

An Economic Theory of Labor Discrimination

Hernán Vallejo

Documento CEDE

12

Marzo de 2021

Serie Documentos Cede, 2021-12 ISSN 1657-7191 Edición electrónica. Marzo de 2021

© 2021, Universidad de los Andes, Facultad de Economía, CEDE. Calle 19A No. 1 – 37 Este, Bloque W. Bogotá, D. C., Colombia Teléfonos: 3394949- 3394999, extensiones 2400, 2049, 2467

infocede@uniandes.edu.co

<http://economia.uniandes.edu.co>

Impreso en Colombia – Printed in Colombia

La serie de Documentos de Trabajo CEDE se circula con propósitos de discusión y divulgación. Los artículos no han sido evaluados por pares ni sujetos a ningún tipo de evaluación formal por parte del equipo de trabajo del CEDE. El contenido de la presente publicación se encuentra protegido por las normas internacionales y nacionales vigentes sobre propiedad intelectual, por tanto su utilización, reproducción, comunicación pública, transformación, distribución, alquiler, préstamo público e importación, total o parcial, en todo o en parte, en formato impreso, digital o en cualquier formato conocido o por conocer, se encuentran prohibidos, y sólo serán lícitos en la medida en que se cuente con la autorización previa y expresa por escrito del autor o titular. Las limitaciones y excepciones al Derecho de Autor, sólo serán aplicables en la medida en que se den dentro de los denominados Usos Honrados (Fair use), estén previa y expresamente establecidas, no causen un grave e injustificado perjuicio a los intereses legítimos del autor o titular, y no atenten contra la normal explotación de la obra.

Universidad de los Andes | Vigilada Mineducación Reconocimiento como Universidad: Decreto 1297 del 30 de mayo de 1964. Reconocimiento personería jurídica: Resolución 28 del 23 de febrero de 1949 Minjusticia.

CEDE

Centro de Estudios sobre Desarrollo Económico

Documento CEDE

Los documentos CEDE son producto de las investigaciones realizadas por al menos un profesor de planta de la Facultad de Economía o sus investigadores formalmente asociados.

An Economic Theory of Labor Discrimination

Hernán Vallejo*

Universidad de los Andes, Facultad de Economía

Bogotá D.C., Colombia

hvallejo@uniandes.edu.co

Abstract

This article presents a theory of labor discrimination based on the behavior of economic agents that maximize utility and profits. The article makes use of a monopsony that hires workers that have the same labor productivity, to focus on perfect discrimination; discrimination by quantities of labor hired; and discrimination by types of labor hired. The article concludes that in such contexts, workers with the same productivity may be discriminated in wages and quantities of labor hired, when firms make use of their market power; when there are differences in the opportunity costs and the wage elasticities of labor supply among workers; when there is asymmetric information, self-selection and adverse selection; and when firms or governments decide not to allow for wage discrimination. First best minimum wages may contribute to improve employment and welfare, but higher minimum wages may not. **JEL Classification:** J31, J42, J71.

Keywords: Monopsony, labor discrimination, asymmetric information, self-selection, adverse selection, market power

*The author thanks Carlos Andrés Camelo and Santiago Neira for their feedback. All remaining errors belong to the author.

Una teoría económica de la discriminación laboral

Hernán Vallejo¹

Universidad de los Andes

Facultad de Economía

Bogotá D.C., Colombia

hvallejo@uniandes.edu.co

Este artículo presenta una teoría de la discriminación laboral basada en el comportamiento de los agentes económicos que maximizan la utilidad y los beneficios. El artículo hace uso de un monopsonio que contrata trabajadores que tienen la misma productividad laboral, para enfocarse en la discriminación perfecta; la discriminación por cantidad de mano de obra contratada; y la discriminación por tipo de trabajo contratado. El documento concluye que en tales contextos, trabajadores con la misma productividad pueden ser discriminados en salarios y cantidades de trabajo contratado, cuando las empresas hacen uso de su poder de mercado; cuando existen diferencias en los costos de oportunidad y en las elasticidades salario de la oferta laboral entre los trabajadores; cuando hay información asimétrica, autoselección y selección adversa; y cuando las empresas o los gobiernos deciden no permitir la discriminación salarial. Los salarios mínimos de primero mejor, pueden contribuir a mejorar el empleo y el bienestar, pero salarios mínimos más altos, puede que no lo hagan.

Clasificaciones JEL: J31, J42, J71

Palabras claves: Monopsonio, discriminación laboral, información asimétrica, autoselección, selección adversa, y poder de mercado.

¹El autor agradece a Carlos Andrés Camelo y Santiago Neira por sus comentarios. Todos los errores restantes pertenecen al autor.

