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Román Andrés Zárate*

Abstract

Economic literature has identified positive effects of peer abilities on individual achievement. However, the intuitive arguments supporting this evidence are not clear. This article presents a specific mechanism: cooperation and competition among group members; more precisely, the presence of positive and negative externalities in human capital accumulation. First, I develop an economic model that incorporates both kinds of externalities and shows the existence of an optimal level of competition between group members that maximizes human capital accumulation. Then, using data from PISA (2000) and an empirical strategy that controls for potential endogeneity issues, I find empirical evidence supporting the main results of the theoretical model. Namely, I find robust evidence of a non-linear effect of competition on academic performance. These results are consistent with the proposed model and the presence of positive technological externalities in educational production functions.

JEL Classification Codes: D62, I21, J24

Keywords: Peer effects, Cooperation, Competition, PISA.

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Efectos de Grupo, Cooperación y Competencia en la Formación de Capital Humano

Resumen

La literatura económica ha identificado efectos positivos de los compañeros de clase sobre los resultados académicos individuales. Sin embargo, existen vacíos respecto a los mecanismos a través de los cuales esta relación opera. Este artículo presenta un mecanismo específico: la cooperación y competencia entre los miembros de un grupo. Es decir, la presencia de externalidades positivas y negativas en la acumulación de capital humano. Para mostrar esto, se desarrolla un modelo teórico que incorpora ambos tipos de externalidades y muestra la existencia de un nivel óptimo de competencia que maximiza la acumulación de capital humano. Adicionalmente, usando datos de PISA (2000) y una estrategia empírica que controla por potenciales problemas de endogenidad, se expone evidencia empírica consistente con los resultados teóricos. En general, se encuentra evidencia robusta de un efecto no lineal de la competencia sobre el desempeño académico. Estos resultados son consistentes con el modelo propuesto y la presencia de externalidades tecnológicas positivas en las funciones de producción de educación.

Códigos de clasificación JEL: D62, I21, J24

Palabras clave: Efectos de grupo, Cooperación, Competencia, PISA.

1 Introduction

Peer effects have been widely studied in economic literature. The importance of identifying them correctly has been the main issue of empirical analyses. Most of the economic literature on the topic has focused on finding an unbiased estimation using a source of exogenous variation. Nevertheless there is no strong empirical or theoretical evidence about the channels or mechanisms through which peer effects operate. This article proposes a particular mechanism: cooperation and competition among the members of a group. Both topics, cooperation and competition, have been studied in economic literature in different contexts, but there is not enough evidence on education outcomes and students' decisions. The existing literature on this topic is not clear for different reasons, mainly due to strong assumptions in theoretical analyses and a lack of causal identification in empirical studies. This paper tries to overcome both problems by using weaker assumptions in the theoretical analysis and an empirical strategy that allows for exogenous variation in the levels of competition and cooperation faced by students.

Different authors have pointed out the importance of peer effects based on decisions made by parents, schools and policy makers' about how children should be allocated to different groups. In particular, under the presence of peer effects, the group in which a student is allocated today affects his or her future cognitive and labor outcomes. Then, it would be desirable to promote certain types of educational systems. For example, the correct estimation of peer effects is important when determining if tracking systems, in which students are allocated with peers of similar abilities, are more efficient than others. The main problem in identifying these effects is self-selection; the group in which a student is allocated depends on his parents' preferences and on restrictions imposed by the educational system. Therefore, the positive effect of peers may be caused by selection decisions instead of a real mechanism in which peers affect students' outcomes. Under this problem, most empirical analyses have focused on finding an exogenous source of variation that explains peer composition but not students' decisions or outcomes.

For example, Zimmerman (2003), Sacerdote (2001) and Foster (2006) use random assignment of roommates as a natural experiment for an unbiased estimation[1]. Other authors use different sources of exogenous variation: Hoxby (2000) uses race and gender variation across adjacent cohorts as an instrument of peer characteristics; Lefgren (2004) uses variation in school tracking policies; and Dills (2005) uses the presence of a magnet school that selects high quality students from throughout the school district. The study of Hanusheck et al. (2009) uses school and grade

[1]Other research has focused on heterogeneous impacts of peer effects. In particular, Brunello, De Paola & Scoppa (2009) use the same source of variation (randomized assignment of roommates) for evaluating peer effects in an Italian University. They find positive peer effects for students enrolled in hard science programs and no effect for those enrolled in humanities or social sciences.

fixed effects in order to control for unobserved characteristics that cause self-selection of students. Other authors have used housing mobility programs to identify peer effects (Rosenbaum (2000) and Souza Briggs (1997)). Although these effects have been identified and widely studied in economic literature, there is still a lack of information about the channels through which peer effects take place. Hoxby (2000) provides some information about potential channels that may explain how these effects operate.

> [...] it is useful to clear about what peer effects include. Peer effects do include students teaching one another, but direct peer instruction is only the tip of the iceberg. A student's innate ability can affect his peers, not only through knowledge spillovers but through his influence on classroom standards. A student determined behavior may affect his peers. For instance, a student who has not learned self-discipline at home may disrupt the classroom. Peer effects may follow lines like disability, race gender, or family income: a learning disabled child may draw disproportionately on teacher time, racial or gender tension in the classroom may interfere with learning, richer parents may purchase learning resources that get spread over a classroom [...] (Hoxby, 2000)

As it can be seen, some intuitive arguments have been mentioned in the literature. However, most of them have not been empirically analyzed. In order to understand those mechanisms it is important to think in terms of agents, exogenous parameters, and endogenous decisions which can alter cognitive outcomes. For example, Duflo, Dupas & Kremer (2011) use a randomized experiment of tracking in Kenya for analyzing both, the effects of peers and tracking systems, on educational outcomes. They develop a theoretical model in which they consider the mechanisms through which this variation occurs are not only peer composition, but also teachers' effort decisions and academic level chosen for tests and learning. Using a regression discontinuity design, the authors find no difference in test scores between students allocated to the lower ability section and those allocated to the higher ability group. The authors show that this result is consistent with their theoretical analysis. However, it is plausible to think that there are other mechanisms or explanations to these results. Specifically, students' preferences and effort decisions based on expected ranking in the final test scores distribution[2].

