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A SHORT GUIDE TO LONG-RUN TIME-PHASED SYSTEMS OF PRODUCTION

Hernando Matallana, Bogotá, August 2008¹

Abstract

The paper discusses briefly several long-run systems of production *vis-à-vis* alternative theories of value and distribution stated by Marx, classical-neoricardian, post-Leontief-neoclassical, and post-Keynes-Sraffa authors during the last hundred and fifty years. All the systems of production considered in the paper have in common the circular production of commodities by means of commodities. A distinctive characteristic of these systems is the standard of value in terms of which the wage rate and the prices of commodities are expressed. In particular, the logical structure of the systems of prices of production implies that different standards of value imply different theories of value and distribution both real and monetary in character. Methodological pluralism suggests that distinct theories of value and distribution reflect different scientific/normative economic paradigms. It appears that dialectics is needed in order to transcend inter-paradigmatic debate.

Key Words: capital theory, distribution theory, economic paradigms, money wage, prices of production, standard of value, Standard system, systems of production

JEL Classification: B12, B14, B24, E12

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Resumen

El artículo discute de manera concisa varios sistemas de producción *vis-à-vis* teorías alternativas del valor y la distribución formulados por Marx, autores clásico-neoricardianos, neoclásicos post-Leontief y economistas post-Keynes-Sraffa en el último siglo y medio. Todos los sistemas tienen en común la producción circular de mercancías por medio de mercancías. Una característica distintiva de estos sistemas es el patrón de valor en términos del cual son expresados el salario y los precios de las mercancías. En particular, la estructura lógica de los sistemas de precios de producción implica que distintos patrones de valor implican diferentes teorías del valor y la distribución de carácter real y monetario. El pluralismo metodológico sugiere que distintas teorías del valor reflejan distintos paradigmas económicos científicos/normativos. Al parecer se requiere de la dialéctica a fin de superar el debate inter-paradigmático.

Palabras clave: teoría del capital, teoría de la distribución, paradigmas económicos, salario monetario, precios de producción, standard de valor, sistema patrón, sistemas de producción

Clasificación JEL: B12, B14, B24, E12

A SHORT GUIDE TO LONG-RUN TIME-PHASED SYSTEMS OF PRODUCTION

1 Introduction

The paper considers the logical structure of several long-run time-phased systems of production *vis-à-vis* alternative theories of value and distribution. All the systems of production under consideration have in common the circular production of commodities by means of commodities, with both the wage rate and the rate of profit/interest, or alternatively the value of labour power and rate of surplus value, explicitly considered. A distinct characteristic of the different systems of production discussed below is the quality of the standard of value in terms of which the wage rate and the prices/labour values of commodities are expressed. The discussion in the paper suggests that the logical structure of the systems of prices of production implies that different standards of value imply different theories of value and distribution, both real and monetary in character.

The notion of the circular production of commodities was well known to pre-classical economists such as Petty in the late 17th century, Cantillon and Quesnay in the 1750s, and to classical economists such as Smith in the late 18th-century, and Ricardo and Torrens in the early 19th-century. However, the formulation of the true time-phased system of prices of production reflecting the economic logic of the capitalist production was not possible as long as the inner relation between the distribution of income between capital and labour, and the prices of production of circularly produced commodities had not been precisely understood.

Marx developed the notion of circular production of commodities for a capitalist economy for the first time in the 1850s and 1860s. Marx's various systems of equations in volume II of *Capital*—the so-called 'reproduction schemes'—were formulated in terms of the labour theory of value. However, Marx did not decompose the aggregate values of the reproduction schemes into the unit values of the commodities and their physical quantities (Samuelson 1970).

After Marx's death, the production of commodities as a circular process of production was the subject matter almost exclusively of the German-Russian economists that engaged in the discussion on Marx's economic ideas in *Capital*. Von Bortkiewicz, Charasoff, Hilferding, Tugan-Baranovsky, and Luxemburg should be mentioned. In particular, Von Bortkiewicz and Charasoff contributed to the formal development of the theory of the prices of production of circularly produced commodities. Later in the 1920s and the 1930s, Remak, Sraffa (1928 unpublished), Von Neumann (1937) and Leontief (1928), all of which were well acquainted with the theoretical work of Marx and Von Bortkiewicz, formulated alternative systems of prices of production.

The publication of the english version of Von Neumann's existence proof of the competitive equilibrium of a classical (subsistence wage) economy in 1945 (Kurz/Salvadori 1995, chap. 13), and of Leontief's theoretical and empirical work on the inter-industry structure of the economy of the United States in the early 1940s revealed the limits of the Walras-Cassel non-circular (one-way factor-goods) linear theory of production upon which neoclassical general equilibrium theory had relied since its formulation by Walras in the 1870s. The incorporation of the notion of circular production into the neoclassical theory of general equilibrium and the development of linear programming in the 1940s and 1950s enriched and widened the theory of production amongst general equilibrium theorists (Dorfman et al. 1958).

However, it was not until Samuelson in 1957 and Sraffa in 1960 (1928 unpublished) published the first time-phased system of prices of production that the hitherto hidden logical relation between circular production, income distribution, prices of production, and the choice of technique could be fixed. Both Samuelson and Sraffa considered explicitly the wage rate *and* the rate of profit, with single production, wages paid *ante festum* and constant returns to scale in Samuelson's system, and both single and joint production, wages paid *post festum* and no specific assumption on returns in Sraffa's. After over two hundred years of theoretical debate, both Sraffa and Samuelson must be credited for having precisely stated the true system of prices of production reflecting the inner economic logic of the capitalistic production of commodities by means of commodities.

The logical structure of the Samuelson-Sraffa time-phased system of prices of production conveys all the capital-theoretical 'perverse effects' that had been discovered by Wicksell a century before and anew by Joan Robinson in the early 1950s. In particular, Sraffa's work pointed at the importance of the critical consideration of the 'reswitching of techniques' and the 'reversal of capital' for the theory of capital and distribution.²

In the 1960s, the so-called 'Cambridge debates in capital theories' revealed the conceptual limitations and the logical contradictions of: (i) the Clark-Wicksteed's notion of capital as a measurable quantity independently of distribution; (ii) the neoclassical marginal-productivity theory of distribution; (iii) the Austrian theory of roundaboutness production (Hayek's wrecked 'mythology of capital'); and (iv) Marx's classical labour theory of value. Yet, after the 'Cambridge debates in capital theories' neither the orthodox marxists, nor the Austrian authors, nor the Clarkian-neoclassical economists, nor the conventional (anti-heterodox) macroeconomists have been willing to take the logical criticism of the pre-Leontief theory of capital that entails the post-Sraffa-Samuelson consideration of the circular production of commodities. These particular issues of the 'Cambridge debates in capital theories' will not be discussed in the paper.³

The paper is organized as follows. Section 2 contains a general discussion on quantities and prices of production of the systems of circular production. This

²See Harcourt (1972).

