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María Alejandra Torres León†

Abstract

What happens to workers of the fossil fuels industry if an energy transition takes place? Even though an energy transition is one of the main objectives in the fight against climate change, it carries several economic and social costs, especially as it has heterogeneous effects on different groups of individuals. This paper introduces a directed technical change model where innovation is focused on the energy sector that demands both skilled and low-skilled labor. In this context, I show how an environmental catastrophe is inevitable if there is not a policy to carry out an energy transition. Once this policy is implemented, there is directed technical change toward the clean sector and workers in the dirty sector bear an extra cost to adapt their abilities to the skills' demand in the new sector. Consequently, the existing income gap is amplified following i) changes in relative labor supply favoring workers in the clean sector and ii) a reduction in disposable income for human capital investment. Government intervention is needed to compensate households and guarantee that economic and environmental gains from the energy transition outweigh its welfare losses.

Keywords: Energy transition, directed technical change, growth, income distribution, labor market.

JEL Codes: J24, O33, O44, Q43.

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¿Si no es verde, pierde? Transición energética, cambio técnico dirigido y desigualdad salarial*

María Alejandra Torres León**

Resumen

¿Qué ocurre con los trabajadores del sector de combustibles fósiles cuando ocurre una transición energética? Aunque la transición es uno de los principales objetivos en la lucha contra el cambio climático, conlleva costos económicos y sociales, especialmente por el efecto heterogéneo que tiene sobre distintos grupos de individuos. Este trabajo introduce un modelo de cambio técnico dirigido donde la innovación se concentra en el sector energético, que demanda trabajo calificado y de baja calificación. Así, muestro cómo una catástrofe ambiental es inevitable sin una política para lograr la transición. Una vez esta se implementa, hay cambio técnico dirigido hacia el sector limpio, por lo que los trabajadores del sector sucio deben cubrir el costo de adaptarse a la demanda de habilidades del nuevo sector. Como consecuencia, la brecha existente de ingreso se amplifica por i) cambios en la oferta relativa de trabajo que favorecen a los trabajadores del sector limpio y ii) una reducción en el ingreso disponible para inversión en capital humano. Entonces, la intervención del gobierno es necesaria para compensar a los hogares y garantizar que las ganancias económicas y ambientales de la transición superen las pérdidas en bienestar.

Palabras clave: Transición energética, cambio técnico dirigido, crecimiento, distribución del ingreso, mercado laboral

Códigos JEL: J24, O33, O44, Q43.

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1 Introduction

What happens to workers of the fossil fuels industry if an energy transition takes place? An energy transition is a shift from a fossil fuels intensive economy to one reliant on the use of alternative sources of energy. This type of environmental policy is gaining more importance nowadays on the local and international agenda, following global decarbonization efforts. Those who defend it do so not only based on its positive direct effects but also on the potential dangers of doing nothing. However, some oppose it under the belief that it is highly costly and might not have the desired environmental effect. Therefore, environmental policy belongs to the set of economic decisions rooted in a cost-benefit analysis. As the benefits of a policy intervention outweigh its costs, there are incentives to implement it. The first way to approach these costs is in terms of the timing in which they occur. An energy transition seems costly in the short term, but its benefits come forward in the long run and are visible for future generations. Yet, a relevant short-term cost is how policy affects different sectors of the population distinctly, generally harming the most vulnerable.

From an economic theory perspective, an energy transition can be considered a creative destruction innovation process, where the consolidation of the clean energy sector displaces the fossil fuel sector. Following the Schumpeterian growth theory, this innovation process will eventually lead to higher growth rates once the turnover and financing costs are offset (Aghion and Howitt, 2009). Since this process involves leaving an economic sector behind, its costs are mainly perceived by individuals belonging to that sector, particularly workers with specific abilities demanded there. Moreover, one can expect that the degree to which they will be affected depends on how easily they can respond to this shock and adapt to changes in the demand for sector-specific abilities. Therefore, there is a welfare problem where there are winners and losers as the transition occurs. Since the effects of the transition are not uniform, there is a distributive problem limiting the extent to which policy can be implemented. To address this, a political economy analysis that goes beyond the scope of this paper is needed to understand how those negatively affected might raise concerns, oppose and even impede the transition from happening, even when its aggregate effects will benefit them in the long run.

However costly, an energy transition is necessary to avoid an environmental catastrophe, which happens once the planetary boundaries are crossed, and environmental damages become irreversible and affect human development (Rockström et al., 2009; Peretto and Valente, 2021). Moreover, these environmental damages also lead to an economic catastrophe, as they affect consumption and economic growth by creating risks on economic activity (Nordhaus, 2019; Stern, 2007; Bretschger and Valente, 2011). As this issue has been studied by the scientific and economic community, it has also gained relevance in the international community. Mainly, with the Paris Agreement and the Sustainable Development Goals, international cooperation efforts are being directed to promote the adaptation of economies to environmental commitments. This raises the importance of studying the direct and indirect effects of pursuing an energy transition, as the energy sector produces around 73% of CO_2 emissions worldwide (WRI, 2020) and it is a socially complex but necessary policy. In that sense, this paper presents a theoretical model following Acemoglu et al. (2012) approach to directed technical change, where innovation is vertical and sector-biased to a clean or a dirty sector, depending on parametric conditions. Energy production demands skilled and low-skilled labor, a sector-specific technology, and natural resources. Skilled and low-skilled labor is supplied by households, modeled under an overlapping generations structure following Diamond (1965) and Samuelson (1958). Depending on the worker's type of skills, they receive a wage that determines the investment in human capital for their offspring. Assuming credit restrictions, there are income and skills gaps that reinforce each other.

The timing in the model goes as follows. First, there is vertical innovation in the two sectors, but technology production is more expensive in the clean sector. Therefore, the demand for dirty energy is higher, and so is the environmental damage from its production. Consequently, the clean sector gets smaller and can even disappear, and the environmental damage ends up causing an environmental catastrophe. To avoid this scenario, a policy is implemented to increase costs in the dirty sector and lower them in the clean one. If the policy is strong enough, it displaces the dirty sector, achieving a complete transition. Consequently, households that belong to the dirty sector face a mobility cost in adapting to the transition. This cost is related to updating their abilities and supplying labor for the clean sector. However, this cost reduces available income, which reduces utility and investment in human capital for the next generation. Taking this into account, a policy is introduced to compensate for the welfare loss caused by the transition policy.

The model has two main results. First, a complete transition without government intervention is not possible because, under initial conditions, innovation is more costly in the clean sector. Consequently, reliance on dirty energy production and use leads to an environmental catastrophe. To avoid this, a transition policy is needed to guarantee the transition by incentivizing investment in the clean sector. This policy reduces costs in the clean sector and increases them in the dirty one, changing the direction of technical change towards the clean sector. A second result is that once this transition happens, there is an increase in labor demand for the new sector, which demands sector-specific abilities. Therefore, households bear a mobility cost that is relatively higher if they are low-income. As a result, their disposable income is affected, and only a proportion of workers can transit to the clean sector. This cost leads to a distributive effect where the income gap increases in the short run following i) changes in relative labor supply and ii) a reduction in skilled human capital investment. As a response, a second policy is desirable to compensate households and improve social welfare.

The relevance of studying the mechanisms under which an energy transition has distributive effects through the labor market is already being seen. So far, the literature has focused on presenting the theoretical and empirical mechanisms of clean technologies adoption and distributive effects of technical change apart. More recently, work has been done on studying the distributive effects on climate policy, particularly of carbon taxes (Castellanos and Heutel, 2019; Marin and Vona, 2019; Aubert and Chiroleu-Assouline, 2019; Weber, 2020). However, the theoretical link between an energy transition, directed technical change, and inequality has not yet been studied. Hence, this work's contribution is to rationalize and characterize the conditions under which technical change in the form of an energy transition is possible and affects workers differently by generating a distributive effect parting from human capital accumulation. Also, it proposes a policy to target the need to re-direct technical change and compensate households.

This work is related to two strands of the literature. First, it contributes to the literature on directed technical change, starting with Acemoglu (2002), where innovations are non-neutral and focused on the most productive sectors. In the context of dirty and clean technologies, a market transition to the clean sector that avoids an environmental catastrophe is possible if the demand for dirty inputs is significantly reduced. This can happen either because knowledge accumulation becomes the engine of growth while resources use is reduced (Peretto, 2021), or because the clean sector becomes more productive than the dirty sector and inputs are substitutes (Afonso et al., 2021; Acemoglu et al., 2019; León, 2019; Valente and Di Maria, 2008; Violante, 2009). However, if the dirty sector is more productive, innovation stays there, and the economy reaches an equilibrium with environmental catastrophe. In this case, state intervention is desirable to internalize the dirty sector externality and redirect innovation to the clean sector, mainly through fiscal policy (Acemoglu et al., 2012; Acemoglu et al., 2016; Greaker et al., 2018; Krass et al., 2013; Peretto, 2009). A first contribution is to generalize the conditions under which an energy transition is a process of directed technical change and can avoid an environmental catastrophe and those where the transition is not possible, and the catastrophe is inevitable.

Second, it contributes to the literature on wage inequality and the distributive effects of technical change. One of the biggest expected effects of an energy transition is job generation (Morgenstern et al., 2002). In that matter, ILO (2019) estimates that around 18 million jobs worldwide will be created with the transition by 2030, around 0,3% more than in a business as usual scenario. However, estimations show that job creation is conditional to market structure and the capacity of workers to reallocate to other sectors, as these can be more intensive in skilled labor (Babiker and Eckaus, 2007; Kammen and Kapadia, 2004; Guivarch et al., 2011; Bezdek et al., 2008; Muro et al., 2019; Chen et al., 2020). This, in turn, depends on workers' abilities acquired throughout their lives and how these can match the demand for specific skills when the transition happens. Moreover, as skilled workers can adapt more easily to disruptions and new technologies (Nelson and Phelps, 1966), the transition will mean higher relative costs to low-skilled workers. Here, literature has focused on a wage premium for different types of workers: the skills acquired by educated workers will be demanded in skilled jobs, regardless of the sector. In contrast, it is harder for low-skilled workers to supply the abilities demanded in the new sector. Even more, skills tend to be sector-specific (Manovskii and Kambourov, 2009; Gathmann and Schönberg, 2010), and technical change can depreciate the human capital of low skilled workers faster (Galor and Moav, 2000). This paper contributes to the literature by modeling this cost for households instead of firms and showing the conditions under which disposable income is affected and generates a distributive effect.

Third, directed technical change is linked to wage premiums and, therefore, to distributive effects on workers (Acemoglu, 2002; Galor and Moav, 2000; Hémous and Olsen, 2020). This is particularly true for the energy sector in the US, where clean energy workers earn higher and more equitable wages compared to other sectors workers (Muro et al., 2019). One of the fundamental mechanisms to explain this is households' investment in human capital. This depends on households' initial income (Galor and Zeira, 1993; He, 2006; Angarita, 2021) and the pace of skill acquisition (Aghion and Commander, 1999). As a consequence, education has higher returns that incentivize the production of complementary technologies to skilled labor (Acemoglu, 2002). Whenever skilled workers - who also tend to have a higher income - receive higher returns, the process of inter-generational transmission of wealth generates income differences that are persistent throughout time (Zuleta, 2015, Alvarez et al., 2019). In the context of an energy transition, potential distributive effects have motivated the creation of the Just transition movement[1], which originates from unions and environmental justice groups, demanding that, as the transition is imminent, it should also avoid adverse effects on lower-income sectors (Alliance, 2021). As literature has shown that technical change is skill-biased and can have persistent effects, a final contribution is to show how distributive effects can be temporary and come from sector-biased technical change instead of skill-biased technical change.

Including the introduction, this paper has three parts. The following presents the theoretical model where an environmental catastrophe can only be avoided through a

[1] More information about the Just Transition movement can be found in https://bit.ly/3CnReCL

state-supported energy transition that has a distributive effect on households. Finally, a section including the conclusions and further remarks of this paper.