This paper focuses on student's decisions and preferences; in particular, preferences for her ranking in test scores distribution and for the total human capital accumulated by herself, as well

[2] Duflo, Dupas & Kremer (2011) argue that the result they find is explained by teachers decisions. They assume that students near the median of the initial test are very similar in other characteristics. Given that they do not find differences on final test scores between students allocated to higher and lower sections, but they do find the presence of positive peer effects, they conclude that teachers' decisions about the academic level of their courses is the mechanism that explains this pattern. My perspective is based on students' preferences and decisions. The student with the lowest ability in the high ability group can exert lower or higher levels of effort due to his expected position in the final test scores distribution. If this mechanism can explain this result, then competitive or cooperative attitudes from students are essential in the analysis of how students should be allocated.

as her decisions on how much effort to exert and the level of cooperation between her and her classmates. Specifically, it analyzes how cooperation and competition can alter students' decisions and cognitive outcomes. On the one hand, it is important to wonder for the presence of positive technological externalities in the educational production functions. Under the presence of these externalities, cooperation decisions of student i can positively affect the educational outcomes of student j. On the other hand, cooperative environments can diminish group educational outcomes if students perceive individual and group effort levels as substitutes. Under this perception, more cooperative environments produce free-riding effects in which each individual expects other students to help her and, as a consequence, reduces her effort. This would result in a lower overall group effort which would have a negative impact on educational performance.

Even though there is little evidence on cooperative or competitive attitudes of a group and their effect on cognitive outcomes, the topic is related to a vast economic literature. For example, it has been suggested that there are positive externalities in human capital accumulation. Specifically, Lucas (1988) introduced average human capital in the society as an argument in the production function. This element captures the existence of positive technological externalities because it shows how the construction and spreading of ideas constitute social processes in which interactions among individuals are essential. Then, if relations among students also have these positive externalities, cooperative attitudes would increase their accumulated human capital, and this would translate into higher academic test scores.

On the other hand, Lazear & Rosen (1981) show that individuals would exert higher effort levels if wages are based on relative positions of production levels. This means that tournament systems in which individual rewards depend on their ranking would increase effort levels. In education processes this can be seen as students having a higher concern for their ranking. However, Lazear (1985) shows that this kind of incentives can create higher levels of sabotage between workers of a firm due to the increases in competition. Given this result, it is possible to think that in educational outcomes, individual concerns for ranking would also have a negative impact on academic performance. This can be explained by the students' incentive to cooperate less. However, simple tournament analysis has omitted other effects of competitive environments. For example, incentives to incur in unethical behaviors and cheating (Schleifer, 2004; Schwieren & Weichselbaumer, 2008), and a strong negative effect on the individuals' emotional state (Brandts, Riedl, & van Winden, 2005).

This article also tries to analyze empirically the effects of cooperative or competitive environments on educational outcomes. For this, I use the PISA dataset which includes test score results of students in mathematics, reading and science. Measures of competitive and cooperative attitudes come from questions regarding how much students are concerned about their ranking in the class and how much they help their classmates. Different sources of endogeneity can be present in this estimation. For example, self selection of parents with higher competitive preferences or simultaneity between academic results and competition and cooperation decisions. Due to this potential endogeneity, it is necessary to identify an exogenous source of variation which explains individual competitive and cooperative attitudes but that is not correlated with peers' test scores. This exogenous source comes from students' relationships at home; the presence of older, younger and same-age siblings may help explain individual attitudes toward competition and cooperation. However, it is less intuitive to think that these variables would alter peers' cognitive outcomes. For these reasons, I use these variables in an instrumental variables empirical approach. Assuming a non-linear functional form, I estimate the effect of a competitive environment on individual test scores. I find empirical support in favor of an inversed U-relationship between academic performance and the degree of competitiveness of the group. This result is similar for mathematics and reading test scores, and robust to different covariates, subsamples and estimation procedures.

Finally, it is important to mention that, despite the fact that there is not much evidence of cooperation or competition decisions on academic outcomes, some authors have focused on the topic. For example, Bigoni et al. (2011) designed an experiment in which incentives are exogenously imposed and students are randomly assigned to three different incentive schemes. A first tournament scheme that fosters competition between students, a cooperative scheme that promotes information sharing and cooperation, and a control treatment in which students neither compete nor cooperate. The authors find that competition induces higher effort levels than cooperation, and that the latter did not show increases in test scores compared to the control group. The problem with this analysis is that the experimental design does not take into account the existence of positive technological externalities[3].

Bratti et al. (2008) develop a theoretical model and an empirical analysis similar to those found in this paper[4]. They show that free-riding incentives lead to an insufficient degree of cooperation between schoolmates. They find empirical evidence that individual competitive attitudes have positive effects on test scores, while student performance increases with average cooperative

[3]Individuals are randomly assigned to each one of the schemes mentioned before and are ex-ante informed about it. They are also randomly assigned to a schoolmate but they don't know his/her identity. During the test, virtual contact information is available in case they decide to communicate between them. This type of cooperative scheme does not internalize positive technological externalities. The presence of these externalities requires communication between students, not at the moment of the test, but when they are studying for it. Under this empirical design, it can be expected that effort levels under the cooperative scheme would be lower. Students perceive their efforts as substitutes and there is no timing for positive externalities to emerge. Then, this result should be addressed with caution due to the incentives design of the authors.