³See Harcourt (1972).

section is technical in character. Section 3 considers the Keynes-Samuelson-Sraffa money wage price system. Although Keynes (1936) did not write a system of prices of production, he must be considered since he suggested the theory of a monetary production economy as an alternative economic paradigm other than the classical and neoclassical visions of capitalism. Section 4 discusses both Sraffa's and Hahn's unit-price-simplex price system. This section contains a digression on Sraffa's Standard system. Section 5 contains the Von Bortkiewicz-Von Neumann-Sraffa subsistence-wage system of prices of production. Section 6 considers Marx's system of labour values and section 7 the latter's unfulfilled twofold transformation of both the value of labour power into wage (i.e. the price of labour) and the labour values into prices of production. Section 8 contains a final summing up.

2 The general setting

2.1 The system of quantities⁴

The production of commodities requires both labour and means of production. Formally, a single-product method of production is represented by a production function:

$$x_j = f_j(L_j, x_{ij}), \quad i = 1, \dots, n \quad (1)$$

where x_j is the quantity of the j -th commodity measured in terms of its own technical units, x_{ij} the quantity of the i -th non-labour input measured in terms of their own technical units and L_j the quantity of labour measured in terms of labour time units (e.g. hours) required to produce x_j . For any 'given' level of production, the division of the production function above yields the average technical coefficients of production:

$$l_j = f_j(a_{0j}, a_{ij}), \quad i = 1, \dots, n \quad (2)$$

where $a_{0j} = L_j/x_j$ is the quantity of labour and $a_{ij} = x_{ij}/x_j$ is the quantity of the i th-input required to produce one unit of the j th-commodity. No specific assumption on returns to scale is made in this *general* functional relation.⁵

The consideration of circular production of commodities implies that the commodities are produced by means of produced commodities and labour. In particular, the means of production are produced by means of produced means of production and labour. The single production technique of circular production is described by a set of interdependent methods of production:

⁴See Dorfman et al. (1958), Pasinetti (1975)

⁵The technical coefficients of *circular* production were introduced by Dmitriev (1977, 1904). The notation used in the main text is borrowed from Dorfman et al. (1958).

$$\begin{aligned}
x_1 &= f_1(L_1, x_{11}, x_{21}, \dots, x_{n1}) \\
&\dots \\
x_n &= f_1(L_n, x_{1n}, x_{2n}, \dots, x_{nn})
\end{aligned} \tag{3}$$

Two assumptions are introduced in order to simplify the discussion:

(i) All produced commodities are Sraffa ‘basics’, i.e. all outputs are used directly or indirectly as inputs for the production of all outputs. This yields a square indecomposable matrix of technical coefficients.

(ii) The technique of production is productive in the sense that a positive net product is obtained after subtracting the inputs from the outputs. This yields a semi-positive square matrix of technical coefficients.

Together the two assumptions yield a square indecomposable semi-positive matrix of technical coefficients. In general, the methods of production described by the matrix will differ from the technical point of view, hence, the individual rows of the matrix will differ from each other.

The system of quantities is given next in matrix-vector notation:

$$x = ax + y \tag{4}$$

and

$$y > 0 \tag{5}$$

and

$$L = a_0x \tag{6}$$

where $a = (a_{ij})$ is the square matrix of technical non-labour input-output coefficients of production, $x = (x_1, \dots, x_n)$ the row vector of gross single-product output of homogeneous commodities, $y = (y_1, \dots, y_n)$ is the semi-positive row vector of the net quantities produced by the economy, and $a_0 = (a_{01}, \dots, a_{0n})$ is the row vector of technical labour-output coefficients of production. In particular, $L_j = a_{0j}x_j$, hence $L = a_0x$.

Both the technical coefficients of matrix a and vector a_0 are average values. In particular, no specific assumption on returns to scale is implied by the technique of production described by the technical matrix (a, a_0) whenever the quantities are assumed not to change.

The productivity assumption implies that matrix a fulfils the Simon-Hawkins conditions (Dorfman et al. 1958, chap. 9). Accordingly, both the inverse matrix a^{-1} and the inverse Leontief matrix $(I - a)^{-1}$ exist; in particular, $\lim_{n \rightarrow \infty} a^n = 0$. It follows that, for $y > 0$:

$$x = (I - a)^{-1}y > 0 \tag{7}$$

and

$$L = a_0 (I - a)^{-1} y > 0 \quad (8)$$

Both the total output x and the total amount of labour L required to produce x can be determined provided the technique of production and the quantity of net output y are known.

It is assumed throughout that the quantities of the system of production are ‘given’. The quotes mean that the quantity variables are not explained here. Put the other way around, it would require to write down the complete equilibrium conditions of the several systems of production corresponding to all alternative theories of value and distribution considered in the paper in order to fully determine in each case the price-quantity solution. This is beyond the intention of the paper, hence, the quantities of the inputs, outputs and labour are assumed to be given.

The ‘givenness’ of the quantities of the system correspond to a long-run stationary equilibrium, i.e. a self-replacement state economy in which (i) for every commodity the *ex ante* demand is equal to the *ex ante* supply and (ii) the net product is fully consumed by the households. In particular,

$$y = c \quad (9)$$

where c is the vector of end consumption. The stationary self-replacement state assumption may be justified by stating that “The model of ‘simple reproduction’, in which all variables repeat themselves over time, is the natural starting place for an exact analysis” (Samuelson 1957: 886).

The stationary production feasibility conditions of x require that:

(i) the sum total of the i -input in all industries is not larger than the ‘given’ self-replacement amount of the i -input available in the economy:

$$\sum_{j=1}^n x_{ij} \leq \bar{x}_i \quad (10)$$

and (ii) the sum total of labour in all industries is not larger than the ‘given’ self-replacement amount of labour available in the economy:

$$\sum_{j=1}^n L_j \leq \bar{L} \quad (11)$$

The production feasibility conditions of x may be restated in matrix-vector notation:

$$a (I - a)^{-1} y \leq \bar{x}_i \quad (12)$$

and

$$a_0 (I - a)^{-1} y \leq \bar{L} \quad (13)$$

The self-replacement state assumption implies the ‘givenness’ of quantities of the system. Hence, the technique of production described by the matrix of

average technical coefficients of production (a, a_0) does not convey a specific assumption on returns to scale. Also, if the quantities do not change, then the marginal productivities of resources cannot be defined either.

2.2 The system of prices of production⁶

The long-run equilibrium price of production of the j -th commodity produced by means of a single-product method of production is given by:

$$\begin{aligned} p_j x_j &= \sum_{i=1}^n p_i x_{ij} (1+r) + w L_j \\ &= \left(\sum_{i=1}^n p_i a_{ij} (1+r) + w a_{0j} \right) x_j \end{aligned} \quad (14)$$

where p_j is the price of the j -th commodity, p_i the price of the i -th input, r the uniform rate of profit, and w the wage-by-time rate, the latter being paid at the end of the production process.