1 Introduction

This article presents a theory of labor discrimination, understood as firms that pay different wages and hire different quantities of labor, when dealing with workers that have the same labor productivity.

The article starts with a brief summary of the previous literature on economic theories of labor discrimination. Then it outlines the general assumptions and the structure of the theory. That is followed by the presentation of the perfect discrimination of labor (first degree labor discrimination); the discrimination of labor by quantities hired (second degree labor discrimination); and the discrimination by wages on segmented workers (third degree labor discrimination).

For each degree of discrimination, detailed descriptions of the optimization process for the employer and the workers, and its consequences in terms of welfare, surpluses and resource allocation, are provided. Then, an application of two public policy interventions under third degree discrimination of labor is included, in order to illustrate how the theory can be used to analyze the roll of governments when dealing with labor discrimination. The article ends with some conclusions.

2 Previous literature

Labor discrimination has long been studied in the economic literature.

Cain (1987) presents a detailed survey on labor discrimination and asks himself among other questions, whether it is inefficient to pay different wages to workers that have the same productivity. In terms of theory, he highlights neoclassical theories as mainly demand side theories; competitive or monopolistic; exact or stochastic. Institutional theories highlight the role of the environment and feedback effects. Marxian theories are mentioned in terms of the exploitation of labor, but without going into the details. He also presents a detailed discussion on empirical evidence.

Cain also mentions Robinson (1933), who constructs and models labor discrimination by postulating that minorities have labor supplies that are more inelastic to wages. Madden (1973) makes a more recent application of Robinson's work. Cain reports considerable

evidence that women have labor supplies that are more elastic to wages. He addresses monopsony models and considers that since monopsony is not prevalent, it is not worth it to further examine differences in labor elasticities. However, he highlights the fact that if there are differences in labor supply, it is important to verify to what extent those differences explain the gaps in observed wages.

Robinson (1933) is also considered the first author to explore third degree price discrimination. Schmalensee (1981) found that an increase in output is a necessary condition for third degree discrimination to increase social welfare, if demands are independent and there are constant marginal costs.

Varian (1985) generalized the results of Schmalensee for third degree price discrimination, while exploring in more detail the welfare effects of such discrimination.

Yoshida (2000) built a model to analyze how discrimination in the input markets can change the output of the final good and affect welfare. He found that although the impacts are ambiguous, an increase in the output of the final good is a sufficient condition for welfare deterioration, as opposed to the results obtained by Schmalensee (1981) and Varian (1985).

Varian (2010) presents a model of second degree discrimination in consumption, based on varying the amounts of quantities sold and their corresponding payments, as mechanism to extract as much surplus as possible, when there is asymmetric information.

Boal and Ransom (1997) present a monopsony model, in which wage discrimination may occur due to different price elasticities of supply, different quit rates and different offer arrivals in the context of job search.

The theory presented here is constructed assuming a monopsony under partial equilibrium, since it is the simplest market structure in which the producer has the greatest market power, and thus, the greatest capacity to extract surplus from its employees.

3 An Economic Theory of the Discrimination of Labor

In order to present the economic theory of labor discrimination proposed in this article, some general assumptions are required. Those are followed by the presentation of the first, second and third degree models of labor discrimination.

3.1 General Assumptions

In order to keep everything as tidy and simple as possible within the economic theory presented, the assumptions used across the different models included in this article, are:

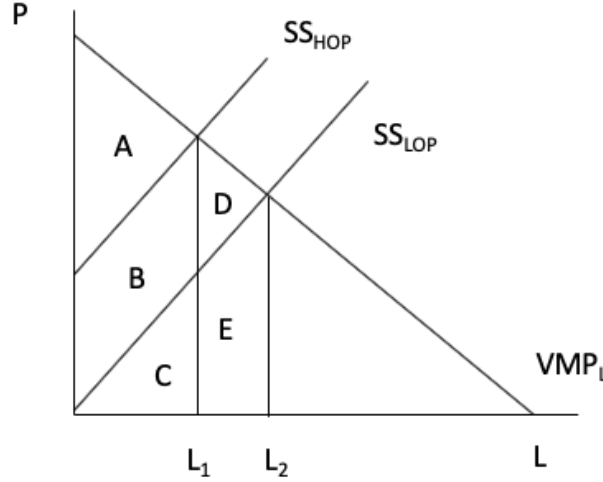
There are many suppliers of labor that for simplicity will be called workers, regardless of whether they actually have a job or not. Workers offer labor and each of them can have one of two different opportunity costs: high (*HOC*) or low (*LOC*). Workers can have different opportunity costs because some can produce as much per hour as the low opportunity cost workers, but with more effort. Or they have children, elderly or ill members of their families to take care of, while the lower opportunity cost workers do not. Or some have to commute to work, while others do not or can work remotely. Or they have to take care of a farm that they own, or a family business, or a social foundation, while the lower opportunity cost workers do not, and so on. In any case, workers know what their opportunity costs are.