[4]The data analyzed by the authors correspond to PISA (2003). In this paper the empirical analysis is based on PISA (2000). See section 3 for differences and further details.

behavior. Their theoretical model has strong assumptions. In particular, they assume preferences for individual and social learning, where social learning is a public good with the traditional functional forms of individuals' contributions[5]. They conclude that individuals are competitive if time dedicated to individual learning is higher than the one dedicated to the public good. In the case of cooperative behavior, the contrary happens. However, they don't consider preferences for ranking independently from individual test score. Their empirical analysis consists of correlations instead of causal inference. It is important to establish potential endogenous biases that make their estimators inconsistent. In particular, the positive effects of individual competitive attitude can be correlated with unobserved characteristics that explain higher test scores results.

In summary, economic literature has focused on identifying peer effects, but the channels through which they operate are still not clear. This paper tries to identify if they take place through a specific mechanism: cooperation and competition among students. This topic is related to positive technological externalities and individual concerns for relative ranking. Even though cooperative and competitive decisions of students have been analyzed by other authors, conclusions are unclear due to strong theoretical assumptions and the lack of an empirical identification of causality or an incorrect incentive design. Using a theoretical framework with weaker assumptions than those used in the existing literature, this paper identifies how cooperative or competitive attitudes affect cognitive outcomes, and shows a non-linear effect of competition on academic performance. Furthermore, using exogenous variation in peer test scores and competitive attitudes, I present robust empirical evidence supporting the main result of the model.

The rest of this paper is organized as follows: section 2 develops a simple theoretical framework in order to explain potential channels through which competition and cooperation affect educational outcomes. Section 3 presents the data and an empirical strategy using an exogenous source of variation for identifying these effects. Section 4 shows the main empirical results. Section 5 presents a discussion based on the results of the previous sections. Finally, section 6 concludes.

2 Theoretical Framework

This section presents a model aimed at formalizing the channels through which cooperation and competition can alter cognitive outcomes. For this purpose, I will consider the arguments of educational production function, the students' utility functions and their optimal decisions regarding effort and cooperation levels.

[5]They assume that production function of the public good S is given by $\frac{1}{\sigma} * \sum a_i * s_i$ where a_i corresponds to initial abilities of individual i and s_i to its contribution. σ represents initial ability dispersion of the group. One of their conclusions is that cooperative behavior may emerge if groups are more homogeneous in initial ability distribution. This result is not surprising if you take into account the functional form assumed in the production of the public good and that more homogeneous groups are more efficient in its production.

2.1 Educational Production Functions

Different papers have addressed the arguments of educational production functions[6]. Educational production functions considered in most of these papers are similar to the following:

$$h_i = h(S, Q, C, A, I) \tag{1}$$

Where h_i corresponds to human capital or skills accumulated by individual i; S is years of schooling; Q is a vector of school and teachers' characteristics; C is a vector of child characteristics; A is a vector of households' characteristics; and I is a vector of inputs under control of parents. In the economics of education's literature it is usual to assume a household utility function and budget constraints that constitutes a classical optimization problem. Parents' decisions would alter the cognitive outcomes of their children. My approach is different; in particular, I am not considering parents as the main agents. Specifically, the main purpose of this paper is to see how effort decisions of individual i can change and, therefore, my educational production function should include effort as an argument. We also consider different child characteristics: it is important to add a parameter that captures the initial abilities of individual i[7]. The educational production function should then be of the following type:

$$h_i = h(\theta_i, e_i, X) \tag{2}$$

Where θ_i is a parameter of initial abilities, and e_i represents the effort levels of individual i. X is a vector of school, teachers, household and other characteristics as described in equation 1. However, this function does not consider peer effects. For this purpose, additional arguments should be included in equation 2. This can be expressed in the following functional form:

$$h_i = h(\theta_i, k_i, \mu_i, \theta_j, \mu_j, X) \quad \forall\, j \neq i \tag{3}$$

Equation 3 considers interactions between individual i and individuals j. θ_i and θ_j represent initial abilities' parameters of individuals i and j, respectively. μ_i and μ_j correspond to the parameters through which I will capture peer effects. More precisely, they represent the fraction of time each individual decides to spend studying in group (e.g. cooperating); k_i corresponds to the fraction of time each individual decides to spend studying by herself. This can be captured in a traditional time constraint:

$$1 = k_i + \mu_i + l_i \tag{4}$$

[6] See Glewwe & Kremer (2006), among others.

[7] This parameter can also be interpreted as complementary factors in educational production functions.

Equation 4 states that overall time of individual i (normalized to 1) is divided between the time she spends studying individually, the time she spends studying in group and the time she dedicates to leisure (l_i). Given the general functional form presented in equation 3, which has both peer effects and cooperation decisions as arguments, it is important to think in plausible and intuitive assumptions to characterize this function.

Assumption A1. *$h(\cdot)$ is a differentiable function. $h_{\theta_i} > 0$, $h_{k_i} > 0$, $h_{\mu_i} \geq 0$, $h_{\mu_j} \geq 0$ and $h_{\theta_j} \geq 0$ $\forall j \neq i$.*

Assumption A1 shows that h is an increasing function in the initial ability parameter and the level of individual effort exerted by agent i. Also, assumption A1 shows the presence of positive externalities due to cooperation decisions. Human capital accumulated by individual i does not depend negatively on the fraction of time each individual decides to study in group and on the initial ability parameter of individuals j. I am explicitly assuming that being with individuals with higher initial abilities or individuals who decide to exert higher levels of cooperation have positive effects on learning. The next set of assumptions should consider the second derivatives of educational production functions presented in equation 3, in order to understand interactions among agents' decisions. The following assumption is necessary to guarantee that the problem of individual i is well defined.