The time-phased single-product Sraffa-Samuelson system of prices of production of commodities produced by means of commodities and homogeneous labour, with the labour wage being paid *post festum*, is given next in matrix-vector notation:

$$px = px_i (1+r) + wL \quad (15)$$

$$= pax (1+r) + wa_0x \quad (16)$$

where $p = (p_1, \dots, p_n)$ is the long-run equilibrium vector of the prices of the commodities. Labour is homogeneous in the sense that the wage-by-time rate is the same throughout the economy.

Reordering the terms in equation (15) and dropping the output vector x yields the prices of production as a function of the technique of production, the rate of profits and the wage rate:

$$p = wa_0 [1 - a(1+r)]^{-1} \quad (17)$$

In particular, the price of production of the j -th commodity is given by:

$$p_j = wa_{0j} [1 - a(1+r)]^{-1} \quad (18)$$

where $a_{0j} = (0, \dots, a_{0j}, \dots, 0)$.

The system of prices does not convey a specific standard of value. Hence, the wage rate and the prices of production are Debreu accounting prices. It will

⁶See Sraffa (1960); Pasinetti (1975); Samuelson (1957, 1982); Burmeister (1968); Riese (1987)

be shown later in the paper that the standard of value must be chosen in order to determine the prices of production, either as relative prices (sections 4 and 5), or as both money and relative prices (section 3). *Mutatis mutandis*, this also applies for Marx’s system of values (section 6).

The system of prices contains n equations, one equation for each commodity, and n^2+2n+2 unknowns, i.e. n prices of production, n^2+n technical coefficients, the wage rate and the rate of profits. If the quantities are given and there is only one way to produce each commodity, then the n^2+n technical coefficients of production of the matrix (a, a_0) are ‘given’. Still, two further equations are needed in order to determine the prices of production.

The existence of alternative techniques of production leaves the n^2+n coefficients underdetermined. This introduces a choice-theoretical problem. In general the choice of technique by the firms will depend on the level of the rate of profit (Sraffa 1960, chap. 12).⁷

Generally speaking, the full determination of the price system requires to consider: (i) the ‘givenness’ of the standard of value of the economy in terms of which the wage and the price of the commodity are expressed; (ii) the formal introduction of the standard of value in the price system, i.e. the numeraire equation; (iii) the ‘givenness’ of the independent variable of distribution, i.e. the distribution equation; and (iv) the choice of technique. Several theoretical determinations of the price system are considered in the following sections.⁸

3 Keynes-Samuelson-Sraffa

3.1 The money wage and the monetary theory of distribution

Keynes (1936, chap. 2) and Samuelson (1957), as probably Sraffa (1960), take for granted that the wage rate is expressed in terms of an abstract money of account which Samuelson calls “dollar”. The qualitative choice of the nominal numeraire implies that:

$$w = w^{nom} \tag{19}$$

⁷This assumption can be abandoned once the optimization of production by means of the choice of technique of production is considered. Accordingly, the optimal n^2+n coefficients of production can be determined. This is illustrated explicitly in section 3.2 for the monetary production economy.

⁸The single-product system of prices of production can be generalized in several directions. This may include: the production of basic and non-basic commodities; joint production, of which fixed capital is the leading case; and the consideration of several heterogenous non-augmentable factors of production, as well as exhaustible resources. These extensions of the basic system of circular production are not discussed in the paper (see Sraffa 1960).

Accordingly, the nominal prices of the commodities are also expressed in terms of the “dollar”:

$$p^{nom} = w^{nom} a_0 [1 - a(1 + r)]^{-1} \quad (20)$$

The monetary prices of production, the relative wage and the relative prices of the commodities are determined once: (i) the level of the dollar wage rate is ‘given’ to the firms, e.g. as a result of Keynes’s bargaining process between the entrepreneurs and organized labour (Keynes 1936, chap. 2); and (ii) the equilibrium rate of profits is ‘given’ to the firms from outside the system by the money rates of interest as suggested by Keynes (1936, chap. 16) and Sraffa (1960, chap. 5):

$$w^{nom} = \bar{w}^{nom} \quad (21)$$

and

$$\begin{aligned} r &= \bar{r}(i^{nom}) \\ &\geq i^{nom} \\ &\equiv \end{aligned} \quad (22)$$

where i^{nom} is the money rate of interest, and \bar{r} is the equilibrium rate of profits. Under free competition the equilibrium rate of profits allows the firms to pay exactly the equilibrium money rate of interest.

Hence, for the j -th commodity:

$$p_j^{nom}(\bar{r}) = \bar{w}^{nom} a_{0j} [1 - a(1 + \bar{r})]^{-1} \quad (23)$$

Ceteris paribus, it follows that the price of production of any commodity in terms of units of labour is determined once the rate of profits is known:

$$\begin{aligned} p_j^L(\bar{r}) &= \frac{p_j^{nom}}{\bar{w}^{nom}} \\ &= a_{0j} [1 - a(1 + \bar{r})]^{-1} \end{aligned} \quad (24)$$

By the same token, the relative wage expressed in terms of a particular commodity, i.e. the quantity of labour that a unit of that particular commodity can buy or Smithian “labour commanded”, is also determined once the rate of profits is known:

$$w_j^{rel}(\bar{r}) = \frac{\bar{w}^{nom}}{p_j^{nom}(\bar{r})} \quad (25)$$

$$= \frac{1}{a_{0j} [1 - a(1 + \bar{r})]^{-1}} \quad (26)$$

The market logic of equation (25) implies that in the (abstract) money wage economy the wage rate does not determine the relative wage, whichever the commodity in terms of which the latter might be expressed. That is, the money wage conveys the numeraire in terms of which the prices of the commodities are expressed, not the distributional equation of the price system.

Also,

(i) the long-run dollar price of any particular commodity is a positive function of the 'given' dollar wage rate:

$$\frac{\partial p_j^{nom}}{\partial \bar{w}^{nom}} > 0 \quad (27)$$

(ii) and a positive function of the rate of profits:

$$\frac{\partial p_j^{nom}}{\partial \bar{r}} > 0 \quad (28)$$

(iii) the relative wage expressed in terms of any particular commodity is a negative function of the monetary rate of profits⁹:

$$\frac{\partial w_j^{rel}}{\partial \bar{r}} < 0 \quad (29)$$

and (iv) the relative prices of the commodities expressed in terms of any particular commodity will change whenever the rate of profit changes:

$$\frac{\partial (p_k^{nom} / p_j^{nom})}{\partial \bar{r}} \neq 0 \quad (30)$$

The preceding results can be stated in terms of the macroeconomic market theory of a monetary production economy. Accordingly,

(i) under competitive conditions, the 'given' money rate of interest determines the equilibrium competitive rate of profits;

(ii) it also determines, *ceteris paribus*, the supply real wage, i.e. the real wage that is in conformity with the equilibrium competitive rate of profits;

(iii) the workers and the firms bargain *ultimately* the money wage; yet, the bargaining process leaves undetermined the equilibrium price level without which the real wage rate cannot be determined;

and (iv) the aggregate money supply price is determined once both the money rate of interest *and* the money wage rate are 'given' from outside the system of production.