The profit maximizing firm has market power and it wants to extract as much surplus from its workers, as possible. It will be assumed that the monopsonist will operate a single plant for simplicity. However, that assumption will be relaxed in the appendix at the end of this article.

3.2 First Degree Labor Discrimination

If the firm has perfect information and it can discriminate labor perfectly, the model can be described as in figure 1:

Figure 1: First Degree Discrimination of Labor



Source: author.

In this case, the producer would hire *HOC* workers to work L_1 and pay them $B + C$; and hire *LOC* workers to work L_2 and pay them $C + E$.

With this arrangements, the producer would hire both *HOC* and *LOC* workers; extract the maximum possible surplus from both types of workers; and maximize social welfare, albeit with the producer capturing all the social surplus $2A + B + D$, and the workers capturing no surplus at all. Regardless, workers with the same productivity would be receiving different total payments and would work different amounts of hours. The *HOC* workers would be hired for less hours and have a higher average wage, while the *LOC* workers would be hired for more hours and have a lower average wage.

3.3 Second Degree Labor Discrimination

The second degree discrimination case is based on the model presented in Varian (2010) for discrimination in consumption, adapted to analyze discrimination in labor markets. In this case, the employer does not have full information and in particular, it cannot distinguish between the *HOC* and the *LOC* types of workers.

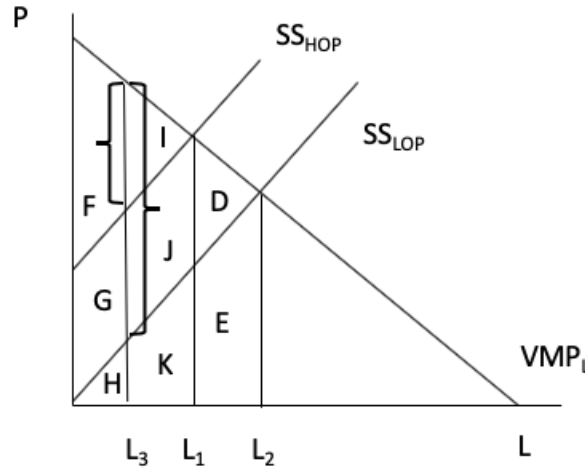
As such, the payment scheme described before in the first degree labor discrimination

case would be untenable, since there would be self-selection on the part of the lower opportunity cost workers, which would behave as if they had higher opportunity costs, by being willing to work L_1 for a payment of $B + C$. That way, they would have B as surplus, while working L_2 for a payment of $C + E$, they would have no surplus at all.

Thus, to solve this self-selection problem, the employer that applies second degree discrimination would have to pay for L_1 , $B + C$ and for L_2 , $=B + C + E$. This way, there would not be a self-selection problem. *LOC* workers would be indifferent between working L_1 or L_2 , since they would have B as surplus in either case.

Since the low opportunity cost workers would be genuinely indifferent, they would work the amount desired by the employer, in this case L_2 . However, with this new payment scheme, the employer would not be extracting the maximum possible surplus, as a second degree discriminator. To see why, consider figure 2.

Figure 2: Increasing Surplus Extraction with Second Degree Discrimination of Labor



Source: author

To increase surplus extraction, the second degree discriminator would have to reduce L_1 , for example to L_3 , and pay for L_3 , $G + H$ and for L_2 , $G + H + K + E$.

Again, there would not be a self-selection problem. *LOC* workers would be indifferent between offering to work L_3 or L_2 , since they would have G as surplus either way.

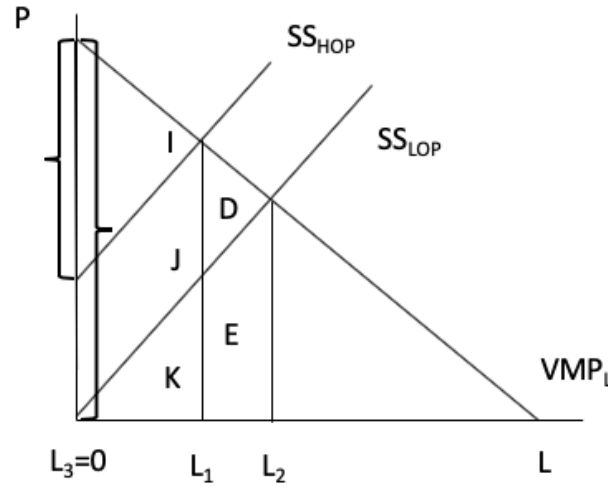
However, the producer wouldn't be extracting the maximum possible surplus, as a second degree discriminator. Every time the producer reduces one unit of labor from L_3 ,

it loses a slice of area F with the high opportunity cost workers, and gains a slice of areas I and J with the low opportunity cost workers.