Assumption A2. $h_{k_i k_i} < 0$, $h_{\mu_i \mu_i} < 0$, $\lim_{\mu_i \to 0} h_{\mu_i}(\cdot) = \infty$, $\lim_{\mu_i \to \infty} h_{\mu_i}(\cdot) = 0$, $h_{\mu_j \mu_j} < 0$

Assumption A2 presents decreasing or constant marginal returns for all arguments of equation 3. Thus, assumption A2 is necessary for obtaining an interior solution to the individual's maximization problem. However, more important is to characterize how do these marginal returns interact with each other. The following assumption address this issue.

Assumption A3. $h_{k_i \theta_i} > 0$, $h_{k_i \mu_i} = 0$, $h_{k_i \mu_j} = 0$, $h_{k_i \theta_j} = 0$.

Assumption A3 states two important things. On the one hand, there is a positive complementarity between individual effort and initial abilities. On the other hand, there is a substitution between individual learning and cooperative or group learning. Specifically, this assumption says that increasing the fraction of time individual i and individual j spend working in group does not increase the marginal return of individual learning. In other words, this assumption explicitly states that there are no complementarities between cooperation and competition in human capital formation. The last element of this assumption states that marginal return of individual effort (k_i) does not depend on the initial ability of other individuals.

Assumption A4. $h_{\mu_i \theta_i} > 0$, $h_{\mu_i \mu_j} > 0$,$h_{\mu_i \theta_j} > 0$.

formation. The last element of this assumption states that marginal return of individual effort (k_i) does not depend on the initial ability of other individuals.

Assumption A4. $h_{\mu_i\theta_i} > 0$, $h_{\mu_i\mu_j} > 0$,$h_{\mu_i\theta_j} > 0$.

This assumption relates the fraction of time individual i spends studying in group, and the other arguments in educational production function presented in equation 3. As it can be seen, I am assuming that if individual i and individual j have higher initial abilities, then studying in group (e.g. cooperating) is more efficient. In the same way, the marginal returns to the fraction of time individual i dedicates to group work is higher when individuals j choose a higher level of cooperation (μ_j).

2.2 Optimization Problem

Once established how educational outcomes are produced, it is important to determine how individuals make their decisions. For this purpose it is necessary to address students' utility functions. I will assume that students derive utility from three different terms. The first one corresponds to the absolute human capital accumulated by themselves. The second term is the expected ranking of individual i in the final human capital distribution. Finally, I will assume that students' utility increases with higher levels of leisure. Specifically, I will assume the following functional form:

$$U_i = \overline{w}h_i + \gamma R_i + f(l_i) \tag{5}$$

Where h_i corresponds to the absolute human capital accumulated by individual i and $\overline{w}$ is a parameter that captures the relative importance of it; R_i corresponds to the relative ranking of individual i in the final human capital distribution; γ, is a parameter that captures utility differentials based on relative ranking. As it can be seen, higher values of γ imply that rewards are more dispersed. Thus, individuals have more incentives to compete. Finally, $f(l_i)$ corresponds to the utility derived from leisure time.

Assumption A5. *$f(\cdot)$ is a differentiable function,* $f_{l_i} > 0$, $f_{l_il_i} < 0$, $\lim_{l_i \to 0} f_{l_i}(\cdot) = \infty$, $\lim_{l_i \to 1} f_{l_i}(\cdot) = 0$.

Assumption A6. *R_i[8] is a differentiable function.* $R_{ik_i} > 0$, $R_{i\mu_i} \leq 0$, $R_{ik_ik_i} < 0$, $R_{i\mu_i\mu_i} < 0$, $\lim_{k_i \to 0} R_{ik_i}(\cdot) = \infty$, $\lim_{k_i \to 1} R_{ik_i}(\cdot) = 0$, $\lim_{\mu_i \to 0} R_{i\mu_i}(\cdot) = -\infty$, $\lim_{\mu_i \to 1} R_{i\mu_i}(\cdot) = 0$.

Assumption A5 shows that $f(\cdot)$ is a differentiable, monotonically increasing, concave function on leisure. Assumption A6 states that R_i is a differentiable function, that ranking increases with

[8]Future research will focus on identifying a functional form that relates human capital accumulated by the whole group and relative position. See Mejía & St-Pierre (2008) for an example.

a higher investment in individual effort and that it decreases or remains constant with increases in cooperation levels. It is important to remember that individuals want a higher ranking in order to maximize utility function given by equation 5. The last two elements of the assumption are necessary in order to find an interior solution to the problem.

Now that the individuals' utility function is determined, it is possible to establish the maximization problem she faces. This problem can be seen by introducing equation 4 in 5:

$$\max_{\{k_i,\mu_i\}} U_i = \overline{w}h_i + \gamma(R_i) + f(1 - k_i - \mu_i) \tag{6}$$

A solution in this problem is constituted by a pair (k_i^*, μ_i^*) for each individual. The first order conditions of this problem are given by the following equations:

$$[k_i]: \ \overline{w}\frac{\partial h_i}{\partial k_i} + \gamma\frac{\partial R_i}{\partial k_i} - \frac{\partial f}{\partial l_i} = 0 \tag{7}$$

$$[\mu_i]: \ \overline{w}\frac{\partial h_i}{\partial \mu_i} + \gamma\frac{\partial R_i}{\partial \mu_i} - \frac{\partial f}{\partial l_i} = 0 \tag{8}$$

Assumptions A1 to A6 guarantee an interior solution to the problem. The first term of equation 7 shows the marginal benefit of increasing individual effort k_i. As it can be seen, this marginal benefit increases when the exogenous parameter $\overline{w}$ is higher and when the differential due to relative ranking (γ) is higher. The second term of equation 7 represents the marginal cost of increasing individual effort. This marginal cost is represented by the time of leisure she loses when studying.

Similarly, the first term of equation 8 shows the marginal benefit of increasing group effort. However, increasing this effort has two possible costs. The first one is associated to the increase of human capital accumulated by individual j which reduces the differential gap between both agents. The second one is the cost associated to the leisure loss.