Hence, the market logic of the monetary production economy implies that the money rate of interest is the independent variable of distribution, and the real wage rate the dependent variable. The money wage supplies the standard of value in terms of which the prices of the commodities are expressed. It is

⁹At the aggregate level this implies that the macroeconomic real wage rate is a negative function of the (equilibrium) rate of profits. See Riese (1987), part A.

Keynes's contention in the *General Theory* that in a monetary production economy the money rate of interest determines the distribution of income between labour and capital.

A complete model of the monetary production economy implies the formulation of: (i) the theory of effective demand and employment (and involuntary unemployment); and (ii) the theory of interest and endogeneous money (Betz 2001).

3.2 Optimization in the constant-returns-to-scale case¹⁰

The firms in the economy may optimize the production process either for a *spectrum* or a *continuum* of techniques.¹¹ First order conditions for the *continuum* of techniques case under constant returns to scale are given for the j -th commodity by:

$$\bar{w}^{nom} = p_j^{nom} \cdot \frac{\partial x_j}{\partial L_j} \quad (31)$$

and

$$p_i^{nom} (1 + \bar{r}) = p_j^{nom} \cdot \frac{\partial x_j}{\partial x_{ij}} \quad (32)$$

where the subindex i refers to the input and the subindex j refers to the produced commodity.

The economic rationale of production optimization implies the efficient allocation of resources by the individual firm. Under competitive conditions, the individual firm optimizes with the money wage and the rate of interest as parameters. Both the long-run equilibrium money prices and the relative prices depend ultimately on the rate of profit. Hence, the marginal productivity of the various inputs and labour depends on the rate of profit, not the other way around. In symbols,

$$\frac{\partial x_j}{\partial L_j} = \frac{\bar{w}^{nom}}{p_j^{nom}(\bar{r})} \quad (33)$$

and

$$\frac{\partial x_j}{\partial x_{ij}} = (1 + \bar{r}) \cdot \frac{p_i^{nom}(\bar{r})}{p_j^{nom}(\bar{r})} \quad (34)$$

A change in the rate of profits (e.g. due to a change of the money rate of interest) alters the equilibrium relative prices of production. The new relative prices leads to the reallocation of resources inside the firm for the optimization

¹⁰See Hahn (1982) and Riese (1987).

¹¹Sraffa (1960) and Pasinetti (1977) discuss the choice of technique for a *spectrum* of techniques.

of the production process to be in conformity with the new market equilibrium conditions. The range of optimal reallocation of resources is fixed by the marginal productivity of labour and the produced inputs.

4 Sraffa-Hahn

4.1 The unit price simplex numeraire

The market logic of non-monetary classical and neoclassical theories of value and distribution does not imply an abstract standard of value. Accordingly, the wage rate and the prices of the commodities may be expressed in terms of a particular commodity, or a (unit) price simplex. Ricardo (1821) and Walras (1954, 1926) chosed a particular (produced) commodity as the numeraire. Von Neumann (1937) introduced for the first time a unit price simplex. Later, the neoclassical economists, Sraffa and the neoricardians have also used alternative versions of the unit price simplex as the numeraire. In particular, both Sraffa (1960) and Hahn (1982) consider two special unit price simplexes when discussing the inner relation between income distribution and prices of production.

Accordingly, the numeraire of the economy may be:

(i) a produced commodity, e.g. the k -th commodity:

$$p_k = 1 \tag{35}$$

(ii) a Hahn (1982) unit price simplex:

$$S = \left\{ p, w \mid \sum p + w = 1, p > 0, w > 0 \right\} \tag{36}$$

or (iii) a Sraffa (1960) unit net product price simplex:

$$S = \{ p \mid py = p(I - a)x = 1, p > 0, (I - a)x > 0 \} \tag{37}$$

In all cases the choice of the numeraire implies *uno acto* (i) the introduction of a new equation into the system of prices and (ii) the expression of the wage and the prices of the commodities as *relative* prices. The analytical advantage of Sraffa's numeraire over both the single-commodity numeraire and Hahn's numeraire is that the former allows to express immediately the wage income and profit income as relative shares of the national income.¹²

Still, in all three cases the system of production has one degree of freedom. An additional distributional equation is required in order to determine the relative prices of the commodities. Both the relative prices of the commodities

¹²This particular point is addressed later in section 4.1.

and the relative wage are fixed once either the wage rate in terms of the chosen standard of value or the rate of profit is 'given' from outside the system of production.

Hahn (1982, section V) offers a neoclassical non-monetary intertemporal consumption preference theory of the rate of interest. This requires him to consider a saving-investment equation in order to determine the equilibrium rate of profits. Consequently, he also assumes full-employment.

By contrast, Sraffa (1960, chaps. 2-5) does not discuss a specific theory of value and distribution. In particular, he does not restate the classical theory of value and distribution. The neoricardians are wrong on this point (Kurz/Salvadori 1995). Nor does he suggest a neoclassical theory of value and distribution with full-employment for the self-replacement state case, unlike Hahn. Sraffa's main purpose is the understanding of the inner logical relation between the distribution of income, the relative prices of production of the commodities produced by means of commodities, and the choice of technique. In particular, Sraffa shows in his book that, in general, a change of income distribution between capital and labour alters both the relative prices of production and the 'quantity of capital' with the self-replacement quantities of the outputs, the inputs and labour unchanged. This allowed him to show precisely that both the Clark-Wicksteedian theory of distribution based on the marginal productivity of "capital" and labour and the Austrian notion of capital are logically and theoretically corrupt (Harcourt 1972). Incidentally, Sraffa suggests at the end of chapter 5 of his book the determination of the rate of profit by the money rates of interest. Yet, this requires him to reject the unit price simplex and to consider the "dollar" wage as discussed in the preceding section (Burmeister 1968; Riese 1987).

In the unit price simplex economy, the firms may optimize the production process either for a *spectrum* or a *continuum* of techniques. In the *continuum* case (Hahn 1982) the optimization of allocation of resources using first order conditions does not differ *in nuce* from that discussed in the subsection at the end of the previous section. Sraffa (1960, chap. 12) and Pasinetti (1977, chap. 6) discuss the choice of technique for a *spectrum* of techniques.

4.2 A digression on Sraffa's Standard system¹³

Sraffa's (1960, chaps. 3-5) Standard system is obtained whenever it is assumed that the proportions in which the commodities are produced are equal to those in which they enter its aggregate means of production. The distinctive characteristic of the Standard system is that the 'quantity of capital', i.e. *pax*, does not change when measured in terms of the Standard numeraire whenever the distribution of income changes. Moreover, the Standard system assumption

¹³To some extent his subsection assumes that the reader is familiar with both Ricardo's (1821, chap. I) and Sraffa's (1960, chaps. 3-4) discussion on an invariable measure of value.

voids the existence of price Wicksell-effects, hence, the possibility of ‘capital reversal’.