Note that the slice of triangle H lost with the low opportunity cost workers as L_3 is moved to the left, is gained with those same workers in area K , so these two effects cancel out.

Thus, if the employer wants to maximize the surplus extraction, it hires only an amount of labor L_2 for which it pays $K + E$ since $G + H = 0$ when $L_3 = 0$, and only low opportunity cost workers are willing to provide such amount of labor, as shown in figure 3.

Figure 3: Maximising Surplus Extraction with Second Degree Discrimination of Labor



Source: author

This means that at equilibrium, the employer does not hire high opportunity cost workers at all, even though they have the same marginal product of labor as the low opportunity cost workers, and even though it would be optimal for the employer to hire them, if it could discriminate labor perfectly. In this sense, there is an adverse selection problem due to the asymmetry of information between the firm and the workers.

3.4 Third Degree Wage Discrimination

In the third degree discrimination case, the employer does not have full information so it cannot discriminate the wage perfectly, but it can distinguish the type of opportunity

cost that each worker has.

For the sake of generality, consider for a moment that workers are segmented in n groups according to the type and level of their opportunity costs.

The profit maximization problem for the monopsonists can be written as:

$$TR = pq(l_1 + \dots + l_n) - w(l_1)l_1 - \dots - w(l_n)l_n$$

The first order conditions in this case would be:

$$\begin{aligned} \frac{\partial TR}{\partial l_1} &= p \frac{\partial q(\sum l_i)}{\partial l_1} - w(l_1) - l_1 \frac{\partial w(l_1)}{\partial l_1} = 0 \\ &\vdots \\ \frac{\partial TR}{\partial l_n} &= p \frac{\partial q(\sum l_n)}{\partial l_n} - w(l_n) - l_n \frac{\partial w(l_n)}{\partial l_n} = 0 \end{aligned}$$

This means that:

$$\begin{aligned} p \frac{\partial q(\sum l_i)}{\partial l_1} &= w(l_1) + l_1 \frac{\partial w(l_1)}{\partial l_1} \\ &\vdots \\ p \frac{\partial q(\sum l_n)}{\partial l_n} &= w(l_n) + l_n \frac{\partial w(l_n)}{\partial l_n} \end{aligned}$$

So it must be that:

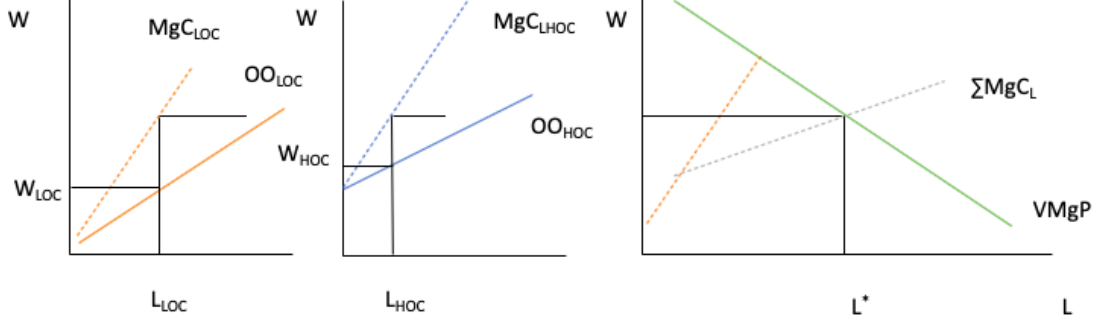
$$p \frac{\partial q(\sum l)}{\partial l} = \left[w(l_1) + l_1 \frac{\partial w(l_1)}{\partial l_1} \right] = \dots = \left[w(l_n) + l_n \frac{\partial w(l_n)}{\partial l_n} \right]$$

Thus, a profit maximizing monopsonist will hire workers with n different opportunity costs, when the value of the marginal product of workers at the plant ($VMgP = p \frac{\partial q(\sum l)}{\partial l}$) equals the level of the marginal cost of hiring each of the n types of workers.

In order to represent the model graphically and again for simplicity, consider that there are only two types of workers (HOC) and (LOC), as before.