2.3 Theoretical Results

It is now important to analyze how these decisions are modified by changes in the parameters of the model. Specifically, the main parameter of interest is the competition parameter (γ). In particular, I am interested in studying how does human capital accumulated by both individuals change if the level of competition between the agents is higher.

Proposition P1. If $\gamma_1 < \gamma_2 \Rightarrow k_{1,i} < k_{2,i}$. Higher values of γ increase individual effort k_i.

Proof: *See the appendix.*

Proposition P2. If $\gamma_1 < \gamma_2 \Rightarrow \mu_{1,i} \geq \mu_{2,i}$. Higher values of γ decrease group effort μ_i.

Proof: *See the appendix.*

The intuition behind P1 is that when γ is higher, there are more incentives for individuals to compete. As the differential due to competition increases there is a higher incentive to increase the levels of individual effort in order to have a higher ranking. Proposition P2 states that when competition is higher individuals decrease group work. The intuition behind this result is that the higher the differential between both agents is, the higher are the costs of increasing group effort. Both propositions lead to the following result:

Proposition P3. There is an interior optimal value of γ ($\tilde{\gamma}$) that maximizes $\sum_{i=1}^{N} h_i$

Proof: *See the appendix.*

$\frac{\partial \sum_{i=1}^{N} h_i}{\partial \gamma} = \sum_{i=1}^{N} \frac{\partial h_i}{\partial k_i} \frac{\partial k_i}{\partial \gamma} + \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial h_i}{\partial \mu_j} \frac{\partial \mu_j}{\partial \gamma}$. As it can be seen, two additive terms show the effect of increasing γ over the overall academic performance. The first term ($\sum_{i=1}^{N} \frac{\partial h_i}{\partial k_i} \frac{\partial k_i}{\partial \gamma}$) represents the marginal benefit of increasing γ, it is the increase in human capital accumulated by the whole group due to the increases in individual effort of all individuals. The second term ($\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial h_i}{\partial \mu_j} \frac{\partial \mu_j}{\partial \gamma}$) represents the marginal costs of increasing γ; namely, the decreases in human capital accumulated by the group due to the reductions of cooperation levels between agents. Given the assumptions, it is possible to conclude the existence of an optimal competition value (γ) that maximizes overall academic performance. Then, there is a trade-off between competition and cooperation in human capital formation.

Now we turn to the empirical strategy in order to test the main prediction of the model; namely, the existence of an interior optimal level of competition, and the presence of positive technological externalities in educational production functions, taking into account that individuals base their decisions on the absolute value of human capital accumulated by her, and her position in the ranking.

3 Data and Empirical Strategy

The data used for the empirical exercise comes from the international database of the OECD Programme for International Student Assesment (PISA) of 2000. In PISA (2000), fifteen-year old students from participating countries are evaluated in three different areas: mathematics, reading and science[9]. This allows me to have comparable test scores across students given that they take the same test in all of the areas. The data consists of a cross section data base in which we can see academic performance for each student. Besides test score results, PISA contains information of children, household and school characteristics. Two additional questionnaires are answered by students at the moment they take the test. This allows me to add covariates and diminish

[9]Science tests are only available in a subsample of the data.

potential omitted variables biases. In addition to student's questionnaires, PISA (2000) has a school questionnaire answered by school's principals. This allows me to add school controls which also affect educational outcomes. Thus, I am able to control for different characteristics which are arguments of the educational production function.

In addition to this information, a questionnaire of cross-curricular competencies is available. In this questionnaire, information about preferences on ranking and group work are identified. In particular, students answer whether they agree or disagree with a list of statements in the questionnaire[10]. For ranking preferences I use the students' level of agreement with the following statement: "I like to try to be better than other students". I construct two different measures of competitive attitudes at the student level. The first measure is a dummy equal to one if students agree or agree somewhat with the mentioned statement, I will call this variable $better_1$. The second measure is equal to one only if the student agrees with such statement. This variable is named $better_2$. For cooperative preferences I use the statement of how much they like to help other people do well in a group. Similarly to competitive attitudes I construct two measures for cooperative attitudes. The variable $help_1$ is a dummy equal to one if students agree or agree somewhat with the statement, and equal to zero otherwise. The second measure, called $help_2$, is a dummy equal to one if the student agrees with the statement and zero otherwise.

Apart from the statements chosen for the measures I developed, there are other questions related to competitive and cooperative attitudes. PISA (2000) data base contains competitive and cooperative learning indexes based on these questions[11]. The problem of using this information is that it contains elements which correspond to students' decisions instead of cooperative or competitive preferences. For example, the competitive index includes the question that asks whether students learn more if they try to do better than their classmates. Even though this information is related with competitive attitudes, it corresponds to an outcome based on students' effort decisions. Therefore, including them in the analysis would be problematic given that preferences and agents' decisions won't be differentiated in the empirical analysis.

PISA tests have been applied every two years since 2000. However, cooperative and competitive attitude questions are only available for 2000 and 2003. For 2003, these questions are specific to the area of mathematics. Due to a change in the questions' wording and the lack of information for other years, empirical exercises are carried out only for the 2000 data. Finally, the cross-curricular

[10]There are four possible answers: 1. Disagree 2. Disagree somewhat 3. Agree somewhat 4. Agree.

[11]There is a positive and significant correlation between these indexes and main independent variables used in this paper. Furthermore, results are robust if competition measures are constructed based on these indexes.

competencies questionnaire is only available for a subsample of participant countries. Then, our empirical analysis will focus only on those countries that have this information available[12].