The composite commodity fulfilling this condition is the ‘Standard composite commodity’. In symbols,

$$\begin{aligned} x &= ax + y \\ &= ax + Rax \end{aligned} \quad (38)$$

where R is the Sraffa’s Standard ratio (a positive scalar number). Accordingly, the Standard net product is given by:

$$\begin{aligned} y &= (I - a)x \\ &= Rax \end{aligned} \quad (39)$$

The Standard unit price simplex can be restated as:

$$S = \{p \mid py = pRax = 1, p > 0, (1 - a)x > 0\} \quad (40)$$

This yields the Standard system of prices of production:

$$\begin{aligned} pax(1 + r) + wa_0x &= px \\ &= pax + p(x - ax) \\ &= pax + py \\ &= pax(1 + R) \end{aligned} \quad (41)$$

Subtracting the value of the means of production, i.e. pax , from both sides of the Standard system in the last equation yields:

$$r(pax) + wa_0x = R(pax) \quad (42)$$

The Standard ratio R is ‘given’ independently of the distribution of income. Hence, it can be shown that the maximal rate of profits of the Standard system is equal to the Standard ratio. If $w = 0$, then all the net product goes to the capitalists. Since in this case $r(pax) = R(pax)$, it follows that $r = \max r = R$. By the same token, if $w = R(pax) = py = 1$, then $r = 0$.¹⁴

Dividing through by the last equation yields the sum of the relative income shares of the profit and the wages:

$$\frac{r(pax)}{R(pax)} + \frac{wa_0x}{R(pax)} = 1 \quad (43)$$

It is possible to normalize the quantity of labour (e.g. by means of a change of the unit of measure!) such that:

¹⁴The Frobenius-Perron theorems on eigenvalues when applied on semi-positive square matrices have been useful for the formal determination of the maximum rate of profits for any single-product Standard technique of production (Pasinetti 1977, chap. 5).

$$L = a_0x = 1 \tag{44}$$

It follows that:

$$\frac{r}{R} + \frac{w}{R} = 1 \tag{45}$$

This yields a *linear* inverse relation between the rate of profit and the wage rate, the latter being expressed in terms of the Standard unit price simplex or Standard commodity.

Let the wage rate be the independent variable of distribution. This yields Sraffa's first Standard relation between the wage (as a proportion of the Standard net product) and the rate of profits:

$$r(\bar{w}) = R(1 - \bar{w}) \tag{46}$$

Let now the rate of profits be the independent variable of distribution. This yields Sraffa's second Standard relation between wages and the rate of profits:

$$w(\bar{r}) = 1 - \frac{\bar{r}}{R} \tag{47}$$

Sraffa asserts that any single-product system of prices of production always contains a lineal inverse relation between the wages and the rate of profits provided the former is expressed in terms of the Standard commodity (i.e. Sraffa's Standard unit price simplex).

5 Von Bortkiewicz-Von Neumann-Sraffa

Unlike Keynes and the neoclassical economists, the classical economists assume that labour is an augmentable commodity by means of a subsistence wage, the latter being the relative price of reproducible labour. Since by assumption the classical workers do not save it follows that:

$$wL = (p\bar{c}^w) L \tag{48}$$

where \bar{c}^w is the exogenously given vector of subsistence consumption. As before, $L = a_0x$. Unlike the previous two sections, here it is assumed that the workers are paid *ante festum*, i.e. before the production process.

Replacing the subsistence-wage equation into the system of prices yields the classical Von Bortkiewicz-Von Neumann-Sraffa system of prices of production¹⁵:

$$pax(1+r) + p(\bar{c}^w a_0)x(1+r) = px \tag{49}$$

¹⁵The origins of this price system are discussed in section 7.

where $(\bar{c}^w a_0)$ is the Seton-Bròdy-Nguyen semi-positive square matrix of technical labour-feeding coefficients of production. Reordering the terms in the last equation yields the Von Bortkiewicz-Von Neumann-Sraffa price system:

$$p(a + \bar{c}^w a_0)x(1+r) = px \quad (50)$$

Leaving aside the output vector x , the following characteristic equation of this price system is obtained:

$$p[I - (a + \bar{c}^w a_0)(1+r)] = 0 \quad (51)$$

The non-trivial solution (i.e. $p \neq 0$) requires the matrix $[I - (a + \bar{c}^w a_0)(1+r)]$ to be singular. The Frobenius-Perron theorems allow to show that the solution of the system yields the maximal uniform rate of profits of the Von Bortkiewicz-Von Neumann-Sraffa system of production. Once the rate of profits is known, it then becomes possible to determine the accounting prices of the commodities. The exact relative prices are determined once a produced commodity (Sraffa 1960) or a unit price simplex is chosen as the numeraire (Von Neumann 1937).

The functional capitalists might optimize the production process. Von Neumann (1937) discusses the choice of technique for a *spectrum* of techniques for the classical steady-state-growth joint-production case.

6 Marx

6.1 The system of labour values

Marx's labour theory of value states that (abstract) human labour produces a quantum of value in a given unit of time when producing commodities to be sold at the market. The Marxian system of values of circularly produced commodities is given by:

$$\begin{aligned} vx &= \Lambda + vax \\ &= \lambda x + vax \\ &= \lambda(I - a)^{-1} \end{aligned} \quad (52)$$

where $v = (v_j, \dots, v_n)$ is the vector of the values of the commodities, $\Lambda = (\Lambda_j, \dots, \Lambda_n)$ is the vector of new value-added by *abstract* human labour in the different industries, vax the aggregate value of constant capital, and $\lambda = (\lambda_j, \dots, \lambda_n)$ the vector of new value-added created during the time required to produce one unit of each one of the commodities. Although the underlying technique of production discussed may be assumed the same (section 2.1), Marx's

system of values differs substantially from the system of prices of production (section 2.2).

The new value-added being distributed between the workers and the capitalists in bourgeois society, it follows that:

$$\Lambda = W + \Pi \quad (53)$$

where W is the vector of the value of (homogeneous) variable capital equal to the wages, and Π the surplus value appropriated by the capitalists in each industry.

The rate of exploitation of labour by capital is measured by the rate of surplus value, the latter being defined as the quotient of the surplus value and the value of variable capital. Under free competition, the rate of surplus value is the same throughout the economy:

$$\begin{aligned} \pi &= \frac{\Lambda - W}{W} \\ &= \frac{\Pi}{W} \end{aligned} \quad (54)$$

where π is a scalar measuring the competitive rate of exploitation. According to Marx, the rate of surplus value can be increased either by extending Λ or reducing W or both.