Graphically, the third degree wage discrimination can be represented as in figure 4:

Figure 4: Third Degree Discrimination of Labor



Source: author

Note that a profit maximizing firm will have a mark-down on each of the types of workers that it hires, since at equilibrium for any labor segment i :

$$VMgP = MgC_{l_i} = w(l_i) + l_i \frac{\partial w(l_i)}{\partial l_i} > w(l_i)$$

So, the allocation of labor will be inefficient in this case.

Note also that a profit maximizing firm will pay higher wages to the more wage elastic workers, and less wages to the less wage elastic workers. This result can be obtained by noting that profit maximization implies that for any segmented labor markets i, j :

$$\begin{aligned} \left[w(l_i) + l_i \frac{\partial w(l_i)}{\partial l_i} \right] &= \left[w(l_j) + l_j \frac{\partial w(l_j)}{\partial l_j} \right] \\ \left[w(l_i) \frac{w(l_i)}{w(l_i)} + w(l_i) \frac{l_i}{w(l_i)} \frac{\partial w(l_i)}{\partial l_i} \right] &= \left[w(l_j) \frac{w(l_j)}{w(l_j)} + w(l_j) \frac{l_j}{w(l_j)} \frac{\partial w(l_j)}{\partial l_j} \right] \\ w(l_i) \left[1 + \frac{1}{\eta_i} \right] &= w(l_j) \left[1 + \frac{1}{\eta_j} \right] \end{aligned}$$

If:

$$\eta_i > \eta_j$$

$$w(l_i) > w(l_j)$$

The main results from the third degree wage discrimination model can be summarized as:

- The firm will employ workers where the level of their marginal product at their production plant equals the level of marginal cost of hiring such workers on any market.
- Usually, the firm that hires workers with the same productivity, will pay different wages and hire different quantities of workers that have different opportunity costs, and different wage supply elasticities of labor.
- The firm will pay higher wages to the workers with higher wage supply elasticity, and less wages to the the workers with less wage supply elasticity.
- The firm will apply a mark-down, and as such, the allocation of labor will not be efficient.

4 Policy Alternatives

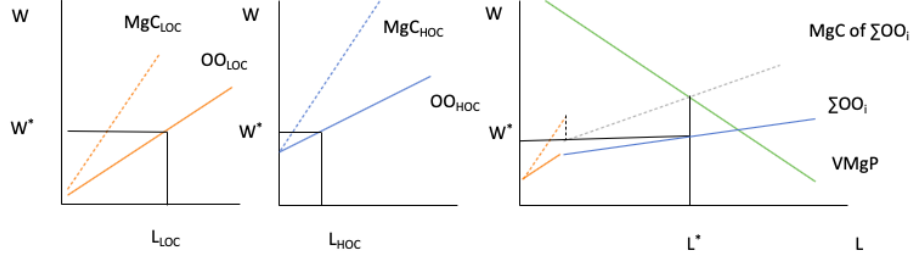
To illustrate how this model can be used to analyze the role of different policy options, consider two alternatives under third degree labor discrimination: forbidding wage discrimination among equally productive workers, and applying a first best minimum wage.

4.1 Forbidden Third Degree Wage Discrimination

If a government dislikes wage discrimination among equally productive workers, it may forbid it. If a firm dislikes wage discrimination among equally productive workers, it may avoid it voluntarily. For brevity, both situations will be referred to as forbidden third degree wage discrimination. In this case and assuming that the government and/or the company board's decisions are complied fully by the economic agents, labor markets are no longer segmented in practical terms, and become one single market. That gives room for three main cases:

The first can be seen as in figure 5:

Figure 5: Forbidden Third Degree Discrimination of Labor
when both (*LOC*) and (*HOC*) are Hired

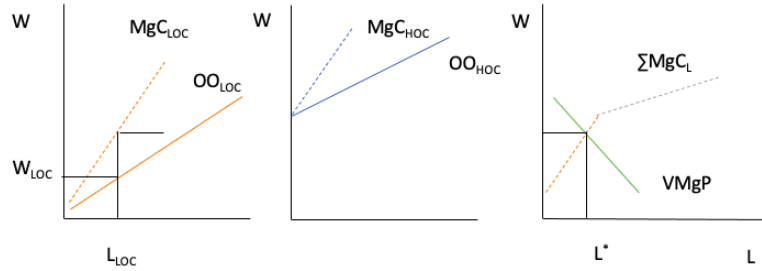


Source: author

In this case 1, the firm hires both types of workers and pays them the same wage, as determined by the Government.