It is important to mention that questions used to construct cooperative and competitive attitudes are not mutually exclusive. Hence, one student can agree that she tries to perform better than others and that she likes others to do well in the group. However, as I explained before, I'm not interested in cooperation or competition alone, but in how are they related, this is why, it is necessary to construct measures of competition in which there exists a trade-off between competitive and cooperative decisions. Specifically, γ captures the relative importance of competitive attitudes. Then, our variable of interest is the proportion of competitive attitudes relative to cooperative ones in a given group. Using the proportion of these constructed variables allows me to construct these relative measures. $Competition_s = \frac{Group_{better_s}}{Group_{better_s}+Group_{help_s}}$ for $s = \{1, 2\}$, where $Group_{better_s}$ corresponds to the fraction of individuals in the group g for whom the variable $better_s$ is equal to one, and $Group_{help_s}$ to the fraction of students for whom the variable $help_s$ is equal to one. With these variables it is possible to construct a measure of the degree of competitiveness in a given group. The following equation shows the relationship to be estimated:

$$\begin{aligned} y_{i,h,g,s,c} = \beta f(competition_{g,s,c}) + \alpha_1 C'_{i,h,g,s,c} + \alpha_2 H'_{h,g,s,c} \\ + \alpha_3 E'_{g,s,c} + \alpha_4 S'_{s,c} + \eta_c + \varepsilon_{i,h,g,s,c} \end{aligned} \tag{9}$$

Where $y_{i,h,g,s,c}$ corresponds to the educational outcome of individual i, of household h, belonging to group g in school s in country's region c. $f(competition_{g,s,c})$ is a flexible functional form of competitive environments generated by other members of the same group. Motivated by the theoretical model, I am assuming a non-linear functional form of competition on academic performance. $C'_{i,h,g,s,c}$ is a vector of child characteristics; $H'_{h,g,s,c}$ a vector of household characteristics; $E'_{g,s,c}$ are characteristics of the environment of the group such as discipline and good relations with teachers; $S'_{s,c}$ is a vector of school characteristics. All these vectors correspond to arguments of the educational production function presented in section 2.1 and introduced in equation 1. η_c is a country-region fixed effect that captures characteristics of this geographic unit that can alter educational outcomes, and $\varepsilon_{i,h,g,s,c}$ is an error term. Tables 1 and 2 present summary statistics of test scores, competition variables, and covariates included in equation 9, and figure 1 test scores distribution for mathematics and reading.

Deriving empirical conclusions in this setting has two main problems. The first one is potential endogeneity presented in competitive attitudes due to self-selection. The second problem is the

[12] Countries for which competitive and cooperative questions are available include: Albania, Austria, Belgium, Brazil, Bulgaria, Chile, Czech Republic, Denmark, Finland, Germany, Hong Kong, Hungary, Ireland, Israel, Italy, Republic of Korea, Latvia, Liechtenstein, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, Romania, Russian Federation, Sweden, Switzerland, Thailand, Macedonia, United Kingdom and United States.

functional form of the effect of competition over test scores. In order to solve the first problem, I propose an instrumental variables estimation. For the second one, it is necessary to assume a particular functional form. Given the theoretical results, I am assuming a quadratic functional form, which will show an inversed U-relationship between group competition and educational outcomes. This functional form is motivated by the theoretical conclusions. Additionally, I perform a non-parametric regression in order to evaluate if this functional form is appropriate.

The main problem in estimating equation 9 is that different sources of endogeneity can bias the coefficients of interest. The first one, as mentioned in section 1, corresponds to selection bias: the allocation to each one of the groups can be correlated with unobserved characteristics not included in the vector $X'_{i,h,s,p,c}$. For example, individuals with higher initial ability parameters can become more competitive due to higher academic standards. Thus, competitive environments can be correlated with higher initial abilities. Hence, estimation of equation 9 can be biased. In order to solve these issues, it is necessary to identify an exogenous source of variation that explains competition environments within a given group, but that is not correlated with individual $i's$ educational outcomes. For this purpose I will use the variation coming from the presence of younger, older and no siblings in the others children's households[13]. There are intuitive reasons pointed out by economic literature to support the presence of positive and negative externalities for having siblings. For example, Knoef & Kooreman (2011) develop a structural model to show how the presence of siblings affects cooperative and competitive decisions within the household. There are intuitive reasons to think that daily contact and resource-sharing positively influence pro-social behavior, or that competition for parents' time and economic resources will increase competition in the classroom. Then, it is expected that individuals with younger, older, same-age or no siblings have different competitive or cooperative attitudes. However, it is not intuitive to think that individual educational outcomes may be affected by the presence of siblings in classmates' households. Due to these reasons, I will use the fraction of students for each group who has older, younger, same age or no siblings in a first stage estimation which predicts exogenous values of competition levels. The following equations show the parameters to be estimated:

$$\begin{aligned} Competition_{g,s,c} &= \delta_0 + \delta_1 group\ \ younger_{g,s,c} + \delta_2 group\ \ older_{g,s,c} \\ &+ \delta_3 group\ \ same\ \ age_{g,s,c} + \delta_4 no\ \ siblings_{g,s,c} + \Delta' X + \varepsilon_{i,g,s,c} \end{aligned} \tag{10}$$

Where $Competition_{g,s,c}$ corresponds to the competition measure previously described and faced by individuals in the same group; $group\ \ younger_{g,s,c}$, $group\ \ older_{g,s,c}$, $group\ \ same\ \ age_{g,s,c}$, and $no\ \ siblings_{g,s,c}$ are the fraction of the group who has younger, older, same age, and no siblings

[13] It is important to state that I am adding the presence of these siblings in each student's own household as covariates.

respectively; X corresponds to a vector of covariates (the same as those presented in equation 9); finally, $\varepsilon_{i,g,s,c}$ is an error term. Using the predicted values of equation 10 in equation 9, I am able to identify an exogenous source of variation of competition levels faced by each student in order to identify the effect of competition on academic performance.