The system of values can now be written:

$$\begin{aligned} vx &= (W + \Pi) + vax \\ &= W(1 + \pi) + vax \end{aligned} \quad (55)$$

Let w be the value of the subsistence wage per unit of labour time. Since $W = wL$ it follows that

$$\begin{aligned} vx &= wL(1 + \pi) + vax \\ &= wa_0x(1 + \pi) + vax \end{aligned} \quad (56)$$

Leaving aside the output vector x , the following system of values is obtained:

$$v = wa_0(1 + \pi) + va \quad (57)$$

If the constant capital is equal to zero, then the inverse Leontief matrix becomes the identity matrix. Hence, the value of the commodities is determined by the new value-added created by the abstract human labour during the time needed to produce one unit of each one of the commodities:

$$\begin{aligned} v &= wa_0(1 + \pi) \\ &= \lambda I \end{aligned} \quad (58)$$

For the positive constant capital case it can be shown that the value of the commodities is equal to the sum of the new value-added and the past abstract labour incorporated in the means of production produced in all the previous periods. Replacing the fore-last equation n times into itself yields:

$$v = wa_0(1 + \pi)(I + a + \dots + a^n) + va^{n+1} \quad (59)$$

The matrix a fulfils the Hawkins-Simon conditions. Accordingly, $\lim_{n \rightarrow \infty} a^n = 0$, hence, the last term vanishes for n very large. Also, the inverse Leontief matrix $(I - a)^{-1}$ exists. Since $(I - a)^{-1} = I + a + a^2 + \dots$, it is possible to write:

$$\begin{aligned} v &= wa_0(1 + \pi)(I - a)^{-1} \\ &= \lambda(I - a)^{-1} \end{aligned} \quad (60)$$

Marx assumes that the value of labour power is equal to the value of the exogenously given subsistence consumption basket, the wage being the price form of the value of labour power. It can thus be written:

$$wL = v\bar{c}^w L \quad (61)$$

where \bar{c}^w is the exogenously given vector of subsistence consumption and $L = a_0x$.

Replacing the last equation into the system of values yields:

$$v = (v\bar{c}^w) a_0(1 + \pi)(I - a)^{-1} \quad (62)$$

After some algebraic manipulation Marx's long-run value system is obtained:

$$v = v(\bar{c}^w a_0)(1 + \pi)(I - a)^{-1} \quad (63)$$

where $(\bar{c}^w a_0)$ is the Seton-Bròdy-Nguyen semi-positive square matrix of technical labour-feeding coefficients of production.

This yields the following characteristic equation of the system of labour values:

$$v \left[I - (\bar{c}^w a_0)(I - a)^{-1}(1 + \pi) \right] = 0 \quad (64)$$

The non-trivial solution (i.e. $v \neq 0$) requires the matrix in equation (64) to be singular. The Frobenius-Perron theorems allow to show that the solution of the system yields the maximal uniform rate of surplus value of Marx's system of labour values. Once the rate of surplus value is known, it is possible to determine the structure of the vector of the accounting values of the commodities.

The length of the vector of the accounting values and the relative values can be obtained once a produced commodity or a unit value simplex is chosen as the numeraire. This is not the case if an abstract standard of value (section 3) is chosen. Let us assume that Marx took for granted the expression of the wage

rate in terms of an abstract standard of value, e.g. Samuelson’s “dollar”. The consideration of the money wage rate into system of values yields:

$$v^{nom} = w^{nom} a_0 (1 + \pi) (I - a)^{-1} \quad (65)$$

and

$$w^{nom} = v^{nom} \bar{c}^w \quad (66)$$

where w^{nom} is the “dollar” wage and v^{nom} the Marxian “dollar” values of the commodities.

The characteristic equation of nominal system of labour values is:

$$v^{nom} \left[I - (\bar{c}^w a_0) (I - a)^{-1} (1 + \pi) \right] = 0 \quad (67)$$

Alas! In equations (64) and (67), the rate of surplus value and the structure of the vector of values are determined without recourse to the labour theory of value, independently of whether the wage and the prices of the commodities are expressed in money terms or not. Still, the choice of a produced commodity or a unit value simplex is needed in order to determine the relative values of the commodities. This makes the abstract standard of value superfluous (though it may play a role in the classical theory of money). Hence, the long-run Marxian values are not money values in the same sense as prices of production are money prices of production in the Keynes-Sraffa-Samuelson economy first discussed (section 3).

The non-trivial solution has been obtained without recourse to the labour theory of value. The theoretical implication of this result should not be underestimated, specially not by orthodox marxists: Marx cannot have at the same time both the labour theory of value *and* the subsistence-wage theory of distribution when determining the long-run system of values.

The labour theory of value can be given a role in determining Marx’s system provided the subsistence wage assumption is abandoned. However, in this case neither the rate of surplus value nor the system of values can be determined through the classical mechanism of distribution. A rule of distribution of the new value-added is missing. All what can be said is that the value of any particular commodity is determined by the quantity of abstract human labour required to produce it. This is exactly the system of labour values of a classless market society as expressed in equation (52). Yet, the distribution of the new value-added between the capitalists and the labourers remains unexplained.

Alternatively, if the uniform rate of surplus value is “given” from outside the system of production, then the relative wage-by-piece expressed in terms of the produced commodity or the unit price simplex can be determined:

$$w a_{0j} = \frac{\lambda_j}{(1 + \bar{\pi})} \quad (68)$$

where $w a_{0j}$ is the wage-by-piece rate and $\bar{\pi}$ the “given” rate of surplus value. However, in this case the subsistence wage does not determine the long-run

relative wage, and the classical long-run equality of the relative wage and the value of labour power in terms of the numeraire is not warranted any longer. Still, a theory of the “given” rate of surplus value is missing.¹⁶

6.2 Optimization in Marx’s economy

Marx’s capitalists may optimize the production process using conventional optimization techniques. For a *continuum* of techniques of production under constant returns to scale the first order conditions are given by:

$$w(1 + \pi) = v_j \frac{\partial x_j}{\partial L_j} \quad (69)$$

and

$$v_i = v_j \frac{\partial x_j}{\partial x_i} \quad (70)$$

Replacing $w = v\bar{c}^w$ into the forelast equation yields:

$$(v\bar{c}^w)(1 + \pi) = v_j \frac{\partial x_j}{\partial L_j} \quad (71)$$

The economic logic of the competitive capitalist production tells that v, \bar{c}^w , and $w(1 + \pi)$ are exogenously given to the individual competitive firm. The optimal allocation of labour and other inputs by the Marxian entrepreneurs implies that the marginal productivity of *concrete* labour (not *abstract* labour) in each industry is endogenously determined:

$$\frac{\partial x_j}{\partial L_j} = (1 + \pi) \cdot \frac{(v\bar{c}^w)}{v_j} \quad (72)$$