2 A model of energy transition and wage inequality[2]

This section describes the agents' behavior in the model and characterizes the equilibrium in the economy. First, environmental quality and natural resources' restrictions are characterized to show how production has environmental effects. Following this, I solve the problem on the supply side, where energy types are demanded depending on their substitutability. Here investment in the clean sector is costly, so innovation is biased towards the dirty sector. As a result, there are conditions under which the two sectors remain, or the clean sector disappears. Either way, the presence of the dirty sector implies that there is an environmental catastrophe. To avoid this, a policy is implemented to redirect innovation to the clean sector and guarantee the transition. On the demand side, households accumulate human capital for the following generation and pay a mobility cost to adapt to the transition. As this cost reduces available income, a policy is introduced to compensate them for the transition's regressivity. In the long run, there is a balanced growth path (BGP) only if the dirty sector is displaced and the catastrophe is avoided. In the setup section, I present the firms' and households' problems. After this, I solve each agent's problem and present the equilibrium in the model.

2.1 Setup

2.1.1 Environmental quality

The evolution of environmental quality is defined by

$$S_{t+1} = (1+\xi)S_t - \delta_d E_{d,t}(N_{d,t}) \qquad 2.1.1$$

$S_t \in [0, \bar{S}]$ is the level of environmental quality, with $\bar{S}$ being the maximum level of environmental quality. ξ is the environmental regeneration rate, δ_d is the environmental impact of dirty energy production, and $E_{d,t}$ is the energy produced in the dirty sector,

[2] A list of the variables and parameters used in the model can be found in appendix 4.1

which is an increasing function in natural resources extraction $N_{d,t}$. If $\frac{E_d}{S} > \frac{\xi}{\delta_d}$, then $\frac{S_{t+1}}{S_t} < 1$. This is, if the negative impact of energy production is higher than recovery, the environment deteriorates. Additionally, if $\frac{E_d}{S} \geq \frac{1+\xi}{\delta_d}$ then environmental quality reaches a trivial steady state of 0 ($\frac{S_{t+1}}{S_t} \leq 1$). Eventually, an environmental catastrophe is reached.

Definition 1. *An environmental catastrophe happens when* $S_{t'} = 0$

Where t' is defined as a moment it time where an environmental catastrophe arrives. If $\frac{E_{d,t'-1}}{S_{t'-1}} = \frac{1+\xi}{\delta_d}$, then $S_t = S_{t'} = 0$ for any $t > t'$.

Assumption 1. $E'_d(N_d) \geq 1$,

From assumption 1, it follows that $E'_d(N_d)\frac{\Delta N_d}{Nd} \geq \frac{\Delta N_d}{N_d}$ and $\frac{E_{d,t+1}}{E_{d,t}} \geq \frac{N_{d,t+1}}{N_{d,t}}$. Therefore, the growth rate of energy production is higher than the one of natural resources extraction.

2.1.2 Natural resources

Dirty production is often intensive on coal, oil, and gas, while cleaner production demands other resources, such as lithium and nickel, but does not have the same environmental impact. For this reason, I assume that natural resources are used separately for the production of dirty (d) and clean (c) energy. $NR_{j,t}$ is the stock of natural resources, where $j \in \{c, d\}$, which depends positively on the environmental quality at the same moment in time. This means that environmental degradation affects the availability of natural resources. The flow of natural resources $N_{j,t}$ is extracted at a rate ζ_j. Resources are exhaustible and necessary for the production of energy. Therefore, even when a higher extraction increases production, it reduces the resource stock faster.

$$NR_{d,t+1}(S_{t+1}) = NR_{d,t}(S_t) - \zeta_d N_{d,t} \tag{2.1.2}$$

$$NR_{c,t+1}(S_{t+1}) = NR_{c,t}(S_t) - \zeta_c(E_{c,t})N_{c,t} \tag{2.1.3}$$

Natural resources used for dirty energy production have a constant extraction rate, while resources used for clean energy production are a function of clean energy defined as

$$\zeta_c = 1 - \frac{E_{c,t}}{1 + E_{c,t}} \tag{2.1.4}$$

Extraction for dirty energy production always needs resources, even when the industry is sufficiently advanced. Clean energy, on the other hand, only needs them for installed capacity building. Hence, the extraction rate is decreasing in clean energy production and eventually becomes 0. As resources are non-renewable, they become exhausted as they are extracted. This introduces an environmental externality caused by a lack of coordination between agents that leads to over-exploitation, as described by Hardin (1968). As environmental quality depends on energy production, it is an increasing function of the natural resources' stock. While resources are extracted, more energy is produced, and the environmental impact is higher. There is a time $\hat{t}$ when natural resources are completely depleted and $NR_{\hat{t}+1} = 0$. From this period on $\zeta_d \frac{N_{d,t-1}}{NR_{d,t-1}} = 1$. Whether total depletion of natural resources happens before or after the environmental catastrophe depends on the natural resources' stock size. A higher stock implies that resources can be used for contaminating production over a long time, and the catastrophe arrives before they are finished. Meanwhile, a small stock implies that it can be completely depleted before the environment collapses. To solve this, I will assume that the dynamic of natural resources is higher than the dynamic of environmental quality, as shown in assumption 2.

Assumption 2. $\frac{E_{d,0}}{S_0} > \frac{\zeta_{d,0}}{\delta_d,0} \frac{N_{d,0}}{NR_0}$.

From assumption 2 and equations 2.1.1 and 2.1.2 it follows that $\frac{S_{t+1}}{S_t} < \frac{NR_{t+1}}{NR_t}$ for any $t > 0$. In other words, environmental damage is faster than extraction. Then, $t' > \hat{t}$ and an environmental catastrophe arrives before complete depletion of natural resources.

Lemma 1. *The use of natural resources for dirty energy production eventually leads to an environmental catastrophe in t', even before complete depletion in $\hat{t}$.*

Proof follows from assumptions 1 and 2.

2.1.3 Firms

Following (Acemoglu, 2002), firms are modeled under a two-sector directed technical change model. Inside each sector, there is Schumpeterian innovation following Aghion and Howitt (2009). In both clean and dirty sectors, innovation occurs as firms devote resources to R&D to innovate in a non-deterministic process. When successful, they sell a patent to monopolistic firms that use it to produce a sector-specific technology. Skilled and low-skilled workers then use this technology to produce energy, which is then used along with the other sector energy to produce a final consumption good.

Final goods A final consumption good Y_t is produced using a CES technology combining fossil fuels energy $E_{d,t}$ and clean energy $E_{c,t}$, with a substitution parameter $\eta \leq 1$. This specification is useful to show how different equilibria can be reached depending on energy types being substitutes or complements.

$$Y_t = (E_{c,t}^{\eta} + E_{d,t}^{\eta})^{\frac{1}{\eta}}$$

Energy production Energy in each sector is produced symmetrically under a Cobb-Douglas technology, with complementarity between inputs and constant returns to scale. These firms operate in competition and demand natural resources $N_{j,t}$. Also, they combine skilled $L_{s,j,t}$ and low-skilled labor $L_{u,j,t}$ through a CES function with a substitution parameter $\rho \in (0,1)$, where θ is the weight of skilled-labor in total labor demand, which is more productive than low-skilled labor in $B > 1$. These are also combined with a continuum of i possible sector-specific technologies $e_{i,j,t}$, all of which have an associated productivity $A_{i,j,t}$. As there is a continuum of technologies, this specification guarantees that if new technologies are produced, they displace already existing technologies that are less productive. Labor and energy production are respectively

$$L_{j,t} = (B\theta L_{s,j,t}^{\rho} + (1-\theta)L_{u,j,t}^{\rho})^{\frac{1}{\rho}} \qquad 2.1.5$$

$$E_{j,t} = N_{j,t}^{\phi_2}(B\theta L_{s,j,t}^{\rho} + (1-\theta)L_{u,j,t}^{\rho})^{\frac{1-\phi}{\rho}} \int_0^1 A_{i,j,t}^{1-\phi_1} e_{i,j,t}^{\phi_1} di \qquad 2.1.6$$

Where $\phi, \phi_1, \phi_2 \in (0,1)$ and $\phi_1 + \phi_2 = \phi$. The market clearing condition for the labor market implies that labor demand for each type of workers $h \in \{s,u\}$ in sector j at the moment t is equal or lower than labor supply, which is normalized to 1.

$$L_{s,c,t} + L_{s,d,t} \leq L_{s,t} \tag{2.1.7}$$

$$L_{u,c,t} + L_{u,d,t} \leq L_{u,t} \tag{2.1.8}$$

$$L_{s,t} + L_{u,t} \leq 1 \tag{2.1.9}$$

Technology production For each sector, technology is produced in monopolistic competition, where producers maximize their benefits[3] $\Pi_{i,j,t}$. Each firm observes the inverse demand function of technology made by energy producers $p_{i,j,t}$. In order to produce technology in both sectors, producers pay a cost $c \in (0,1)$. For the fossil fuels sector:

$$\Pi_{i,d,t} = p_{i,d,t}e_{i,d,t} - ce_{i,d,t}$$

In the clean energy sector, producers pay an extra fixed cost F, representing the necessary infrastructure investment to build clean technologies. This specification follows (Krass et al., 2013) and is related to setup, acquisition, and installation costs. Also, it is related to the assumption that the clean sector is not as developed as the dirty sector. Therefore, as this sector is more advanced, the fixed cost becomes less relevant.

$$\Pi_{i,c,t} = p_{i,c,t}e_{i,c,t} - ce_{i,c,t} - F$$

Ideas production In both sectors, ideas producers invest in R&D and choose resources $R_{i,j,t}$ with a cost[4] r_t to maximize the probability of having a successful innovation μ_j. If the firm is successful, it receives the payment of a patent which is equal to the positive benefits of the technology producer $\Pi_{j,t}$. Success probability is defined as

$$\mu_j = 2\left(\frac{R_{j,t}}{A^*_{j,t}}\right)^{\frac{1}{2}}$$

[3]Note that this is a static problem for multiple firms where there are positive benefits used to buy a patent that lasts one period. A similar analysis could be made following a Romer 1990 structure, so the firm maximizes the present value of its benefits. In any case, the qualitative results needed for a directed technical change analysis do not change.

[4]The assumption of a marginal cost of ideas increasing the interest rate has been used in the growth literature as in Davila (2020) to assure that the credit market clears.

If innovations are successful, average productivity for sector j is multiplied by $\gamma > 1$ and reaches objective productivity: $A_{i,j,t} = A^*_{j,t} = A_{j,t}$. If not, the average productivity for the sector is the same as in the last period. The productivity dynamic is

$$A_{j,t+1} = \begin{cases} \gamma A_{j,t} & P(\mu_j) \\ A_{j,t} & P(1-\mu_j) \end{cases} \tag{2.1.10}$$

Consistently, expected productivity growth is defined as $\frac{A_{j,t+1}}{A_{j,t}} = \mu_j(1-\gamma)$ and can also be understood as the frequency of innovations. Even if there is no innovation in a period t, firms will take advantage of productivity in $t-1$.