4 Results

Figure 2 presents a visual motivation of the question addressed in this paper. The graph shows the mean test score for each area of knowledge for each competition level[14]. The graph shows that for lower competition levels, increases in competition generate positive effects on educational outcomes. However, for higher levels of it, it has a detrimental effect. Specifically, this result is consistent for the three areas of knowledge. Even though this result is interesting, it does not represent any formal analysis that tests the main hypothesis presented in the previous section.

OLS estimations for equation 9 for mathematics and reading test scores are presented in table 3 for measure 1 of competition[15]. Column 1 presents a correlation between competition and test scores, column 2 adds individual covariates, column 3 household characteristics, column 4 region fixed effects, column 5 school environment covariates and column 6 school characteristics. As it can be seen, there are two effects in opposite directions. On the one hand, there is a positive significant effect of the linear term. An increase in one percentage point of competition increases mathematic test scores between 0.006 and 0.010 standard deviations, and reading test scores between 0.004 and 0.008 standard deviations. On the other hand, there is a negative significant effect of the quadratic term. Estimations show an effect between -0.008 and -0.014 standard deviations for mathematics, and between -0.009 and -0.014 standard deviations for reading test scores.

In addition, the table presents the optimal level of competition that maximizes academic test scores for each one of these estimations[16]. It can be seen that there is an interior solution for the optimal value of competition that maximizes academic performance. This value range between 0.318 and 0.401 for mathematic test score and between 0.168 and 0.34 for reading test score. In the same way, the table presents two hypotheses for evaluating the existence of an interior optimal value of competition. The first one, is that $\gamma = 0$ which represents that in proportion everyone cooperates and nobody competes. The second one is $\gamma = 1$ which represents the opposite. The

[14] We define competition as the fraction of individuals who compete relative to those who cooperate in the group. Specifically, $Competition = \frac{\sum better_1}{\sum better1 + help_1}$

[15] Results for science test scores are no statistically significant. For the second measure of competition, I also find a non linear effect of competition on academic performance. These results are no presented due to space reasons, but available upon request.

[16] Optimal values of competition are denoted γ and equal to $\frac{\beta_1}{2\beta_2}$. Where β_1 and β_2 are the coefficients associated to the linear and the quadratic term of competition respectively.

table shows that in all specifications for both, mathematics and reading test scores, it is possible to reject the null hypothesis that this value is equal to zero or equal to one. Then, it is possible to conclude the existence of an interior value of competition that maximizes academic performance. In general, we find that in order to maximize human capital accumulated by the group, students should study individually only the 40% for mathematics and the 36% for reading of the whole time they dedicate to study.

One possible problem of the empirical results presented in table 3 is the functional form I am assuming on the effect of competition on academic performance. In order to show the pertinence of this functional form, I perform a non parametric estimation, specifically, a local linear regression of this effect. The results can be seen in figure 3. The graph shows that a quadratic functional form with the signs of the linear and the quadratic term of table 3 is appropriate. In particular, the graph shows an inversed U-relationship between competition and academic performance for mathematics test scores, which is consistent with the empirical results found for the quadratic functional form.

Even though, these estimations suggest that the theoretical conclusions derived in section 2 actually operate, it is necessary to consider potential endogeneity issues as were described in the previous section. In order to diminish this endogeneity, I propose an instrumental variables estimation using children's interactions at home as an exogenous source of variation for competition between students. Then, the first step is to test whether having older, younger, same age or no siblings is correlated with cooperative and competitive decisions in the classroom. Table 4 presents first stage estimations for the linear and the quadratic term of competition using as instruments the fraction of the classroom who has no siblings and those who has older siblings[17]. The results are robust and similar throughout all empirical specifications. In general, column 6 shows that an increase in one percentage point in the fraction of students with no siblings increases the linear term of competition in 0.00025 points and the quadratic term in 0.00022 points respectively. We find the opposite, but weaker relationship between the proportion of children who has older siblings and competition variables. Specifically a negative effect on the linear term and a positive one on the quadratic term. Even though this effect is not significant in column 6 it is similar to estimations in other specifications. It can be concluded from these results that only children are relatively more competitive than their peers, which causes higher values of competition faced by the students of their class-rooms. Finally, the table presents the F-test for excluded instruments. The null hypothesis of this test is that instruments' coefficients are equal to zero. As it can be

[17]Results are robust to different combinations of the initial four instruments.

seen, for both, the linear and the quadratic term, the F-test is higher than ten which means that these instruments explain the endogenous variables and are jointly statistically significant. Thus, they are relevant enough to be considered as good instruments for competition.

The next step is to estimate the relationship between competition and academic performance using the instrumental variables approach. Results for the first measure of competition on mathematics and reading test scores are presented in table 5. As it can be seen, the coefficients of both, the linear and the quadratic term are significantly higher in absolute terms. Panel A shows the results for the mathematics test scores. The results show that an increase in one percentage point of competition level increases mathematics test scores in approximately 0.709 standard deviations linearly and decreases this same test score in 0.772 standard deviations quadratically. These results are consistent with the idea of a nonlinear effect of competition over academic performance. Similarly, panel B presents the results for reading test scores. Like mathematics, I find a positive effect of the linear term of 0.746 standard deviation and a negative effect of the quadratic term of 0.768 standard deviations in column 6.

It is important to explain the reasons why coefficients of both, the linear and the quadratic terms, are higher. An instrumental variables empirical strategy is a local estimation over those competition levels that vary due to changes in the instruments. The values of the predicted competition variable range between 0.35 and 0.55, as it can be seen in figure 4, then this result is a local estimation of a predicted variable with standard deviation significant lower than the original competition variable. Furthermore, the table shows that the optimal competition values don't change too much from the OLS estimation of table 3 to the IV estimation of table 5. Specifically, with the instrumental variables approach, I find that the optimal values of competition are approximately equal to 0.449 for mathematics and 0.486 for reading test scores respectively. This means that only the 44% and 48% of the students who compete and cooperate should compete in order to maximize mathematics and reading test scores of the whole group. The table 5 also shows the hypotheses that this optimal value is equal to zero or equal to one. Most of all specifications, excepting column 2, show that both null hypothesis can be rejected at 5% of significance level. Then, it can be concluded the existence of an interior competition value that maximizes total academic performance as it is predicted by proposition P3.