¹⁶Samuelson (1971) suggests that a rule of distribution such as a value-added tax on the labour payments may play the same role of the exogenously ‘given’ rate of surplus value. However, if the subsistence wage assumption is reintroduced, then Samuelson’s value-added tax is the dependent variable of distribution. In this case the rate of exchange between the commodities and the State rate of exploitation will not differ from Marx’s values and rate of surplus value already determined without recourse to the labour theory of value. Alternatively, if as suggested by Samuelson the State introduces a turn-over tax, then the rule of distribution applies on both the labour payments and the price of the inputs. In this case one obtains a Samuelson-Hahn-Sraffa system of prices of production. Again, if the subsistence wage assumption is reintroduced, the system of prices transforms into the Von Bortkiewicz-Von Neumann-Sraffa system with no relation to Marx’s labour values. Formally, Samuelson is right. However, he should also have mentioned that: (i) the consideration of a money tax implies the introduction of a political institution, i.e. the State, and the determination of money as the (endogenous) medium of deferred payments, both of which are alien to classical-neoclassical orthodoxy; and (ii) the rate of profit (interest) is the specific rule of distribution of the value-added in a capitalistic society, not the tax system. Incidentally, together (i) and (ii) imply a monetary production economy. Samuelson’s orthodox notion of money as means of exchange did not allow him to see this and hindered him to transcend his neoclassical scholarship.

and

$$\frac{\partial x_j}{\partial x_i} = \frac{v_j}{v_i} \quad (73)$$

Post-Leontief neoclassical economists have proved that the existence of the Walras-Arrow-Debreu general equilibrium does not depend on marginal productivity of resources. Conversely, heterodox post-Keynes-Kalecki authors have shown that a monetary production economy can readily incorporate the optimization techniques (section 3). The latter is also true for Marx's labour value theory of capitalism. This due to the fact that the *real* notion of marginal productivity is not a *substantial* moment of a theory of capital.

7 Marx's unfulfilled twofold transformation

Marx was aware of the fact that the practical (i.e. bourgeois) economic categories refer to: (i) the money wage as the price of labour, not to the money form of the value of labour power; and (ii) the prices of production of circularly produced commodities and not to a system of labour values.

Accordingly, Marx aimed in *Capital* (amongst other things) to show dialectically:

"how the value of a commodity transformed into its price of production in which 1. the *whole of the labour appears paid for* in the form of wages; 2. the surplus-labour, however, or surplus-value, assumes the form of an *addition to the price*, and goes by the name of interest, profit, etc. *over and above the cost-price* (=price of the constant part of capital + wages).

Answering this question presupposes: I. That the *transformation* of, for example, the *value of a day's labour power* into *wages or the price of a day's labour* has been explained. This is done in *Chapter V* of this volume [i.e. actually in section 6 of volume I]. II. That the *transformation of surplus-value* into *profit*, and of *profit into average profit*, etc., has been explained. This presupposes the *process of circulation of capital* has been previously explained, since the turnover of capital, etc., plays a part here. This matter cannot therefore be treated prior to the 3rd book (*Volume II* is to contain books 2 [i.e. volume II of Engel's edition] and 3 [i.e. volume III])."¹⁷ (square brackets added)

¹⁷Quoted from a letter from Marx to Engels dated June 27,1867. Marx adds after the quotation in the main text:"Here it will be shown how the philistines's and vulgar economists' *manner of conceiving things* arises, namely, because the only thing that is ever reflected in their minds is the immediate *form of appearance* of relations, and not their *inner connection*. Incidentally, if the latter were the case, we would have not need of *science* at all."[ibid.] This sentence also appears in volume III of *Capital*.

Before addressing these two problems, it should be reminded that Marx showed in section 1 of volume I of *Capital* that money is: (i) the standard of value in terms of which both the prices and debts are expressed; (ii) the means of exchange and the store of value; and (iii) the medium of deferred payments, i.e. that by means of which debts in the economy are discharged. In particular, the standard of value may be an abstract money of account (fixed by the state) as in section 3 of the paper.

In section 6 of volumen I of *Capital* Marx shows how the value of a day's labour power is transformed into wages both practically and theoretically such that the whole of the labour time appears paid for in the form of wages. Accordingly, the wage being practically the "price of labour", e.g. the wage-by-time or the wage-by-piece, it must not be related to the value of labour power. In chapter XVII of volume I he notes: "The wage-form thus extinguishes every trace of the division of the working-day into necessary labour and surplus-labour, into paid and unpaid labour. All labour appears as paid labour.[Marx (1867, chap. 17)]¹⁸

Marx kept the subsistence wage assumption from volume I through volumen III of *Capital*. This allowed him to develop his theory of surplus value, and to determine the rate of surplus value, too. However, if the subsistence wage rate assumption is kept and a produced commodity or a unit price simplex is chosen as the numeraire, then it follows that the rate of surplus value and the long-run value in exchange between the produced commodities can be exactly determined without recourse to the labour theory of value. One might consider either Marx's rate of surplus value as in Marx's labour-value economy (section 6.1 of this paper), or the rate of profit as in the Bortkiewicz-Von Neumann-Sraffa economy (section 5). In either case both the rate of surplus value or the rate of profits, and the long-run rates of exchange of the commodities are determined by the matrix of the technical coefficients of production a and the matrix of technical labour-feeding coefficients of production $(\bar{c}^w a_0)$.

Marx and the orthodox marxists after him proved unwilling and unable to transform the system of labour values into the true system of prices of production of a capitalistic economy in volume III of *Capital*. Neither were ready to transcend the labour theory of value, nor to reject the subsistence wage assumption, nor to put in its place an alternative monetary theory of value and distribution. Yet, the correct transformation of values into prices would have put Marx on the track towards the formulation of the Sraffa-Samuelson system of prices of production.

In 1907 the russian economist Von Bortkiewicz solved correctly Marx's value-price transformation problem (Samuelson 1970). Ironically, he was never aware of the correctness of his solution! Bortkiewicz, like Marx, and unlike Dmitriev and Von Neumann, never decomposed Marx's labour value aggregates into values/prices and quantities. Also like Marx, Von Bortkiewicz assumed the wage

¹⁸The original german text is quoted here: "Die Form des Arbeitslohns löscht also jede Spur der Teilung des Arbeitstags in notwendige Arbeit und Mehrarbeit, in bezahlte und unbezahlte Arbeit aus." [Marx (1867): 563]

rate to be equal to the value of the subsistence consumption basket. Alas! His solution to Marx's problem led him unknowingly to the Von Bortkiewicz-Von Neumann-Sraffa system of prices of production. As a matter of fact, Von Neumann formalized the generalized Bortkiewicz solution to Marx's transformation problem in his 1937 (1928) paper.¹⁹

Still, in order to formulate the 'true' system of prices of circular production of a capitalist economy, two conditions must be fulfilled: (i) the wage and the rates of interest must be 'given' in terms of the abstract standard of value; and (ii) the subsistence wage assumption must be abandoned.