2.1.4 Households' human capital investment with a mobility cost

Following Samuelson (1958) and Diamond (1965), two types of household are modeled under an overlapping generations structure. Households' problem is defined as

$$\begin{aligned} \max_{\{c_t^{y,f}, c_{t+1}^{o,f}, s_t\}} \quad & U(c) = \log c_t^{y,f} + \beta \log c_{t+1}^{o,f} \\ s.t. \quad c_t^{y,f} + s_t + \psi^f_{j,t+1} + \kappa\psi^f_{j,t} & \leq w^f_{j,t}(1-\lambda) \\ c_{t+1}^{o,f} & \leq s_t r_{t+1} + \lambda w^f_{j,t+1} \\ s_t & \geq 0 \end{aligned}$$

Each household comprises two generations: young (y) and old (o). Young individuals devote their income to consume $c_t^{y,f}$, save s_t^f, invest in human capital for the next generation ψ^f_{t+1} and transfer resources to their parents λ. Also, households face credit constraints, so they cannot finance human capital investment with debt [5]. While being young, workers receive a wage $w^f_{j,t}$ depending on the sector j they work on. When investing in human capital, parents decide the sector where their offspring will belong and whether they are skilled or low-skilled. An important assumption is that the cost of human capital is the same across sectors[6] $\psi^f_{c,t+1} = \psi^f_{d,t+1}$, but is higher for skilled human capital $\psi^s_{j,t+1} > \psi^u_{j,t+1}$. Likewise, $\psi^f_{j,t+1}$ is assumed to be positive. This implies that parents

[5] An extension where this assumption is relaxed is shown in the appendix 4.5

[6] This is a simplifying assumption. If it were to be relaxed, the qualitative results of the model wouldn't change since the important analysis is on costs relative to earnings

observe wages for both sectors and invest in education for the sector with a higher wage. By investing in specific education, workers enter the market with specific abilities for each sector. Consequently, if they want to move to another sector, they face a cost $\kappa\psi_{j,t}$. This cost is related to the need for workers to adapt to the skills demand in the new sector when the transition takes place. That is, $\kappa > 0$ when the transition happens and the worker perceives that cost, while $\kappa = 0$ when this does not happen. As the decision of human capital investment is made by parents in $t-1$, each worker pays the cost associated with their human capital in t. Therefore, low-skilled workers cannot become skilled by paying this cost, but skilled workers can transit and become low-skilled workers if the cost is too high. Old individuals receive the return to their savings at a rate r_t, the transfer made by the young generation, and use them to consume $c^{f,o}_{t+1}$. The source of heterogeneity is wage income $w^f_{j,t}$ with $f = \{s, u\}$, which determines the investment in human capital for the next generation $\psi^f_{j,t+1}$, as a form of inter-generational wealth transmission. Here,

$$w^f_{t+1} = \begin{cases} w^s_{j,t+1} \quad if \quad \psi^s_{j,t+1}(w^f_{j,t}) \\ \\ w^u_{j,t+1} \quad if \quad \psi^u_{j,t+1}(w^f_{j,t}) \end{cases} \tag{2.1.11}$$

Equation 2.1.11 shows that human capital investment is a discrete variable. If parents' income is high enough, they can invest in skilled human capital for their offspring. Consequently, the skill level determines income in $t+1$.

2.2 Firms optimization

Final goods Producers operate in competition and choose both types of energy to maximize their benefits $\Pi_{y,t}$

$$\max_{E_{c,t},E_{d,t}} \Pi_{y,t} = (E^\eta_{c,t} + E^\eta_{d,t})^{\frac{1}{\eta}} - p_{c,t}E_{c,t} - p_{d,t}E_{d,t}$$

From the first order conditions, equation 2.2.1 shows that the marginal substitution rate between energy types equals the price ratio.

$$\frac{p_{c,t}}{p_{d,t}} = \left(\frac{E_{c,t}}{E_{d,t}}\right)^{\eta-1} \tag{2.2.1}$$

Relative demand depends on relative prices, so the firm demands more of the least expensive type of energy. Together with this, the value of η determines whether there is an interior or a corner solution in energy demand.

Energy production Energy firms operate in competition and maximize their benefits $\Pi_{E,j,t}$ by combining labor, technology and natural resources through a Cobb-Douglas technology. The price for natural resources is exogenous.

The maximization problem is defined as

$$\begin{aligned}\max_{N_{j,t},L_{s,j,t},L_{u,j,t},e_{i,j,t}} \Pi_{E,j,t} = p_{j,t}\left[N_{j,t}^{\phi_2}(B\theta L_{s,j,t}^{\rho} + (1-\theta)L_{u,j,t}^{\rho})^{\frac{1-\phi}{\rho}} \int_0^1 A_{i,j,t}^{1-\phi_1} e_{i,j,t}^{\phi_1} di\right] \\ - p_{N,j,t}N_{j,t} - \int_0^1 p_{i,j,t}e_{i,j,t}di - w_{s,j,t}L_{s,j,t} - w_{u,j,t}L_{u,j,t}\end{aligned}$$

From the first-order conditions and using (2.1.5) and (2.1.6), the demands for natural resources, technology, and skilled and low skilled labor can be obtained.

$$p_{N,j,t} = \frac{\phi_2 p_{j,t} E_{j,t}}{N_{j,t}} \tag{2.2.2}$$

$$p_{i,j,t} = \phi_1 p_{j,t} A_{i,j,t}^{1-\phi_1} e_{i,j,t}^{\phi_1 - 1} N_{j,t}^{\phi_2} L_{j,t}^{1-\phi} \tag{2.2.3}$$

$$w_{s,j,t} = \frac{p_{j,t} B\theta(1-\phi)E_{j,t}L_{s,j,t}^{\rho-1}}{L_{j,t}^{\rho}} \tag{2.2.4}$$

$$w_{u,j,t} = \frac{p_{j,t}(1-\theta)(1-\phi)E_{j,t}L_{u,j,t}^{\rho-1}}{L_{j,t}^{\rho}} \tag{2.2.5}$$

The functional form of energy production makes all marginal productivities decreasing and equal to their respective market price. The firm chooses the amount of extracted natural resources and pays a given price[7] equal to $\frac{N_{j,t}}{NR_{j,t}}$. Following equations 2.1.2 and

[7] Appendix 4.3 shows how this price is optimal for an extractive firm and is consistent with the Hotelling

2.1.3, once resources used for dirty energy production are completely exhausted, the sector disappears. This is not the case for the clean sector, where resources are not completely depleted. However, as these resources are extracted, the energy firm pays a higher price for them. On the other hand, the firm demands i technologies and pays the respective price for each one. New and more productive technologies can displace old ones and, by complementing the remaining, will make them more productive. Finally, in regular times, wages are symmetrical between sectors but different between types of workers. Equation 2.6.5 shows how the wage ratio between skilled and low skilled workers from the same sector j, while equation 2.2.7 shows the wage ratio between sectors for type h workers.

$$\frac{w_{s,j,t}}{w_{u,j,t}} = \frac{B\theta}{1-\theta}\left(\frac{L_{u,j,t}}{L_{s,j,t}}\right)^{1-\rho} \tag{2.2.6}$$

$$\frac{w_{h,c,t}}{w_{h,d,t}} = \frac{p_{c,t}}{p_{d,t}}\frac{E_{c,t}}{E_{d,t}}\left(\frac{L_{d,t}}{L_{c,t}}\right)^{\rho}\left(\frac{L_{h,d,t}}{L_{h,c,t}}\right)^{1-\rho} \tag{2.2.7}$$

A higher low-skilled labor supply increases the intra-sector wage ratio. In other words, if energy production is more intensive in low-skilled labor, skilled workers benefit more from higher relative wages. This is a consequence of decreasing marginal productivity of labor, which assures that wages are equal once labor is reallocated. On the other hand, since the problems for both sectors are symmetric as long as no reallocation takes place, both skilled and low-skilled workers go to the sector with higher wages until they are equal.

Technology production After seeing the demand for technology made by energy producers, optimal choices for technology producers lead to

$$p_{i,j,t} = \frac{c}{\phi_1} \tag{2.2.8}$$

$$e_{i,j,t} = \left(\frac{p_{j,t}N_{j,t}^{\phi_2}\phi_1^2 L_{j,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{i,j,t} \tag{2.2.9}$$

rule.

$$\Pi_{i,d,t} = \left(\frac{p_{d,t} N_{d,t}^{\phi_2} \phi_1^2 L_{d,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{i,d,t} \left(\frac{c(1-\phi_1)}{\phi_1}\right) \qquad 2.2.10$$

$$\Pi_{i,c,t} = \left(\frac{p_{c,t} N_{c,t}^{\phi_2} \phi_1^2 L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{i,c,t} \left(\frac{c(1-\phi_1)}{\phi_1}\right) - F \qquad 2.2.11$$

Since firms operate in monopolistic competition, they can fix a price higher than their marginal cost c. This price considers the technology share of energy production ϕ_1, which shows that technologies are not perfect substitutes. Optimal quantities and benefits depend on each technology-specific productivity. The benefits are positive and allow firms to acquire a patent for the technology they produce. This patent lasts one period and is the source of monopolistic power. Note that benefits are specific for each i. Therefore, aggregate benefits for each sector depend on aggregate productivity[8] $A_{j,t} = \int_0^1 A_{i,j,t} di$. Also, it is worth noting that if the fixed cost for clean technology firms is higher than their mark-up, they do not have the benefits to buy a patent. Therefore, there is no clean technology production.

Lemma 2. $F < \left(\frac{p_{c,t} N_{c,t}^{\phi_2} \phi_1^2 L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{c,t} \left(\frac{c(1-\phi_1)}{\phi_1}\right)$ *is a necessary condition for innovation in the clean sector.*

Proof follows directly from equation 2.2.11. Without positive benefits the firm can't acquire a patent and produce technology for innovation.

Ideas production The firm chooses resources to maximize their benefits

$$\max_{R_{i,j,t}} \Pi_{A,j,t} = \mu_{i,j,t} \Pi_{j,t} - R_{j,t} r_t$$

[8] Analyzing aggregate productivity in each sector implies the non-deterministic nature of Schumpeterian models. Even when some firms lose their benefits, the sector perceives the rise in productivity that comes with innovation. Therefore, monopolistic power remains even when an individual firm loses its patent and there is turnover for firms and technologies. Also, following Aghion and Howitt (1992) a period is a time between two successful innovations, which is another reason for monopolistic power to remain.

In interior solution, optimal probabilities of success in innovation are given by

$$\mu_d = \frac{2}{r_t}\left[\left(\frac{p_{d,t}N_{d,t}^{\phi_2}\phi_1^2L_{d,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\left(\frac{c(1-\phi_1)}{\phi_1}\right)\right] \tag{2.2.12}$$

$$\mu_c = \frac{2}{r_t}\left[\left(\frac{p_{c,t}N_{c,t}^{\phi_2}\phi_1^2L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\left(\frac{c(1-\phi_1)}{\phi_1}\right)-\frac{F}{A_{c,t}}\right] \tag{2.2.13}$$

From equations 2.2.12 and 2.2.13, a firm is more likely to be successful in innovating if the payment it receives from the patent –the benefits from the technology producers –are higher. For the same reason, the success probability in the clean sector is lower, as the fixed cost F lowers these benefits. The probability ratio can be obtained using 2.2.12 and 2.2.13.

$$\frac{\mu_c}{\mu_d} = \left[\left(\frac{N_{c,t}}{N_{d,t}}\right)^{\eta\phi_2}\left(\frac{L_{c,t}}{L_{d,t}}\right)^{\eta(1-\phi)}\right]^{\frac{1}{1-\phi_1}}\left(\frac{A_{c,t}}{A_{d,t}}\right)^{\eta-1} - \frac{F}{A_{c,t}\left(\frac{p_{d,t}N_{d,t}^{\phi_2}\phi_1^2L_{d,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\left(\frac{c(1-\phi_1)}{\phi_1}\right)}$$

Define the second term on the right-hand side of the equation as a transition cost, which is a consequence of the fixed cost F in the clean sector. A higher transition cost means fewer incentives to invest in the clean sector. Regarding the first term of the right-hand side, if $\eta = 0$, energy types are complements, so extraction and productivity ratios become irrelevant for innovation, which always happens in both sectors. If $\eta = 1$, then energy types are perfect substitutes, so there is a corner solution where innovation in the clean sector, which is more expensive, disappears due to the transition cost. Now, if $\eta \in (0,1)$ then

$$\frac{\partial\frac{\mu_c}{\mu_d}}{\partial\frac{N_{c,t}}{N_{d,t}}} > 0, \quad \frac{\partial\frac{\mu_c}{\mu_d}}{\partial\frac{L_{c,t}}{L_{d,t}}} > 0, \quad \frac{\partial\frac{\mu_c}{\mu_d}}{\partial\frac{A_{c,t}}{A_{d,t}}} < 0$$