The second case possible when the Government forbids third degree labor discrimination can be seen as in figure 6:

Figure 6: Case 2 when Forbidding Third Degree Discrimination of Labor
and only (*LOC*) are hired

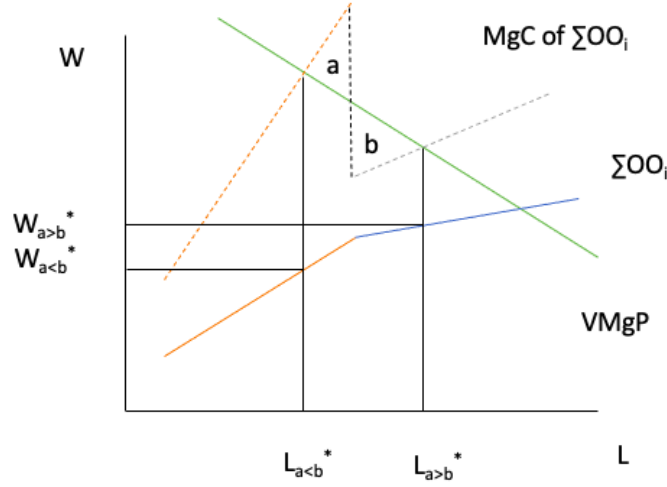


Source: author

In this case, the monopsonist hires only one type of worker, and the other type of worker is not employed.

The third possible case when the Government forbids the third degree wage discrimination can be seen as in figure 7, with an augmented scale to see the details:

Figure 7: Case 3 when Forbidding Third Degree Discrimination of Labor



Source: author

In this case, there are three possibilities: $a > b$, and the monopsonist only hires *LOC* workers. The *HOC* workers are unemployed; $a < b$ and the monopsonist hires both types of workers, and pays them the same wage; $a = b$ and the monopsonist is indifferent between hiring only *LOC* workers, or hiring both *LOC* and *HOC* workers and paying them the same wage.

Thus, it is possible to conclude that if a government or a firm forbids wage discrimination among equally productive workers, under certain circumstances (case 1 or case 3 with $a \leq b$), the profit maximizing monopsonist may comply hiring both types of workers, but under other circumstances (case 2 or case 3 with $a \geq b$), the profit maximizing monopsonist may hire only the workers with the lowest opportunity cost.

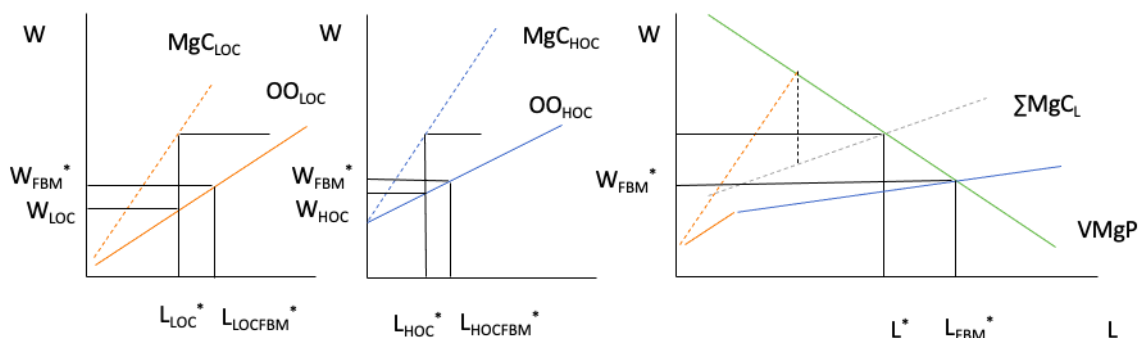
4.2 First Best Minimum Wage When There is Third Degree Wage Discrimination

If a government sets a first best minimum wage (*FBMW*), that is a wage that replicates the wage that would prevail under perfect competition, labor segmentation would

also disappear. The impacts of the *FBMW*, would depend on the third degree discrimination case that prevailed before such minimum wage was established.

If both *HOC* and *LOC* workers were hired initially, employment and welfare would increase unambiguously with the *FBMW*, and there would be no unemployment, as shown in figure 8. Welfare would increase since now the marginal product of the last units of work hired is equal to the *FBMW*, meaning that there is no longer a mark-down at equilibrium.