Early neoclassical economists such as Walras and Jevons rejected the subsistence wage assumption at the time Marx published volume I of *Capital*. However, Walras followed classical economist Ricardo and chose a particular (produced) commodity as the standard of value, e.g. silver. Later Wald and Debreu in the 1940s and 1950s, and much later Hahn in the 1980s replaced Walras's numeraire with Von Neumann's 1937 unit price simplex. Yet, in neither case the wage and the prices were expressed in terms of an abstract money of account.

Keynes's rejection of the subsistence wage assumption and his theory of the money wage in a monetary production economy in the *General Theory* opened the way towards the formulation of the Keynes-Samuelson-Sraffa system of prices of production. This was done first by Sraffa in 1928, however, the latter's manuscript remained unpublished during the next three decades. Alas, the economic trade had to wait until the publication of Samuelson's critical essay on Marx's economics in 1957 and Sraffa's *Production of commodities by means of commodities* in 1960 for the theory of prices of production to move from the classical conception of production to the monetary theory of prices of production, with the neoclassical post-Leontief theory of production in-between!

8 A summing up

Marx's system of labour values is the point of departure of the journey towards the formulation of true time-phases systems of prices of production reflecting the economic logic of the capitalistic production of commodities by means of commodities.

To move from Marx's value system to the Sraffa-Von Neumann price system it is necessary to substitute the Sraffa-Samuelson system of prices of production

¹⁹ Von Neumann's paper contains the first existence proof of a classical economy with circular joint-production (Kurz/Salvadori 1995). Apparently von Neumann wrote the existence proof paper in 1928 at the time he wrote his seminal paper on game theory. In this year a post-Von Bortkiewicz german-russian theoretical seminar on economic systems of production took place at the University of Berlin. Von Neumann was not an economist, but he attended the seminar (Weintraub 1983). Later he presented the paper at a mathematical seminar at Princeton University in 1932. The paper was published first in german in 1937 and later in english in 1945.

for the Marx's system of values. In either system (i) the long-run values and the long-run (normal) prices of production are determined by the technique of production and subsistence wage; and (ii) a produced commodity or a unit price simplex is the numeraire. The transformation of the system of values into the Von Bortkiewicz-Von Neumann-Sraffa system of prices of production implies the rejection of the labour theory of value.

To move from the post-Marx Bortkiewicz-Sraffa-Von Neumann price system to the Sraffa-Hahn price system substitute the post-classical Jevons-Walras assumption of labour being a non-augmentable-original factor of production for the Cantillon-Quesnay-Smith-Ricardo reproducible-labour-subsistence-wage assumption. Here both the wage and the prices of the commodities are expressed in terms of unit price simplex, still.

Finally, to move from Sraffa-Hahn price system to the post-neoclassical Keynes-Sraffa-Samuelson moneyed system of prices of production substitute the monetary abstract standard of value in Keynes's *Treatise on Money* and *General Theory* (but also in Marx's volume I of *Capital*) for either the one-commodity numeraire or Sraffa's unit price simplex or Hahn's unit price simplex. Also, assume that the rate of profits is determined from outside the system of production by the Keynes-Sraffa money rates of interest.

Once this is done the twofold transformation of the Marxian labour values into money prices of production is completed, the adequate order of movement from one system of political economy to the next (not followed here) being that suggested by the dialectics of the economic categories as intended by Marx in unfinished "*Das Kapital*".

References

- [1] Betz, K. (2001) *Jenseits der Konjunkturpolitik*. Marburg: Metropolis Verlag.
- [2] Burmeister, E. (1968) "On a theorem of Sraffa," *Economica*, Feb., 35 (137), 83-87.
- [3] Dmitriev, V. K. (1977, 1904) *Ensayos económicos sobre el valor, la competencia y la utilidad*. México: Siglo Veintiuno Editores.
- [4] Dorfman, R., Samuelson, P.A., Solow, R.M. (1958) *Linear Programming and Economic Analysis*. New York: MacGraw-Hill.
- [5] Hahn, F.H. (1982) "The Neo-Ricardians," *Cambridge Journal of Economics*, 6, 353-374.
- [6] Harcourt, G.C. (1972) *Some Cambridge controversies in the theory of capital*. Cambridge: Cambridge University Press.

- [7] Keynes, J.M. (1936) *The General Theory of Employment, Interest and Money. Vol. VII. of The Collected Writings of John Maynard Keynes.* London and Basingstoke: The Macmillan Press, 1973.
- [8] Kurz, H.D., Salvadori, N. (1995) *Theory of production. A long-period analysis.* Cambridge New York Melbourne: Cambridge University Press.
- [9] Marx, K. (1867, 1885, 1894) *Das Kapital. Kritik der politischen Ökonomie. Marx Engels Werke.* Vols. 23-25. Berlin: Dietz Verlag, 1964.
- [10] Morishima, M., Seton, F. (1961) "Aggregation in Leontief Matrices and the Labour Theory of Value", *Econometrica*, 29 (2), April, 203-220.
- [11] Nguyen, D. (1982) "Notes on Professor Samuelson's Analysis of the Marxian Transformation Problem", *Southern Economic Journal*, 49 (1), July, 1-10.
- [12] Pasinetti, L.L. (1977) *Lectures on the Theory of Production.* Cambridge: Cambridge University Press.
- [13] Ricardo, D. (1821) *On the Principles of Political Economy and Taxation. Vol. I. of The Works and Correspondence of David Ricardo.* Cambridge: Cambridge University Press, 1951.
- [14] Riese, H. (1987) *Hahn, die Neoricardianer und Keynes - Ein Beitrag zur Marktlogik des Kapitals.* Freie Universität Berlin. Unpublished paper.
- [15] Samuelson, P.A. (1957) "Wages and Interest: A Modern Dissection of Marxian Economic Models," *American Economic Review*, Dec., 47 (6), 884-912.
- [16] Samuelson, P.A. (1970) The "Transformation" from Marxian "Values" to Competitive "Prices": A Process of Rejection and Transformation, in: Samuelson, P.A. *The Collected Scientific Papers of Paul A. Samuelson.* Vols. III, 309-311.
- [17] Samuelson, P.A. (1970) "Understanding the Marxian Notion of Exploitation: A Summary of the So-Called Transformation Problem Between Marxian Values and Competitive Prices", *Journal of Economic Literature*, 9 (2), 399-431.
- [18] Sraffa, P. (1960) *Production of Commodities by Means of Commodities.* Cambridge: Cambridge University Press.
- [19] Neumann, J. von (1937) "Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpuntsatzes," *Ergebnisse eines mathematischen Kolloquiums 1935-1936*, 8, 73-83.
- [20] Weintraub, E.R. (1983) "On the Existence of a Competitive Equilibrium: 1930-1954", *Journal of Economic Literature*, 21 (1), March, 1-39.