In this case, relative probability is increasing in relative natural resources extraction. This happens because natural resources are necessary for energy production, and firms benefit from extraction as the time horizon for resources to be scarce is far. Similarly, the relative probability is increasing in relative labor supply. In equilibrium, labor allocation depends on its returns and productivity. Since the dirty sector is more productive, wages are higher, and workers have no incentives to move. Finally, relative probability is decreasing in relative productivity, which is a result of equation 2.2.1. Whatever happens with the clean sector depends on the demand for each type of energy. If $\eta \in (0, 1)$, energy types are substitutes, and the clean sector shrinks as there is a virtuous cycle in the dirty sector. Here, higher productivity attracts more investment, which keeps increasing innovation in the dirty sector. Moreover, each time is less likely that the condition in lemma 2 is fulfilled as revenues will be lower for clean energy firms. Consequently, clean energy becomes more expensive to producers of final goods, but higher prices do not translate into productivity. A similar process happens if energy types are complements, but productivity in the clean sector will be higher than if they are substitutes. In any case, higher productivity in the dirty sector implies that the catastrophe arrives faster. A corner solution where the clean sector disappears would be seen for $\eta = 1$

Lemma 3. *If $\eta \in (-\infty, 1)$ the clean sector coexists with the dirty sector, but clean energy demand is lower due to higher prices. If $\eta = 1$ the clean sector disappears.*

Proof in appendix 4.2.1

2.3 A trivial equilibrium with environmental catastrophe

A direct consequence of the dirty sector being more productive is that there are no market incentives for dirty energy production to cease and, therefore, reduce its environmental impact. This implies that private benefits are high enough to discourage clean energy production. Two possible cases can be seen.

i. The clean sector disappears completely, and final goods production only uses dirty energy. Assuming that the natural resources' stock used for dirty energy production is high enough, the environmental impact of production outweighs environmental regeneration, and an environmental catastrophe is reached.

ii. The two sectors coexist, but the demand for dirty energy is higher than the one for clean energy (Lemma 3). This is a consequence of an interior solution for final goods production, where there are increasing costs in clean technology production.

Therefore, environmental damage remains higher than regeneration, and there is an environmental catastrophe. Higher demand for clean energy can slow this process, but increasing dirty energy production makes the catastrophe inevitable.

Lemma 4. *Even when there is a clean energy sector operating, the existence of a dirty energy sector leads to an environmental catastrophe.*

Proof in annex 4.2.2

2.4 Achieving an equilibrium with an energy transition

To avoid an environmental catastrophe, it is necessary to achieve an energy transition that eliminates the dirty sector. For this to happen, there must be incentives to innovate in the clean sector. Particularly, the following conditions must be satisfied:

1. Clean technology producers must have positive benefits.
2. Incentives to invest in clean technology must be at least equal to incentives to invest in dirty technology: $\Pi_{c,t} \geq \Pi_{d,t}$.
3. Technical change must be directed to the clean sector: $\frac{\Delta A_{c,t}}{A_{c,t}} > \frac{\Delta A_{d,t}}{A_{d,t}}$

As the market equilibrium leads to an environmental catastrophe, government intervention is needed to guarantee the transition. I consider two types of policy intervention: a Pigouvian tax on dirty energy production and a lump-sum subsidy to clean technology production. Even if the tax is not optimal and does not intend to internalize an externality, it is Pigouvian because it discourages innovation in the dirty sector by raising its cost. Meanwhile, the lump-sum transfer to clean technology production encourages innovation in this sector. To be effective, the subsidy must be at least equal to the fixed cost faced by this sector to guarantee positive benefits for clean technology firms.

Tax on dirty energy production

A value-added tax τ is now paid by dirty energy producers. The problem is defined as

$$\max_{N_{d,t},L_{s,d,t},L_{u,d,t},e_{i,d,t}} \Pi_{E,d,t} = (1-\tau)p_{d,t}\left[(N_{d,t}^{\phi_2})(B\theta L_{s,d,t}^{\rho} + (1-\theta)L_{u,d,t}^{\rho})^{\frac{1-\phi}{\rho}} \int_0^1 A_{i,d,t}^{1-\phi_1} e_{i,d,t}^{\phi_1} di\right]$$
$$- p_{N,d,t}N_{d,t} - \int_0^1 p_{i,d,t}e_{i,d,t}di - w_{s,d,t}L_{s,d,t} - w_{u,d,t}L_{u,d,t}$$

Where $\tau \in (0,1)$ is the tax to dirty energy production. Optimal conditions are now

$$p_{N,d,t} = \frac{(1-\tau)p_{d,t}\phi_2 E_{d,t}}{N_{d,t}} \quad 2.4.1$$

$$w_{s,j,t} = \frac{(1-\tau)p_{d,t}B\theta(1-\phi)E_{d,t}L_{s,d,t}^{\rho-1}}{L_{d,t}^{\rho}} \quad 2.4.2$$

$$w_{u,d,t} = \frac{(1-\tau)p_{d,t}(1-\theta)(1-\phi)E_{d,t}L_{u,d,t}^{\rho-1}}{L_{d,t}^{\rho}} \quad 2.4.3$$

$$p_{i,d,t} = (1-\tau)\phi_1 p_{d,t}A_{i,d,t}^{1-\phi_1}e_{i,d,t}^{\phi_1-1}N_{d,t}^{\phi_2}L_{d,t}^{1-\phi} \quad 2.4.4$$

With the introduction of the tax, the perceived marginal productivity of factors in the dirty sector is lower, which implies lower remuneration to factors. Consequently, the price of natural resources is also reduced, and so is extraction. Even if the tax is not designed to internalize the environmental externality, it reduces revenues for extractive firms, which reduces their extraction and damage to the environment. Wages also become lower for both skilled and low-skilled workers in the sector. From equations (2.4.2) and (2.4.3):

$$\frac{w_{h,c,t}}{w_{h,d,t}} = \frac{p_{c,t}}{p_{d,t}}\frac{E_{c,t}}{E_{d,t}}\left(\frac{L_{d,t}}{L_{c,t}}\right)^{\rho}\left(\frac{L_{h,d,t}}{L_{h,c,t}}\right)^{1-\rho}\frac{1}{1-\tau} \quad 2.4.5$$

By introducing the tax, relative wages in the clean sector are higher, giving workers incentives to move to this sector. Under perfect mobility, workers reallocate to the other sector, and decreasing marginal productivity eventually results in equal wages for both sectors. However, if mobility is costly, the tax generates a wage premium in the short run. The tax also decreases technology demand, affecting benefits for dirty technology producers. The reduction comes from a lower technology demand, affecting firms' mark-up and capacity to acquire a patent.

$$\Pi_{i,d,t} = \left(\frac{p_{d,t}(1-\tau)N_{d,t}^{\phi_2}\phi_1^2 L_{d,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{i,d,t}\left(\frac{c(1-\phi_1)}{\phi_1}\right) \qquad 2.4.6$$

Transfer to clean technology production

Even if the tax on dirty energy production can re-direct investment to the clean sector, as long as the fixed cost on clean technology production is too high, there are no positive benefits in the sector or innovation. In this case, the policy must include a lump-sum transfer ω^c to clean energy producers. This transfer guarantees that clean technology firms have positive benefits if $\omega^c \geq F$. Benefits are now defined as

$$\Pi_{i,c,t} = \left(\frac{p_{c,t}N_{c,t}^{\phi_2}\phi_1^2 L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{i,c,t}\left(\frac{c(1-\phi_1)}{\phi_1}\right) - F + \omega^c \qquad 2.4.7$$

To guarantee that incentives to invest in the clean sector are higher, it must happen that $\Pi_c > \Pi_d$. A tax rate that guarantees this is

$$\tau > 1 - \frac{p_{c,t}}{p_{d,t}}\left(\frac{N_{c,t}}{N_{d,t}}\right)^{\phi_2}\left(\frac{L_{c,t}}{L_{d,t}}\right)^{1-\phi}\left(\frac{A_{i,c,t}}{A_{i,d,t}}\right)^{1-\phi_1} + \frac{F-\omega^c}{\left(\frac{p_{d,t}N_{d,t}^{\phi_2}\phi_1^2 L_{d,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} A_{d,t}\left(\frac{c(1-\phi_1)}{\phi_1}\right)} \qquad 2.4.8$$

Lemma 5. *A combination of policies τ and $\omega^c \geq F$ may guarantee that $\Pi_c > \Pi_d$ and, all things equal, $\mu_c > \mu_d$.*

Proof: follows from equation 2.4.8

2.5 Possible long-run equilibria for production

Four possible equilibria exist depending on the values of η and whether there is a transition policy. Results are resumed in figure 2.5

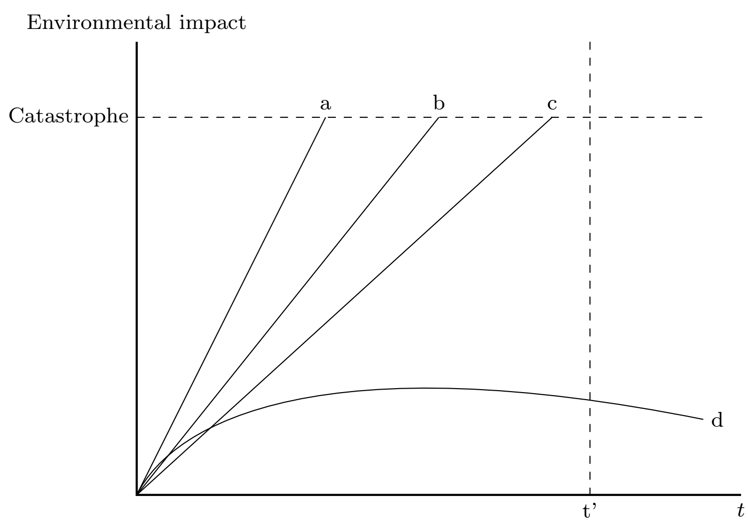
a. $\eta = 1$ and there is no policy: The clean sector disappears completely, and there is an environmental catastrophe due to dirty energy production.

b. $\eta \in (-\infty, 1)$ and there is no policy: Both sectors coexist, but demand for dirty energy implies that there is a catastrophe, even if it arrives later (See proof for Lemma 3).

c. $\eta < 0$ and there is a transition policy: Both sectors coexist, but dirty energy production is more costly. Since it is still being produced, there is also a catastrophe, later than in cases a. and b.

d. $\eta \in (0, 1]$ and there is a transition policy: The clean sector becomes more productive, and there is a complete transition where the dirty sector disappears. As there is no environmental degradation, the catastrophe is avoided.

Figure 2.5: Environmental outcomes from energy production



a: $\eta = 1$ and no policy, b: $\eta \in (\infty, 1)$ and no policy, c: $\eta < 0$ and there is a policy, d: $\eta \in (0, 1]$ and there is a policy.

Figure 2.5 resumes the possible scenarios for production depending on the value of η and whether there is a transition policy or not. Time is on the x-axis, where the dashed line t' is the moment natural resources are completely depleted. The environmental impact of production under each scenario is shown on the y-axis, where the dashed line represents the moment where the catastrophe happens. Note that from Lemma 1, t' is later than any t, but the last depends on each scenario.

Lemma 6. *A necessary condition for avoiding an environmental catastrophe is a transition policy to incentivize investment in the clean sector. A sufficient condition is that energy types are substitutes and technical change is re-directed so the dirty sector disappears.*

Proof follows from Lemmas 1, 3 and 5.