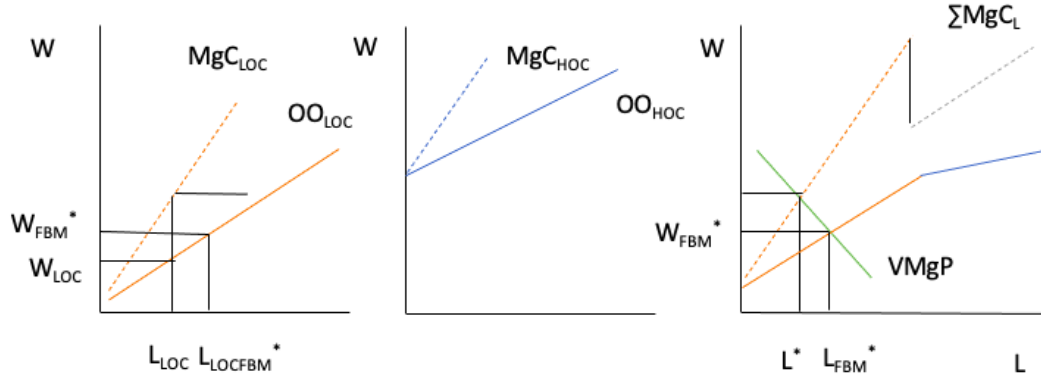
Figure 8: First Best Minimum Wage Applied to Third Degree Discrimination
with Employment for *HOC* and *LOC*



Source: author

If only LOC workers were hired initially and after the *FBMW*, employment and welfare would increase unambiguously with the *FBMW*, as shown in figure 9. Welfare would increase as before, since now the marginal product of the last unit of work hired is equal to the *FBMW*, meaning that there is no longer a mark-down at equilibrium.

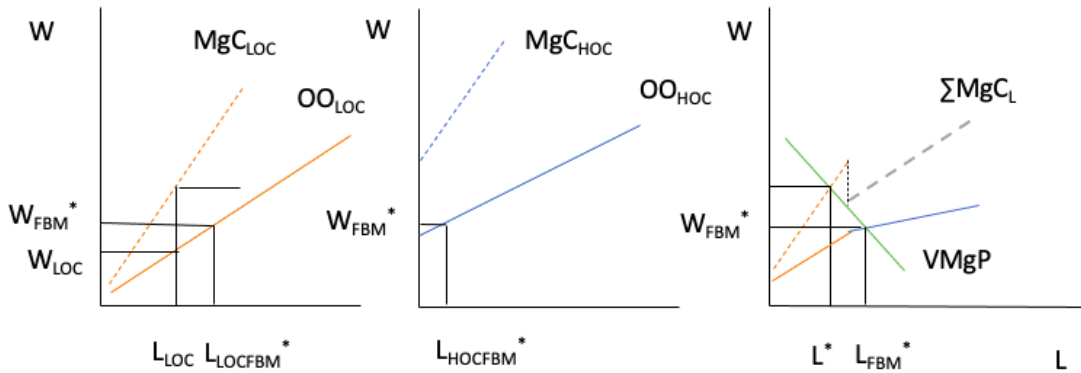
Figure 9: First Best Minimum Wage Applied to Third Degree Discrimination
with Employment for *LOC* only



Source: author

However and depending for example on the size of the areas a and b , a *FBMW* may induce a monopsonist to hire both *LOC* and *HOC* workers, when it only hired *LOC* workers without the *FBMW*. An example of such case is shown in figure 10.

Figure 10: First Best Minimum Wage Applied to Third Degree Discrimination
going from employing only *LOC*, to employing both *LOC* and *HOC*



Source: author

In this case, welfare would increase since now the marginal product of the last units of work hired, is equal to the first best minimum wage, meaning that there is no longer a mark-down.

The results of the first best minimum wage when there is third degree price discrimination can be summarized as follows: in general, efficiency would increase and be maximum since there would be no mark-down; employment would increase unambiguously and eventually, workers with higher opportunity cost that were not employed before, would be employed with the introduction of the *FBMW*.

However, the higher the minimum wage is set with respect to the the first best minimum wage, the higher the unemployment, and the lower the increase in welfare and employment. Eventually, both employment and welfare will fall with respect to the third degree discrimination case, if the minimum wage is high enough when compared to the *FBMW*.

Conclusions

This article has presented an economic theory of labor discrimination. It has been argued that if a firm with power in the labor market could perfectly discriminate workers, it would maximize efficiency and social welfare, but workers would have no surplus at all. In this case, the firm would hire workers with the same productivity, but pay them different amounts for different quantities of work, depending on their opportunity costs.

In the case of second degree discrimination, if a monopsonist does not have perfect information and in particular, if it cannot distinguish between high and low opportunity cost employees, the firm won't be able to extract all the surplus from its workers among other things, because it has to deal with the self selection of low opportunity cost workers, behaving as high opportunity cost workers.

In this case, the producer may use the amount of labor hired and its corresponding payments, as a mechanism to extract as much surplus as possible. In the model proposed, the producer can extract more surplus from its workers, by offering an amount of work and a payment that is not accepted by the high opportunity cost workers, and that extracts

all the surplus of the low opportunity cost workers.

