greatest adjustment is on the intercept (β_0). The constant adjusted model essentially displaces the power law model positively over the speed axis and corrects slightly on the mean of the differences (β_1).

4.3 Functional Adjustment Model

The constant adjustment model corrects the central tendency on the mean of the real data. Unlike the power law, the correction by constant is better adjusted to the actual mean of the data. However, the model can be improved. Following the results from the constant adjustment model we fit the functional adjustment model as below:

$$\boldsymbol{Y} - \boldsymbol{\mu}^{\boldsymbol{C}}(\boldsymbol{x}) = \boldsymbol{\alpha_0} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \boldsymbol{\alpha_i u_i(z)}$$

$$\boldsymbol{Y} - f(v_{1i}, z_{1i}, z_{2i}) + 38.86 + 1.48 * \left(f(v_{1i}, z_{1i}, z_{2i}) - \bar{f}\right) = \sum_{\boldsymbol{i=0}}^{\boldsymbol{m}} \boldsymbol{\alpha_i u_i(z)}$$

We proceed to execute the algorithm

4.3.1 Theta Posterior Distribution $P(\theta|y)$

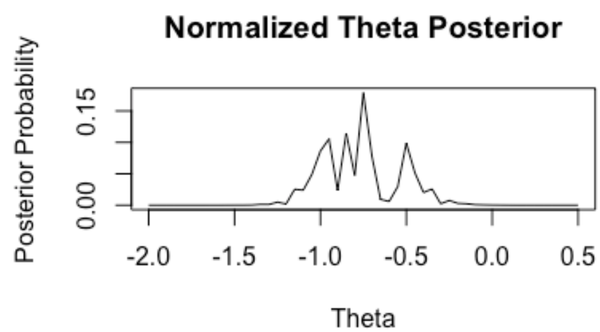

Figure 10 Normalized θ posterior distribution

Just like the previous model, the estimation of the posterior normalized distribution of beta is congruent with the initial thought, where the increase in height implies a greater variance in the velocity distribution.

4.3.2 Posterior Parameter distribution

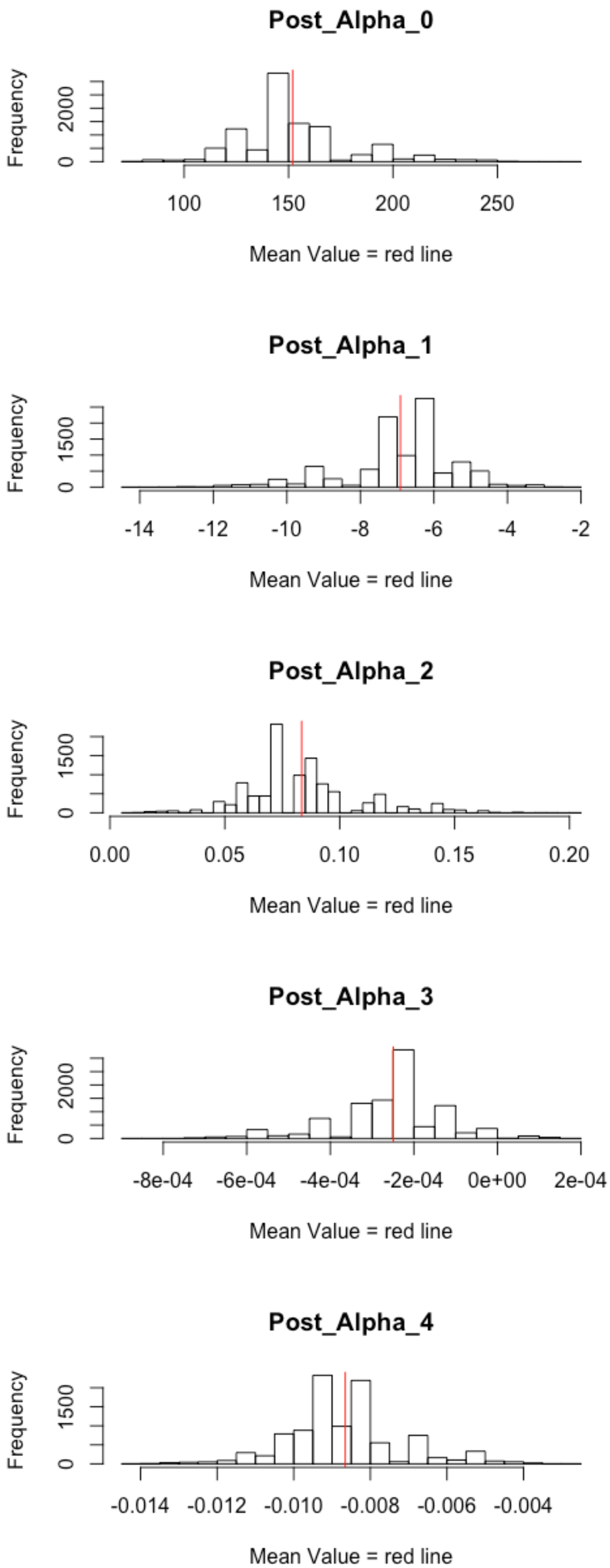

Figure 11 Posterior distribution $P(\alpha|\sigma, \theta, y, \hat{\beta}) \sim N(\hat{\alpha}, \sigma^2 V_\beta)$ for the estimated parameters for $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$.

With the data estimated for the subsequent betas, we proceed to estimate the functional adjusted model and evaluate its performance.