2.6 Directed technical change towards the clean sector

As the policy guarantees that there is a clean sector, there is a steady-state for relative productivity, and $\frac{\mu_c}{\mu_d} = 1$. From this initial condition, technical change is re-directed to the clean sector, eventually becoming more advanced. Let us suppose that $\eta \in (0,1)$ since it is the only case where there is no environmental catastrophe. Using 2.1.6 and 2.2.1, the productivity ratio is defined as

$$\frac{A_{c,t}}{A_{d,t}} = \left(\frac{N_{c,t}}{N_{d,t}}\right)^{\frac{\phi_2 \eta}{(1-\phi_1)(1-\eta)}} \left(\frac{L_{c,t}}{L_{d,t}}\right)^{\frac{\eta(1-\phi)}{(1-\phi_1)(1-\eta)}} \left(\frac{1}{1-\tau}\right)^{\frac{\eta}{(1-\phi_1)(1-\eta)}} - \Omega \tag{2.6.1}$$

Where Ω is the transition cost weighted by relative productivity.

$$\Omega = \left[\frac{\phi_1(F-\omega^c)}{c(1-\phi_1)A_{c,t}} \left(\frac{c}{p_{d,t} N_{d,t}^{\phi_2} \phi_1^2 L_{d,t}^{1-\phi}}\right)^{\frac{1}{1-\phi_1}}\right]^{\frac{1}{1-\eta}} \frac{A_{c,t}}{A_{d,t}} \tag{2.6.2}$$

If energy types are substitutes, the productivity ratio is increasing in relative natural resources use and relative labor supply. A reasonable assumption is that the dirty sector uses more resources than the clean sector $N_{d,t} > N_{c,t}$. However, the tax compensates for this effect. Also, relative productivity is increasing in relative labor supply, which is related to a scale effect where workers take advantage of better technologies to produce. It is worth noting that the transition cost is decreasing in clean sector productivity; therefore, it disappears in the long run as the clean sector is more developed and $\Omega = 0$[9]. Intuitively, as the sector's relative size is higher, fixed costs are less important than revenues. Also, in the long run, a higher part of the infrastructure needed for technology production is built, so this cost is also less relevant. The result is that investment incentives are reallocated to the clean sector. A stronger policy shrinks the dirty sector to the point that it disappears.

[9] See proof in annex 4.2.5

Proposition 1. *Under a policy that guarantees that the three conditions on section 2.4 hold and energy types are complements, $\eta < 0$, there is a partial transition where the two sectors coexist until there is an environmental catastrophe. If the conditions hold and energy types are substitutes $\eta \in (0, 1]$, there is a complete transition where the dirty sector disappears.*

Proof follows from Lemmas 1, 3, 4 and 5. The proof is similar to the one on 4.2.1.

Intuitively, complementary between the two types of energy, $\eta < 0$, implies that there is no possibility of a complete transition because the two types of energies are always needed. Substitutability, on the other hand, implies that it is possible to replace dirty energy sources with clean sources and, for this reason, a complete transition is possible. As the transition takes place and Ω converges to zero, the productivity ratio becomes the one defined on equation 2.6.3. Also, from equations 2.2.1, 2.4.5 and 2.6.3, the wage ratio between sectors is defined on equation 2.6.4

$$\frac{A_{c,t}}{A_{d,t}} = \left(\frac{N_{c,t}}{N_{d,t}}\right)^{\frac{\phi_2 \eta}{(1-\phi_1)(1-\eta)}} \left(\frac{L_{c,t}}{L_{d,t}}\right)^{\frac{\eta(1-\phi)}{(1-\phi_1)(1-\eta)}} \left(\frac{1}{1-\tau}\right)^{\frac{\eta}{(1-\phi_1)(1-\eta)}} \qquad 2.6.3$$

$$\frac{w_{h,c,t}}{w_{h,d,t}} = \left(\frac{N_{c,t}}{N_{d,t}}\right)^{\frac{\phi_2 \eta}{1-\eta}} \left(\frac{L_{c,t}}{L_{d,t}}\right)^{\frac{\eta(1-\phi)}{1-\eta}-\rho} \left(\frac{L_{h,d,t}}{L_{h,c,t}}\right)^{1-\rho} \left(\frac{1}{1-\tau}\right)^{\frac{2+\eta}{1-\eta}} \qquad 2.6.4$$

Considering labor demand defines in equation 2.1.5, it can be seen that relative wages are increasing in skilled and low-skilled labor supply from the clean sector if $\eta > \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$. The marginal effect on wages of increasing labor supply of each type is

$$\begin{array}{cccc}
\frac{\partial \frac{w_{s,c,t}}{w_{s,d,t}}}{\partial L_{s,c,t}} > 0, & \frac{\partial \frac{w_{s,c,t}}{w_{s,d,t}}}{\partial L_{u,c,t}} > 0, & \frac{\partial \frac{w_{s,c,t}}{w_{s,d,t}}}{\partial L_{s,d,t}} < 0, & \frac{\partial \frac{w_{s,c,t}}{w_{s,d,t}}}{\partial L_{u,d,t}} < 0 \\
\frac{\partial \frac{w_{u,c,t}}{w_{u,d,t}}}{\partial L_{s,c,t}} > 0, & \frac{\partial \frac{w_{u,c,t}}{w_{u,d,t}}}{\partial L_{u,c,t}} > 0, & \frac{\partial \frac{w_{u,c,t}}{w_{u,d,t}}}{\partial L_{s,d,t}} < 0, & \frac{\partial \frac{w_{u,c,t}}{w_{u,d,t}}}{\partial L_{u,d,t}} < 0
\end{array}$$

As labor supply for this sector increases relative to supply for the dirty sector, there are incentives to produce technology to be used by these workers. While using it, workers become more productive and receive a higher wage in return. This, in turn, generates incentives for the entry of new workers to the clean sector, making it grow even further. This effect is reinforced by the tax on dirty energy production, as marginal productivity in that sector is lower, and so are wages. If $\eta < \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$, there is a neoclassic effect of decreasing marginal productivity of labor on wages. In this case, an increase in labor supply in one sector reduces wages until they equal those of the other sector. Consequently, there is not a sector that attracts more labor and incentivizes innovation over the other. As the wage ratio within each sector does not depend on productivity, there is no skilled-biased technical change effect. However, for any $\rho \in (-\infty, 1)$, increases in low-skilled labor supply increase wages for skilled workers relative to low-skilled workers. In the long run, wages become equal, and the allocation of workers in either skilled or low-skilled labor will depend on their human capital investment[10].

$$\frac{w_{s,j,t}}{w_{u,j,t}} = \frac{B\theta}{1-\theta}\left(\frac{L_{u,j,t}}{L_{s,j,t}}\right)^{1-\rho} \tag{2.6.5}$$

Proposition 2. *If energy types are substitutes and* $\eta > \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$*, policy can generate technical change biased towards the clean sector. In this case, there is a wage premium between sectors but there is no premium between skilled and low-skilled labor within each sector.*

Proof in annex 4.2.3

2.7 Households

Recall that households choose present and future consumption and savings to maximize their utility. From the optimal conditions for both households

$$\frac{c^{o,f}_{t+1}}{c^{y,f}_{t}} = \beta r_{t+1} \tag{2.7.1}$$

$$c^{y,f}_{t} = \left(w^{f}_{j,t}(1-\lambda) + \lambda\frac{w^{f}_{j,t+1}}{r_{t+1}} - \kappa\psi^{f}_{j,t} - \psi^{f}_{j,t+1}\right)\frac{1}{1+\beta} \tag{2.7.2}$$

[10]Note that in both inter-sector and intra-sector premiums, there is the possibility of wages being equal. This is only possible in the long run once mobility costs disappear and wages for skilled and low-skilled labor are high enough to cover any human capital cost.

$$s_t = \left(w^f_{j,t}(1-\lambda) - \kappa\psi^f_{j,t} - \psi^f_{j,t+1} \right) \frac{\beta}{1+\beta} - \lambda \frac{w^f_{j,t+1}}{r_{t+1}} \frac{1}{1+\beta} \tag{2.7.3}$$

$$c^{o,f}_{t+1} = \left(w^f_{j,t}(1-\lambda) + \lambda \frac{w^f_{j,t+1}}{r_{t+1}} - \kappa\psi^f_{j,t} - \psi^f_{jt+1} \right) \frac{\beta r_{t+1}}{1+\beta} \tag{2.7.4}$$

Equation 2.7.1 presents the Euler equation, where intertemporal consumption decisions depend on the discount and interest rates perceived by households. Since both households have the same savings decision, this equation is the same for both. Equations 2.7.2, 2.7.3 and 2.7.4 show that present consumption, savings and future consumption are fixed proportions of households lifetime income. Though human capital investment reduces consumption and savings, it increases income in the next period. Since parents receive the return of this investment in the form of a transfer λ, they maximize their lifetime utility.

2.7.1 Human capital investment without a transition

In the absence of an energy transition, $\kappa = 0$ and parents' decision to invest in human capital only depends on their income in t. In this setting, the decision is made while awaiting the return of the investment in the form of the transfer λ. Accordingly, their decision is based on the intertemporal trade-off between investing and receiving that return.

$$\frac{\lambda}{r_{t+1}}(w^s_{j,t+1} - w^u_{j,t+1}) = \psi^s_{j,t+1} - \psi^u_{j,t+1} \tag{2.7.5}$$

If equation 2.7.5 holds, then households are indifferent between investing in skilled or low-skilled human capital. This happens if the transfer they will receive in $t+1$ is higher than the cost of investing. If future income is higher than investment, parents will prefer to invest in skilled human capital. If investing costs are higher than future income, they will prefer to invest in low-skilled human capital. A similar analysis can be made with parents deciding the sector they want their offspring to be in.

On one side, following the household restriction in t, skilled parents always have a high enough income to invest in skilled human capital and $w^s_t > \psi^s_{t+1}$. If skilled parents were to invest in low-skilled human capital, skilled labor supply would decrease, and skilled wages would be higher, which would give them the incentives to invest in skilled human capital.

Then, as wages in $t+1$ are also from skilled labor, the lifetime income when investing in skilled human capital is higher, and so is the household utility. Meanwhile, low-skilled parents' income must be at least equal to the skilled human capital cost for them to invest in it. If it is lower, they invest in low-skilled human capital. Also, low-skilled parents have no incentive to reduce their consumption and get more available income for skilled human capital because of decreasing marginal utility of consumption. Table 2.7.1 summarizes these conditions.

Table 2.7.1: Human capital decision in the absence of a transition

	Skilled_t	Low skilled_t
Skilled_{t+1}	Always invests	$\psi^s_{t+1} < w^u_t$
Low skilled_{t+1}	Never invests	$\psi^f_{t+1} \geq w^u_t$

Lemma 7. *Income and utility for households with initial income w^s_t is higher than for households with initial income w^u_t in t. If $\psi^f_{t+1} < w^u_t$, there is social mobility in the next period, but if $\psi^s_{t+1} \geq w^u_t$, inequality persists.*

Proof in appendix 4.2.4

2.7.2 Human capital investment with a transition

When the transition begins, households bear the extra cost, which affects their available income. There are three possible outcomes[11]:

i. Inequality persists if $w^s_{j,t} > \psi^s_{j,t+1} + \kappa\psi^s_{j,t}$ and $w^u_{j,t} > \psi^s_{j,t+1} + \kappa\psi^u_{j,t}$.[12] In this case, households face higher costs but their decision is as the one presented in table 1, because of the mobility cost during only one period.

ii. Skilled households are worse off if $\kappa\psi^s_{j,t} > w^s_{j,t}$. In this case, they pay $\kappa\psi^u_{j,t}$ and start receiving a wage $w^u_{j,t}$. This puts them in the same position as initially low-skilled workers that decide between skilled or low-skilled human capital for their offspring according to their wage. If $\kappa\psi^u_{j,t} > w^s_{j,t}$, skilled workers don't adapt to the transition.

[11]The detail on these outcomes is summarized in Annex 4.4

[12]Note that if the wage is high enough to invest in skilled human capital, it is also enough for low-skilled human capital.

iii. Low-skilled workers are worse off if $\kappa\psi^u_{j,t} > w^u_{j,t}$. Since they do not have the alternative to transit with even lower human capital, they do not adapt to the transition.