As such, the equilibrium with second degree discrimination of labor is inferior to the competitive equilibrium and to the perfect discrimination of labor equilibrium, since the welfare that society could obtain from hiring high opportunity cost workers is lost, due the asymmetry of information and the subsequent problem of adverse selection.

In the third degree discrimination case, it has been argued that if the producer hires both workers with high and low opportunity costs, it will pay higher wages to the workers with higher elasticity of supply, and lower wages to the workers with the lower elasticity of supply. It is also argued that such equilibrium is inefficient, since there is always a mark-down.

If the Government decides to forbid third degree wage discrimination, or the firm decides on its own not to discriminate among equally productive workers, under certain circumstances the employer may end up not hiring the high opportunity cost workers. If the Government sets up a first best minimum wage, that replicates the wage that would prevail under perfect competition, it would increase unambiguously the level of employment and welfare, and under certain conditions, the employer may hire segments of the labor market with higher opportunity costs, that were not hired before the minimum wage was established.

However, the minimum wages above the first best minimum wage will increase unemployment. Eventually, if the minimum wage is high enough when compared to the first best minimum wage, both employment and welfare will fall with respect to the third degree discrimination case.

The models presented here also suggest that empirical studies that compare wage gaps among equally productive workers, should take into consideration differences in the quantity of labor hired; the opportunity costs of workers and their wage elasticities of labor supply. They should also consider the impact of discrimination between those employed, and those that are not employed due for example, to asymmetric information, self selection, and forbidden -or self avoided- discrimination between equally productive workers.

References

- Cain, G. (1986) “The economic analysis of labor market discrimination: A survey” *Handbook of Labor Economics* Volume 1, Pages 693-785.
- Madden, J. F. (1973) *The economics of sex discrimination*. Lexington, Mass.: D.C. Heath and Co.
- Robinson, J. (1933) *The economics of imperfect competition*. London: Macmillan.
- Varian, H. R. (2010) *Intermediate Microeconomics: A Modern Approach* Norton, Eighth Edition.
- Varian, H. R. (1985) “Price Discrimination and Social Welfare” *The American Economic Review*, Sep., Vol. 75, No. 4, pp. 870- 875.
- Yoshida, Y. (2000) “Third-Degree Price Discrimination in Input Markets: Output and Welfare” *The American Economic Review*, Mar., Vol 90, No. 1., pp. 240-246.

Appendix

Multiplant Monopsonist with Third Degree Discrimination of Labor

Consider the following extension of the third degree labor discrimination model discussed before. There are n labor markets that are segmented according to the type and level of the opportunity cost, and now the monopsonist uses the labor it hires in m different plants. In this case, its maximization problem is as follows:

$$TR = pq(\sum l)_1 + \dots + pq(\sum l)_m - w(l_1)l_1 - \dots - w(l_n)l_n$$

The first order conditions in this case would be:

$$\begin{aligned}\frac{\partial TR}{\partial l_1} &= p \frac{\partial q(\sum l)_1}{\partial l_1} + \dots + p \frac{\partial q(\sum l)_m}{\partial l_1} - w(l_1) - l_1 \frac{\partial w(l_1)}{\partial l_1} = 0 \\ &\vdots \\ \frac{\partial TR}{\partial l_n} &= p \frac{\partial q(\sum l)_1}{\partial l_n} + \dots + p \frac{\partial q(\sum l)_m}{\partial l_n} - w(l_n) - l_n \frac{\partial w(l_n)}{\partial l_n} = 0\end{aligned}$$

This means that:

$$\begin{aligned}p \frac{\partial q(\sum l)_1}{\partial l_1} + \dots + p \frac{\partial q(\sum l)_m}{\partial l_1} &= w(l_1) + l_1 \frac{\partial w(l_1)}{\partial l_1} \\ &\vdots \\ p \frac{\partial q(\sum l)_1}{\partial l_n} + \dots + p \frac{\partial q(\sum l)_m}{\partial l_n} &= w(l_n) + l_n \frac{\partial w(l_n)}{\partial l_n}\end{aligned}$$

So we can conclude that:

$$p \frac{\partial q(\sum l)_1}{\partial l} + \dots + p \frac{\partial q(\sum l)_m}{\partial l_n} = \left[w(l_1) + l_1 \frac{\partial w(l_1)}{\partial l_1} \right] = \dots = \left[w(l_n) + l_n \frac{\partial w(l_n)}{\partial l_n} \right]$$

Thus, the monopsonist will maximize its profits, when the level of the marginal

revenue is equal in each plant, and that level is also equal to the level of the marginal cost of hiring each type of labor.