4.3.3 Final Functional Adjustment Model

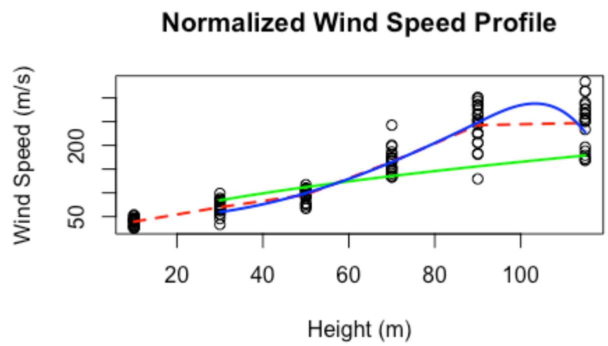

Figure 12 In Green: Power law engineering model. In Red: Real mean of wind speed profile. In Blue: Constant Adjusted Model

As a result, we obtain the functional adjustment model with

$$\mu(x)^F = \mu(x)^C + \sum_{\boldsymbol{i=0}}^{\boldsymbol{m}} \boldsymbol{\alpha_i u_i}(z)$$

With $\alpha_0 = 154.21$, $\alpha_1 = -7.02$, $\alpha_2 = 0.0853$, $\alpha_3 = 0.000359$, $\alpha_4 = 0.00855$ and:

$$MI = \frac{1}{100}\sum_{i=1}^{100}(y_i - \hat{\mu}^F)^2 = 1237.35$$

In this case, the interpretation of the coefficient of the functional bases is not so intuitive. From the results obtained it would seem that the main correction occurs on the intercept (α_0) of the base and again on the normal value of the height (α_1). However, the additional adjustment provided by the bases of z^2, z^3 and the correction by the node $(\boldsymbol{z} - \boldsymbol{\xi_i})_+^3$ which add flexibility to the curve cannot be disregarded.

The corrected final model to estimate the wind speed profile on the transformation of the Weibull is:

$$Y = f(v_{1i}, z_{1i}, z_{2i}) + 193.07 + 1.48 * \left(f(v_{1i}, z_{1i}, z_{2i}) - \bar{f}\right) - 7.02 z_{1_i} + 0.0853\, z_{1_i}^2 + 0.000359\, z_{1_i}^3 + 0.00855\left(z_{1_i} - 90\right)_+^3$$

4.4 Returning to original Data

Finally, we return to the original values using the $\lambda = 2$ from the Box- Cox transformation, the results are shown in the figure below:

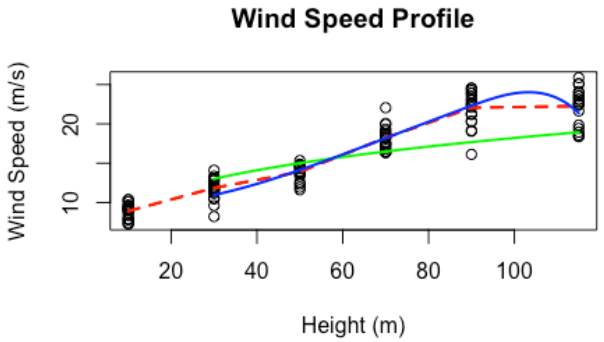

Figure 13 In Green: Power law engineering model. In Red: Real mean of wind speed profile. In Blue: Constant Adjusted Model

It can be seen that the final model fits almost perfectly on the actual average of the data. It can be affirmed that the Bayesian heteroskedastic model manages to successfully correct the power law on the case study.

5. CONCLUSIONS

The results show that implementing the constant adjusted model considerably improves the fitting of the statistical engineering model. Subsequently, incorporating the functional adjustment allows an almost perfect adjustment on the mean of the real data. Finally, we obtain a regression model that incorporates the power law in the calculation that gives a pretty good approximation for the wind speed profile. The general conclusions are presented below:

- The statistical engineering model manages to capture the behavior of the velocity profile in a satisfactory way. As an important fact, in computational terms and time it is not a demanding methodology, which is a satisfactory result because it is easy to implement.
- The estimated model allows to extrapolate the wind speed profile above all the range of heights, which is desirable given the data is generally scarce and very difficult to obtain.
- The proposed methodology is desirable since it adopts the engineering model as a basis and prevents the model from overfitting on the sample data. This allows us to extrapolate on the variable of interest without fear of obtaining results too far from reality.

Finally, engineering models and statistical models have both benefits and cons. The proposed model out performs the engineering model when the height increases giving a pretty good extrapolation for the wind velocity estimation without the need of a deeper analysis in the surface or atmospheric conditions. The statistical adjustments correct the engineering model without trying to understand the what is the mistake in the empirical estimation. Finally, it is quickly to develop a model that yields more realistic predictions.

6. FUTURE WORK

In the Colombian context, there are several interesting. One of them, the Jouktai Wind Farm in the Cabo de la Vela District of the La Guajira department, is currently under construction and will soon initiate operation. Developed by ISAGEN, an estimated initial capacity of 31.5MW is expected, requiring the investment of approximately US$60 million. The project is still at an early stage, but progress in field and power plant development is expected in the next years. Finally, the Ipapure Wind Farm project, with an estimated initial capacity of 200MW, is currently undergoing preliminary studies in the La Guajira department (Norton Rose Fulbright, 2016).

Based on the foregoing, as future work it would be ideal to implement the proposed methodology on real data, if possible ISAGEN data. When implemented on real data, the effectiveness of the developed model could be evaluated. Having a good performance on real data is expected to develop a functional tool that helps in the evaluation and viability of wind energy projects in the future. Additionally, for future work, a suitable methodology for the calibration of the shear parameter would be sought. Given that this depends in large part on the characteristics of the site, it would be interesting to develop a methodology that would allow calibrating the parameter with additional information and integrate it into the Bayesian model.

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