It is worth noting that the previous results are a consequence of a complete transition but would apply as well with a partial transition. Also, this only affects those households whose parents decided to invest in human capital for the dirty sector, and it is a consequence of the assumption of credit constraints. An important aspect is that a period for households lasts an entire generation. This means that even when the transition lasts multiple years, workers will have to pay the mobility cost either way. As long as parents continue to invest in human capital for the dirty sector, workers will keep facing the transition. While the two sectors coexist, some workers stay in the dirty sector, and the impact of the transition is limited to them receiving a lower wage. However, as there is a complete transition where the dirty sector disappears, workers that cannot adapt to it are left unemployed. Likewise, the cost is even higher as wages from the dirty sector are lower due to the transition policy. A second result is that skilled human capital is lower in $t+1$ due to the extra cost born by households. Therefore, low-skilled labor supply increases, and, following equation 2.6.5, so do relative wages between skilled and low-skilled workers inside the clean sector in the short run. Thus, even when initially skilled households are worse off, the income gap increases in the short run. This is also reinforced because workers initially allocated in the clean sector do not bear this cost. Hence, there are two channels for a raise in inequality: i) there are workers initially allocated in the clean sector who benefit from the wage premium and do not bear any mobility cost. And ii) an increase in low-skilled labor supply in the clean sector reduces relative wages.

Proposition 3. *The income gap increases due to i) a wage premium between sectors that benefit workers in the clean sector and ii) the mobility cost causing a raise in low-skilled labor supply, which reduces relative wages for low-skilled workers.*

Proof follows from lemma 2.

2.8 Government

Since the mobility cost born by households amplifies the income gap in the short run, it is desirable for the Government to implement a policy to compensate them for the effect of the transition. For this, assuming that the Government has perfect information on the effect on households, it gives a transfer to those that do not have enough income to cover the cost of the transition. Therefore, all households can pay the mobility cost, and their decision is once again reduced to choosing the level of human capital investment for their offspring. By putting together the transition policy and the compensation policy, the Government's restriction has fiscal balance and is defined by

$$\tau = \omega^c + \omega^h \tag{2.8.1}$$

Even when these policies are not optimal, they improve social welfare by i) making the transition possible and thus, avoiding a climate catastrophe and ii) compensating households. In terms of compensation, the Government can go beyond and choose a transfer for all low-income households, regardless of their ability to cover the mobility cost. In this case, there would be an increase in skilled labor supply in $t+1$, but it would carry a higher fiscal cost. Consequently, more households would likely be affected by the transition since wages from the dirty sector would be even lower, and more households would perceive that their available income would not be enough to pay for the mobility cost.

2.9 Balanced growth path after an energy transition

As shown in proposition 6, the only equilibrium where there is not an environmental catastrophe is if the dirty sector disappears. Also, this is the only equilibrium where there is a balanced growth path defined as a set of prices $\{p_{y,t}, p_{j,t}, p_{N,j,t}, p_{i,j,t}, r_t, w_{h,j,t}\}_{t=0}^{\infty}$ and quantities $\{Y_t, E_{j,t}, e_{i,j,t}, \Pi_{i,j,t} A_{i,j,t}, R_{i,j,t}, L_{h,j,t}, N_{j,t}, s_t, c_t^f, c_{t+1}^f, N_{j,t}\}_{t=0}^{\infty}$ where

1. Theres a constant growth rate $g = \frac{\Delta A_t}{A_t} = \frac{\Delta E_t}{E_t} = \frac{\Delta N_t}{N_t} = \frac{\Delta R_t}{R_t} = \frac{\Delta s_t}{s_t} = \frac{\Delta c_t}{c_t} = \frac{\Delta w_{j,t}}{w_{j,t}}$

2. Markets clear: The consumption, energy, labor, natural resources, patents and savings markets clear.

3. All agents optimize: Final goods, energy, extractive, technology and ideas firms, households and the Government.

From equation 2.1.10, the productivity growth rate is defined as

$$\frac{\Delta A_t}{A_t} = \mu_c(\gamma - 1) = 2\left[\left(\frac{c(1-\phi_1)}{\phi_1}\right)\left(\frac{\phi_1^2 N_{c,t}^{\phi_2} L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\frac{1}{r}\right](\gamma - 1)$$

Since energy production, wages, and final goods production end up depending on A, their growth rates are defined by

$$\frac{\Delta Y_t}{Y_t} = \frac{\Delta E_t}{E_t} = \frac{\Delta N_{c,t}}{N_{c,t}} = \frac{\Delta w_{s,t}}{w_{s,t}} = \frac{\Delta w_{u,t}}{w_{u,t}} = (1-\phi_1)\frac{\Delta A_t}{A_t}$$

It is worth noting that the wage level is not equal because of the differences in productivity and human capital investment. However, growth rates for both wages are equal. Likewise, following the savings market clearing condition, it is also true that resources from the ideas sector, savings, wages, and consumption grow at the same rate.

$$\frac{\Delta R_t}{R_t} = \frac{\Delta s_t}{s_t} = \frac{\Delta c_t}{c_t} = \frac{\Delta w_{s,t}}{w_{s,t}} = \frac{\Delta w_{u,t}}{w_{u,t}}$$

Also, there is a long-run interest rate consistent with this growth rate where the resources to productivity relation is constant and there is not population growth.

$$r = \frac{R}{A}^{-\frac{1}{2}}\left[\left(\frac{c(1-\phi_1)}{\phi_1}\right)\left(\frac{\phi_1^2 N_c L_c}{c}\right)\right]^{\frac{1}{1-\phi_1}} \qquad 2.9.1$$

The balanced growth rate of the economy[13] is then defined as

$$\frac{\Delta A_t}{A_t} = \mu_c(\gamma - 1) = 2\left[\left(\frac{c(1-\phi_1)}{\phi_1}\right)\left(\frac{\phi_1^2 N_{c,t}^{\phi_2} L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\frac{1}{r}\right](\gamma - 1)(1+\phi_1)$$

The growth rate is constant and has a scale effect from $L_{c,t}$ related to the size of the workers that use technology to produce clean energy. There is a similar effect from $N_{c,t}$ where the use of natural resources is complemented by technology and generates growth. However, this effect ceases in the long run as energy production stops needing natural resources, following equation 2.1.4. Even when growth would not be as high, natural

[13] See annex 2

resources would not end, and the problem of the tragedy of the commons would not be seen.

2.10 Equilibrium in the labor market

Equation 2.1.7 shows the labor market clearing condition. Households determine labor supply according to their capital investment decisions, which depend on their available income. From equations 2.6.5 and 2.7.5

$$\frac{L_{u,t}}{L_{s,t}} = \left[\frac{1-\theta}{B\theta}\left(1+\frac{r_{t+1}}{w_{u,t+1}}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}\right)\right]^{\frac{1}{1-\rho}} \quad 2.10.1$$

In the short run, relative labor supply for low-skilled workers depends on the decision of human capital investment[14]. Equation 2.10.1 shows relative labor supply if households are indifferent between investing in skilled or low-skilled human capital. There are three possible cases in the short run:

i. If $w^s_{t+1} - w^u_{t+1} = r_{t+1}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}$, $\frac{L_{u,t}}{L_{s,t}} = \left[\frac{1-\theta}{B\theta}\left(1+\frac{r_{t+1}}{w^u_{t+1}}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}\right)\right]^{\frac{1}{1-\rho}}$ and lifetime income for both households is the same.

ii. If $w^s_{t+1} - w^u_{t+1} > r_{t+1}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}$, $\frac{L_{u,t}}{L_{s,t}} > \left[\frac{1-\theta}{B\theta}\left(1+\frac{r_{t+1}}{w^u_{t+1}}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}\right)\right]^{\frac{1}{1-\rho}}$ and lifetime income for skilled households is higher. This is a consequence of low-skilled households being unable to afford investments in skilled human capital.

iii. If $w^s_{t+1} - w^u_{t+1} < r_{t+1}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}$. This case is not possible because the decision to not invest in skilled human capital lowers skilled labor supply, which increases low-skilled labor supply. In this process, wages for low-skilled labor increase until the condition stated in i. holds again.

Even when the transition has a short-run effect, the labor market gets back to its equilibrium in the long run. This means that the distributive effect is eventually dissipated, and the economy returns to its growth path. This is because, as wages grow over time, all households can eventually afford skilled human capital, given that it has a constant cost. In this scenario, the decision to invest is as in case i., where households are indifferent.

[14] See the procedure on annex 4.7

Moreover, in the long run a constant interest rate with growing wages will lead to $w_{s,t} = w_{u,t}$[15]. This implies two important effects: i) a transition's cost on workers and the economy is not permanent. Even more, the strengthening of the clean sector has the potential for job creation that benefits households, the environment, and the economy as a whole. ii) Regardless of the transition's cost, economic growth is guaranteed in the long run. A trade-off between growth, environment preservation, and welfare is often seen, but a good transition policy can guarantee all.

3 Conclusions and further remarks

An energy transition is one of the most important topics on the recovery agenda. However, eagerness to achieve it often leads to disregarding its costs, particularly the economic cost of making clean energy attractive and the distributive cost on workers. I build a directed technical change model to show how an energy transition between the dirty and clean sectors is possible. The first prediction is that an environmental catastrophe is inevitable unless there is a state-supported energy transition. On the demand side, I build an overlapping generations model where households invest in human capital accumulation and face a mobility cost of adapting to the skills demand of the new sector as the transition takes place. The second prediction is that, as this process happens, there is a distributive effect on workers rooted in the mobility cost, which amplifies existing income disparities. As a result, a transition policy should not only aim to achieve the needed technical change, but it should be comprehensive to compensate households for their welfare loss.

As costly as a transition can be, avoiding an imminent environmental catastrophe is also necessary. However, this process cannot be a *whatever it takes* policy that puts its costs over the most vulnerable. On the contrary, a just transition is the best way to achieve decarbonization, as households are not only being compensated for an extra burden but receive the support to pass to the clean sector and contribute to raising the possibilities of reaching a faster transition. Moreover, successful policies need support, and by considering the potential losers, the likelihood of achieving them is higher.

[15]See proof on annex 4.7

This paper explores how technical change is involved in an energy transition process and the cost it has for workers. The result is a theoretical prediction where a transition policy should compensate households for the distributive effects of leaving the fossil fuels sector behind. The presented mechanism considers the cost of adaptation to the skills demands of the new sector and how the cost is relatively different depending on initial income. The first step in further research is to examine further the general equilibrium implications of the transition on the supply side and the mobility cost on the side of the households. This analysis should give more insight into how extra costs for households affect human capital accumulation and how that might affect labor supply and, consequently, the pace of directed technical change. A limitation of this paper is that households decide only on human capital, so their allocation in the dirty or clean sector is perceived as exogenous for firms and depends on each sector's labor remuneration. Similarly, this paper only considers wage differences from technical change but not from skills between sectors. Another step in further research should be to explore whether the clean sector is skill-biased and how that can intensify the distributive effects of the transition.

Similarly, the theoretical predictions are testable and allow for empirical analysis. Further research should explore the distributive effects of a transition to understand their magnitude and propose policy alternatives accordingly. Acknowledging the difficulty of a regression analysis due to limited data availability, a good approach is to simulate the predictions at a micro-level. This can be done by extending the model to incorporate multiple sectors and characterize their exposure to the transition depending on their linkages to the fossil fuels sector. Following the theoretical prediction, workers from a more linked sector would be more affected by the transition and would need to reallocate to another sector. This reallocation will depend on workers' characteristics and whether they are similar to workers in other sectors. Very different workers will bear the cost of transforming their characteristics (i.e., their abilities or productivity) to fit in a new sector. Then, this cost can be estimated and used to calculate income loss for households. At the aggregate level, changes in the income gap can be tested.

On the other hand, political opposition is no stranger to the transition policy. Even when support for maintaining the status quo on energy policy comes from the fossil fuel industry and climate change deniers, the opposition has also risen by groups that perceive themselves as potentially harmed. Other groups have found their place in a more acknowledging position by supporting the transition but demanding just conditions for those potentially affected. This shows how distributive effects of a transition can also be studied under the scope of a political economy problem. The theoretical modeling approach involves understanding the transition in a social welfare setting and finding a policy that i) maximizes the likelihood of achieving the transition in a cost-effective way and ii) minimizes the distributive impact on households. From a policy recommendation approach, this shows the need to build consensus and consider the position of those potentially affected to avoid opposition to a much necessary policy.

Finally, theoretical models leave aside many important aspects needed to shape better policies. An aspect that can be considered is the timing and intensity of the policy. The debate on climate change has paid attention to the dangers of delaying climate policy, putting pressure on a fast and effective policy. Nevertheless, as a more intense policy might have the desired environmental effects, it can also amplify the social and distributive impacts and opposition from different sectors. Also, attention must be put on the reliance of production and public finances on the fossil fuels sector. A solid policy to discourage the sector might produce adverse economic effects that can also difficult the transition process. In short, even if there is a call for putting the environment over economic priorities, both need to be considered a matter of social and economic sustainability that can increase environmental sustainability.

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4 Appendix

4.1 List of variables and parameters

Subindexes

j	Dirty (d) or clean (c) sector
h	Skilled (s) or low-skilled (u) labor
f	Skilled (s) or low-skilled (u) human capital

Variables

S_t	Level of environmental quality
$N_{j,t}$	Flow of extracted natural resources on sector j
$NR_{j,t}$	Stock of natural resources used on sector j
$\Pi_{y,t}$	Benefits of the final goods producer
Y_t	Final good
$E_{j,t}$	Energy from sector j
$p_{j,t}$	Energy price from sector j
$\Pi_{E,j,t}$	Benefits of the energy producer on sector j
$L_{h,j,t}$	Type h labor on sector j
$p_{N,j,t}$	Price of natural resources for sector j
$p_{i,j,t}$	Price of technology for sector j
$w_{h,j,t}$	Type h worker wage on sector j
$\Pi_{N,j,t}$	Benefits for extractive firms in sector j
$\Pi_{e,j,t}$	Benefits of the technology producer on sector j
$\Pi_{A,j,t}$	Benefits for R&D firms in sector j
$e_{i,j,t}$	Technology for energy production on sector j
$A_{i,j,t}$	Productivity of technology i on sector j
$R_{j,t}$	Resources destined to R&D on sector j
$A^*_{j,t}$	Productivity of technology on sector j if research succeeds
r_t	Interest rate
$c_t^{y,f}$	Consumption of the younger generation on type h household
$c_{t+1}^{o,f}$	Consumption of the older generation on type h household
s_t^f	Savings rate

w_t^f	Labor income on type f household
Parameter	
ξ	Environmental regeneration rate
δ_d	Environmental impact of dirty energy production
ζ_j	Extraction rate of natural resources on sector j
η	Substitution parameter on energy demand
θ	Proportion of skilled workers in energy production
B	Skilled labor associated productivity
ϕ_1	Technology share on energy production
ϕ_2	Natural resources share on energy production
$1-\phi$	Labor share on energy production
ρ	Substitution parameter on labor demand
c	Marginal cost of technology production on sector j
F	Fixed cost on clean technology production
μ_j	Success probability in the production of ideas for sector j
γ	Rise in productivity when innovation is succesful
τ	Tax on dirty technology production
ω^c	Transfer of clean technology production
Ω	Transition cost parameter
β	Discount factor
ψ_t^f	Human capital cost for type f household
λ	Transfer from younger generation to older generation
κ	Mobility cost
ω^h	Transfer to lower income households
g	Balanced growth rate

4.2 Proofs

4.2.1 Proof of Lemma 3

Lemma 3. If $\eta \in (-\infty, 1)$ the clean sector coexists with the dirty sector, but clean energy demand is lower due to higher prices. If $\eta = 1$ the clean sector disappears.

Proof: Recall that final goods production is defined as a CES technology that combines clean and dirty energy.

$$Y_t = (E_{c,t}^{\eta} + E_{d,t}^{\eta})^{\frac{1}{\eta}}$$

Marginal productivities for clean and dirty energy demand are, respectively:

$$\frac{\partial Y_t}{\partial E_{c,t}} = (E_{c,t}^{\eta} + E_{d,t}^{\eta})^{\frac{1}{\eta}-1} E_{c,t}^{\eta-1} = p_{c,t}$$
$$\frac{\partial Y_t}{\partial E_{d,t}} = (E_{c,t}^{\eta} + E_{d,t}^{\eta})^{\frac{1}{\eta}-1} E_{d,t}^{\eta-1} = p_{d,t}$$

Claim 1. *If $\eta \in (-\infty, 1)$ the clean sector coexists with the dirty sector, but clean energy demand is lower due to higher prices.*

If $\eta \in (-\infty, 1)$ there is an interior solution for energy demand where marginal productivity is decreasing.

From equation 2.2.1,

$$\frac{p_{c,t}}{p_{d,t}} = \left(\frac{E_{c,t}}{E_{d,t}}\right)^{\eta-1} \qquad 4.2.1$$

For all finite positive values of $\frac{p_{c,t}}{p_{d,t}}$ there is a $\frac{E_{d,t}}{E_{c,t}} \geq 0$. To show this, assume that there is a finite m such that for all values of $\frac{p_{c,t}}{p_{d,t}}$, $\frac{E_{d,t}}{E_{c,t}} \leq m$. Now, choose a value q such that $\frac{p_{c,t}}{p_{d,t}} \leq q$. Then, $\frac{E_{d,t}}{E_{c,t}} \leq q^{1-\eta}$. Define $m = q^{1-\eta}$. Then both $\frac{p_{c,t}}{p_{d,t}} > 0$ and $\frac{E_{d,t}}{E_{c,t}} > 0$ and there is an interior solution.

This implies that the demand for each type of energy depends on its price, which is higher for clean energy. This is a consequence of equations 2.2.2 and 4.3.2. $p_{N,j,t}$ is decreasing in $N_{j,t}$. As $NR_{j,t} \to 0$, $p_{N,j,t} \to \infty$. From assumption 1, $p_{j,t}$ is higher, and from equation 2.2.1, relative energy demand changes. Lemma 1 implies that the natural resources stock is high enough that $p_{d,t}$ doesn't approach to 0, so $p_{d,t} < p_{c,t}$.

Since innovation in the clean sector is more costly due to the fixed cost of technology production, investment will be more profitable in the dirty sector. As a consequence, innovation will keep increasing and attracting more resources there. There is a virtuous cycle of innovation in this sector where prices lower and demand rises. Then, clean energy becomes less attractive, and the final good producer has fewer incentives to demand it each time. Since the final goods firm keeps demanding both types of energy, but clean energy is more expensive, its relative demand falls.

It is worth noting that demand for clean energy will be higher if $\eta < 0$ relative to $\eta \in (0, 1)$ due to the degree of complementarity between energy types.

Claim 2. *If $\eta = 1$ the clean sector disappears*

If $\eta = 1$, there is a corner solution because energy types are perfect substitutes. In this case,

$$Y_t = E_{c,t} + E_{d,t}$$

Then,

$$\frac{\partial Y_t}{\partial E_{d,t}} = \frac{\partial Y_t}{\partial E_{c,t}} = 1$$

From equation 2.2.1, if $\frac{p_{c,t}}{p_{d,t}} \neq 1$. then there is a corner solution where the firm will demand the less expensive energy type, which is the dirty one.

4.2.2 Proof of Lemma 4

Lemma 4. Even when there is a clean energy sector operating, the existence of a dirty energy sector leads to an environmental catastrophe.

Proof: Following the proof for Lemma 3, if $\eta \in (-\infty, 1)$, both types of energy are demanded. Also, higher costs in the clean sector imply no incentives to produce using only clean energy. Therefore, final goods will necessarily be produced using dirty energy, and so environmental quality will be reduced, as shown in equation 2.1.1. Following definition 1, there is a moment t' where the economy reaches an environmental catastrophe and

$\frac{\Delta S_t}{S_t} \leq 0$. By contradiction, assume that

$$\frac{S_{t+1}}{S_t} > 1$$
$$(1+\xi) > \delta_d E_{d,t}(N_{d,t})$$
$$\frac{\delta_d}{1+\xi} > E_{d,t}(N_{d,t})$$

Under this, it must hold that

$$0 > \frac{\Delta E_{d,t}(N_{d,t})}{E_{d,t}(N_{d,t})}$$
$$0 > \phi_2 \frac{\Delta N_{d,t}}{N_{d,t}} + (1-\phi_1)\frac{\Delta A_{d,t}}{A_{d,t}}$$

However, the energy growth rate is positive, as shown in section 2.9. This implies that eventually

$$0 < \frac{\Delta E_{d,t}(N_{d,t})}{E_{d,t}(N_{d,t})}$$

As energy production grows with time, eventually, the economy reaches the moment t' where $S_t = S_{t'} = 0$. According to definition 1, at this moment, the economy has reached an environmental catastrophe. This will happen even when a clean sector is operating because of increasing relative demand.

4.2.3 Proof of Proposition 2

Proposition 2. If energy types are substitutes and $\eta > \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$, policy can generate technical change biased towards the clean sector. In this case, there is a wage premium between sectors but there is no premium between skilled and low-skilled labor within each sector.

Proof is a consequence of Lemma 3. Equation 2.6.4 can be rewritten as

$$\frac{w_{h,c,t}}{w_{h,d,t}} = \left(\frac{N_{c,t}}{N_{d,t}}\right)^{\frac{\phi_2\eta}{1-\eta}} \left(\frac{B\theta L^{\rho}_{s,c,t} + (1-\theta)L^{\rho}_{u,c,t}}{B\theta L^{\rho}_{s,d,t} + (1-\theta)L^{\rho}_{u,d,t}}\right)^{\frac{\eta(1-\phi)}{\rho(1-\eta)}-1} \left(\frac{L_{h,d,t}}{L_{h,c,t}}\right)^{1-\rho} \left(\frac{1}{1-\tau}\right)^{\frac{2+\eta}{1-\eta}}$$

For a sector-biased technical change process to happen, relative wages should be increasing in relative labor supply from the clean sector. This is, if

$$\frac{\partial \frac{w_{h,c,t}}{w_{h,d,t}}}{\partial \frac{L_{h,c,t}}{L_{h,d,t}}} > 0$$

Using natural logarithms,

$$\frac{\partial \ln\left(\frac{w_{h,c,t}}{w_{h,d,t}}\right)}{\partial \ln\left(\frac{L_{h,c,t}}{L_{h,d,t}}\right)} = \left(\frac{\eta(1-\phi)}{\rho(1-\eta)} - 1\right) \ln\left(\frac{B\theta L_{s,c,t}^{\rho} + (1-\theta)L_{u,c,t}^{\rho}}{B\theta L_{s,d,t}^{\rho} + (1-\theta)L_{u,d,t}^{\rho}}\right) + (1-\rho)\ln\left(\frac{L_{h,d,t}}{L_{h,c,t}}\right) > 0$$

The derivative is possitive if

$$\frac{\eta(1-\phi)}{\rho(1-\eta)} - 1 > 1 - \rho$$

Solving for η, the condition holds as $\eta > \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$. Under this, relative wages are increasing in relative labor supply, and there is a directed technical change effect that creates an inter-sector wage premium. For $\eta < \frac{2\rho-\rho^2}{1-\phi+2\rho-\rho^2}$, a neoclassical effect dominates. Suppose that relative supply in the clean sector rises. Decreasing marginal productivity lowers skilled wages, reducing incentives to invest in this sector. This causes labor mobility until wages rise back or in the dirty sector are reduced. The same mechanism is seen inside each sector for skilled and low-skilled workers, for all values of ρ.

4.2.4 Proof of Lemma 7

Lemma 7. Income and utility for households with initial income w_t^s is higher than for households with initial income w_t^u in t. If $\psi_{t+1}^f < w_t^u$, there is social mobility in the next period, but if $\psi_{t+1}^s \geq w_t^u$, inequality persists.

Claim 3. *Income and utility for households with initial income w_t^s are higher than those with initial income w_t^u in t.*

Assuming $w_t^s > w_t^u$ and that utility is an increasing function of consumption, $U^s > U^u$ if for any $f = \{s, u\}$

$$w_t^s(1-\lambda)+\lambda\frac{w_{t+1}^f}{r_{t+1}}-\kappa\psi_{j.t}^f-\psi_{t+1}^f > w_t^u(1-\lambda)+\lambda\frac{w_{t+1}^f}{r_{t+1}}-\kappa\psi_{j,t}^f-\psi_{t+1}^f$$

Suppose that $\kappa = 0$ and both households invest in ψ_{t+1}^s, then

$$w_t^s > w_t^u$$

Now suppose that $\kappa = 0$ and low-skilled parents invest in ψ_{t+1}^u, then

$$w_t^s(1-\lambda)+\lambda\frac{w_{t+1}^s}{r_{t+1}}-\psi_{t+1}^s > w_t^u(1-\lambda)+\lambda\frac{w_{t+1}^u}{r_{t+1}}-\psi_{t+1}^u$$

From equation 2.6.5, $w_t^s > w_t^u \ \forall t$. Therefore, lifetime income is also higher.

Claim 4. *If $\psi_{t+1}^f < w_t^u$, there is social mobility in the next period, but if $\psi_{t+1}^f \geq w_t^u$, inequality persists.*

- If $\psi_{t+1}^f < w_t^u$, low-skilled households have enough income to invest in skilled human capital for their offspring. Then, in $t+1$, $w_{t+1}^f = w_{t+1}^s$ and that generation will once again be able to invest in skilled human capital.
- If $\psi_{t+1}^f \geq w_t^u$, income for low-skilled households is not enough to invest in skilled human capital, so they invest in low-skilled human capital instead. In $t+1$, the next generation decides over human capital investment under the same criteria.

4.2.5 Proof of $\Omega = 0$

$\Omega = 0$ in the long run as a result of productivity growth in the clean sector.

From equation 2.6.2 recall that

$$\Omega = \left[\frac{\phi_1(F-\omega^c)}{c(1-\phi_1)A_{c,t}}\left(\frac{c}{p_{d,t}N_{d,t}^{\phi_2}\phi_1^2L_{d,t}^{1-\phi}}\right)^{\frac{1}{1-\phi_1}}\right]^{\frac{1}{1-\eta}}\frac{A_{c,t}}{A_{d,t}} \qquad 4.2.2$$

Using the L'Hoppital rule for $A_{c,t}$

$$\lim_{t\to\infty}\Omega = \lim_{t\to\infty}\frac{1}{\frac{1}{1-\eta}A_{c,t}^{\frac{\eta}{1-\eta}}} = 0$$

4.3 The case of optimal natural resources extraction

Natural resources can be extracted optimally by firms that operate in competition. Since natural resources are exhaustible, firms maximize the present value of extraction at a rate r_t. Marginal costs are increasing since exploration is more costly as resources are extracted. The problem is defined as

$$\Pi_{N,j,t} = \sum_{t=0}^{N} \left(p_{N,j,t} N_{j,t} - \frac{N_{j,t}^2}{2NR_{j,t}} \right) \frac{1}{(1+r_t)^t} \tag{4.3.1}$$

From optimal conditions, the price of natural resources is

$$p_{N,j,t} = \frac{N_{j,t}}{NR_{j,t}} \tag{4.3.2}$$

Under this setting, prices are decreasing in the natural resources stock. This is a result of scarcity that follows from equations 2.1.2 and 2.1.3. As resources are depleted, exploration is more costly, and the price that energy producers pay for them is higher. This result is consistent with the Hotelling rule where $\frac{\Pi_{N,j,t+1} - \Pi_{N,j,t}}{\Pi_{N,j,t}} = r_t$. A higher price means that firms have an incentive to reduce extraction over time without reaching a level of zero extraction. Following assumption 2 and lemma 1, positive extraction implies that the environmental catastrophe arrives before resources are depleted.

4.4 Human capital investment after the mobility cost.

Once $\kappa > 0$, human capital investment choices depend on available income after this cost. The conditions are summarised in

	Skilled$_t$	Low skilled$_t$
$\psi^s_{j,t+1} + \kappa\psi^s_{j,t}$	$w^s_{j,t} > \psi^s_{j,t+1} + \kappa\psi^s_{j,t}$	NA*
$\psi^s_{j,t+1} + \kappa\psi^u_{j,t}$	$\psi^s_{j,t+1} > w^s_{j,t} - \kappa\psi^s_{j,t}$	$w^u_t > \psi^s_{j,t+1} + \psi^u_{j,t}$
$\psi^u_{j,t+1} + \kappa\psi^s_{j,t}$	$\kappa\psi^s_{j,t} > w^s_t > \psi^s_{j,t+1}$	NA*
$\psi^u_{j,t+1} + \kappa\psi^u_{j,t}$	$\psi^s_{j,t+1} + \kappa\psi^s_{j,t} > w^s_{j,t}$	$w^u_{j,t} > \psi^u_{j,t+1} + \kappa\psi^u_{j,t}$
ψ^s_{t+1}	NA**	NA**
ψ^u_{t+1}	NA**	NA**
$\kappa\psi^s_{j,t}$	$\psi^u_{j,t+1} > w^s_{j,t} > \kappa\psi^s_{j,t}$	NA*
$\kappa\psi^u_{j,t}$	$\psi^u_{j,t+1} + \kappa\psi^s_{j,t} > w^s_{j.t}$	$\psi^u_{j,t+1} > w^u_{j,t} > \kappa\psi^u_{j,t}$

NA*: Low-skilled workers cannot become skilled after the transition. As parents make human capital investments, low-skilled workers cannot become skilled.
NA**: Workers are left unemployed and lose their income. If workers do not adapt to the transition, they stay unemployed and lose all their income. Therefore, they cannot invest in human capital for their offspring.

4.5 Human capital investment in the absence of credit constraints

In the absence of credit constraints, households can obtain a secondary source of income through debt D_t.

$$\max_{\{c^{y,f}_t, c^{o,f}_{t+1}, s_t\}} U(c) = \log c^{y,f}_t + \beta \log c^{o,f}_{t+1}$$

$$\begin{aligned} s.t. \quad c^{y,f}_t + s_t + \psi^f_{j,t+1} + \kappa\psi^f_{j,t} &\leq w^f_{j,t}(1-\lambda) + D_t \\ c^{o,f}_{t+1} &\leq s_t r_{t+1} + \lambda w^f_{j,t+1} \\ D_t &\leq \varepsilon w^f_{j,t} \end{aligned}$$

D_t is the debt households can acquire through the financial system, and ε is the amount of the income they spend on it. Assuming that credit markets are perfect and $s_t = -D_t$, it is clear that $s_t \geq -\varepsilon w^f_{j,t}$. By allowing households to enter the credit market without a transition, they will always be able to invest in skilled human capital, regardless of their wage. Therefore, this decision will only depend on which wages are higher. When the transition happens, the qualitative results do not change from the ones presented in section 2.7.2. However, fewer workers will be affected by the transition, as debt is an available source of income to finance the mobility cost. The number of workers benefiting from this will depend on the amount of debt they can acquire.

4.6 Balanced growth rate

The balanced growth rate is defined as

$$\frac{\Delta A_t}{A_t} = \mu_c(\gamma - 1) = 2\left[\left(\frac{c(1-\phi_1)}{\phi_1}\right)\left(\frac{\phi_1^2 N_{c,t}^{\phi_2} L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}} \frac{1}{r}\right](\gamma - 1)$$

Assuming $\int_0^1 A_{i,d,t} = A_{d,t}$ and $e_{i,d,t} = 1$. Since there is only a clean sector, production only depends on it and clean energy production becomes the numeraire. Energy production is now defined as

$$E_{d,t} = N_{c,t}^{\phi_2} L_{c,t}^{1-\phi} A_{d,t}^{1-\phi_1}$$

The growth rate is

$$\frac{\Delta Y_t}{Y_t} = \frac{\Delta E_t}{E_t} = \phi_2 \frac{\Delta N_{c,t}}{N_{c,t}} + (1-\phi_1)\frac{\Delta A_t}{A_t}$$

Wages for skilled and low-skilled labor are

$$w_{s,c,t} = \frac{B\theta(1-\phi)p_{d,t}\zeta_d N_t E_{d,t} L_{s,d,t}^{\rho-1}}{L_{d,t}^{\rho}}$$

$$w_{u,c,t} = \frac{(1-\theta)(1-\phi)p_{d,t}\zeta_d N_t E_{d,t} L_{s,d,t}^{\rho-1}}{L_{d,t}^{\rho}}$$

$$\frac{\Delta w_{s,t}}{w_{s,t}} = \frac{\Delta w_{u,t}}{w_{u,t}} = \frac{\Delta E_t}{E_t}$$

For households, as skilled and low-skilled wages grow at the same rate, $\frac{w_{t+1}}{w_t} = g + 1$.

The savings market clearing condition is $R_t = s_t$

$$
\begin{aligned}
R_t = s_t &= (w_t(1-\lambda) - \psi_{t+1})\frac{\beta}{1+\beta} - \lambda\frac{w_{t+1}}{r_{t+1}}\frac{1}{1+\beta} \\
s_t &= (w_t(1-\lambda) - \psi_{t+1})\frac{\beta}{1+\beta} - \lambda\frac{w_t(1+g)}{r_{t+1}}\frac{1}{1+\beta} \\
s_t &= \frac{1}{1+\beta}\left(w_t\left(\frac{\beta r(1-\lambda) - \lambda(1+g)}{r}\right) - \beta\psi_{t+1}\right)
\end{aligned}
$$

Therefore, the growth rate of resources is

$$
\frac{\Delta R_t}{R_t} = \frac{\Delta s_t}{s_t} = \frac{\Delta w_{s,t}}{w_{s,t}} = \frac{\Delta w_{u,t}}{w_{u,t}}
$$

The growht rate of the economy is then

$$
\begin{aligned}
\frac{\Delta A_t}{A_t} = \frac{\Delta E_t}{E_t} = \frac{\Delta R_t}{R_t} = \frac{\Delta s_t}{s_t} = \frac{\Delta c_t}{c_t} = \frac{\Delta w_{s,t}}{w_{s,t}} = \frac{\Delta w_{u,t}}{w_{u,t}} \\
= 2\left[\left(\frac{c(1-\phi_1)}{\phi_1}\right)\left(\frac{\phi_1^2 N_{c,t}^{\phi_2} L_{c,t}^{1-\phi}}{c}\right)^{\frac{1}{1-\phi_1}}\frac{1}{r}\right](\gamma - 1)
\end{aligned}
$$

4.7 Equilibrium in the labor market

The within sector wage premium is defined as

$$
\frac{w_{j,t}^s}{w_{j,t}^u} = \frac{B\theta}{1-\theta}\left(\frac{L_{u,j,t}}{L_{s,j,t}}\right)^{1-\rho}
$$

The condition for indifference in human capital investment is defined as

$$
\frac{\lambda}{r_{t+1}}(w_{t+1}^s - w_{t+1}^u) = \psi_{t+1}^s - \psi_{t+1}^u
$$

Putting together these conditions, relative labor supply is defined

$$w^u_{t+1}\left[\frac{B\theta}{1-\theta}\left(\frac{L_{u,t+1}}{L_{s,t+1}}\right)^{1-\rho}-1\right]=r_{t+1}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}$$

$$\left[\frac{B\theta}{1-\theta}\left(\frac{L_{u,t+1}}{L_{s,t+1}}\right)^{1-\rho}\right]=1+\frac{r_{t+1}}{w_{u,t+1}}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}$$

$$\frac{L_{u,t}}{L_{s,t}}=\left[\frac{1-\theta}{B\theta}\left(1+\frac{r_{t+1}}{w^u_{t+1}}\frac{\psi^s_{t+1}-\psi^u_{t+1}}{\lambda}\right)\right]^{\frac{1}{1-\rho}}$$

In the long run $w^u_{t+1}\to 0$, then relative labor supply depends on constant parameters and by replacing it on the wage ratio, it equals 1.

$$\frac{L_{u,t}}{L_{s,t}}=\left[\frac{1-\theta}{B\theta}\right]^{\frac{1}{1-\rho}}$$

$$\frac{w_{s,t}}{w_{u,t}}=1$$