In addition to these results, table 5 presents the endogeneity test of the competition in the empirical specification of equation 9. The null hypothesis of this test is that the variable of interest is exogenous. The results show that in all the six columns and for both, mathematics and reading test scores, it is possible to reject this null hypothesis. Thus, it is possible to conclude that an instrumental variables empirical strategy is good. In the same way, I present the Hansen statistic for testing if the instruments are effectively exogenous[18]. The null hypothesis is that both instruments effectively are. The table shows that for both, mathematics and reading, it is only possible to reject this null hypothesis in the columns 1 and 2. Then, it can be concluded that when enough covariates are added the presence or absence of siblings in classmates' household is exogenous to individual academic performance. Finally, figure 4 presents a non-parametric estimation of the predicted values of competition on academic performance. As it can be seen in the graph, there is strong evidence that the quadratic functional form I am assuming is appropriate to estimate this relationship[19].

Hitherto, two different empirical strategies are consistent with the theoretical results derived in section 2. An OLS and an IV strategies show a non linear effect of competition over academic performance and the existence of an interior optimal value of competition in order to maximize human capital accumulated by the whole group. It is important to think if the intuitive arguments of the theoretical model are consistent with this result. My main argument is technological due to the presence of positive externalities in educational production functions. If this is the channel through which competition may have a detrimental effect over test scores results, then it is expected that the optimal competition values for those in the lower part of the test scores distribution are lower than those of the higher parts. This, due to the fact that the ones who benefit the most of these externalities are the former ones.

Table 6 shows the optimal competition values for mathematics and reading test scores, separating the sample between those students above and below the median value of the dependent variable, for an OLS estimation. The competition variable is constructed using the answers of all students in the group. As it can be seen, in most of all the specifications, and for both areas, the optimal competition values are lower for those below the median value. This is consistent with the presence of positive technological externalities in educational production functions. In column 6, panel A and B of table 6 show that the optimal competition value for mathematics for those below the median is equal to 0.32, while for those above is equal to 0.43. Furthermore, it can be seen that the null hypothesis that the optimal value of competition γ is equal to zero or to one is rejected for students below the median, but it can't be rejected for those above. For reading test scores, that corresponds to panel C and D of table 6, the results I find are very similar. In particular, column 6 shows an optimal value of 0.21 for those below and of 0.35 for those above. However, both null hypotheses can be rejected for all groups through all specifications.

[18] For doing these tests it is necessary to include more instruments than endogenous variables. I am adding the fraction of the group who has younger siblings.

[19] The predicted values of a first stage have to be used linearly in a second stage. Other functional form biased the coefficients. This is known as the *forbidden* regression problem. However this graphic show intuitively that it is correct to assume a quadratic functional form.

The same result using an IV approach is presented in table 7. The table shows the same pattern of the OLS estimations of table 6. However, optimal values of competition are slightly higher. For those below the median of the dependent variable is approximately equal to 0.41 and for those above 0.52, as can be seen in column 6 of table 7 in panels A and B. The null hypothesis that this optimal value is equal to one can be rejected for those below the median value, but not for those above. Results for reading test scores, in panels C and D, show the same pattern of lower optimal competition values for those in the left part of the test scores distribution. However, standard errors of these values are higher for this group.

In summary, two different empirical strategies, an OLS and an IV, are consistent with proposition P3 of the theoretical model presented in section 2: the existence of an interior optimal value of competition in order to maximize human capital accumulated by the whole group. We find strong evidence of the presence of positive technological externalities separating the sample between those in the higher and the lower parts of the test scores distribution. This result verifies the intuitive arguments presented in the theoretical model, and the presence of positive technological externalities in educational production functions.

5 Discussion

The allocation of students has been one important topic for parents, schools and policy makers. Given the fact that peer effects exist, it is important to establish how groups must be conformed in order to maximize each student's potential. This paper shows that it is important not only how they are allocated, but also how incentives are designed in order to maximize educational outcomes and interactions among students. The parameter and variable of interest in this paper was competition between the members of a group. Even tough this parameter is given by students' utility functions, it is important to notice that teachers, funding systems and scholarship programs, and university and college admissions processes have direct impact on how these preferences are modified.

Specifically, teachers can modify students' preferences through grading systems. There is no clear evidence on what conditions influence some grading systems to be more efficient than others[20]. However this paper presents insights about it. In general, we can find three different grading

[20]Literature has shown evidence favoring cooperative systems over neutral ones. Aliye & Erdem (2009) finds positive effects of cooperation in test scores and other variables such as: positive relations between different ethnics, mutual concern between students and self-esteem. In the same way, evidence in high schools has been found (Astin, 1993; Kadel et al., 1992; Johnson et al., 1991).In relation to higher education, Yamarik (2007) show higher test scores in macroeconomic test scores of students who work in group. In the other hand, it is not clear which system is most efficient between neutral systems and competitive ones. Becker & Rosen (1992) argue that competition among students increases effort if that effort is rewarded. Under this perspective, if loss risk is too high, individual effort can diminish.

systems. First, systems where student grades depend exclusively on their individual performance on academic tests. Second, systems promoting group work and cooperation among students, in which grades are the result of aggregated decisions of group members. Finally, a third grading system, based on relative positions occupied by students in their group, also called a tournament grading system. Those grading systems are related to the competition level students are facing. In particular, systems based on individual performance are neutral because grades are not affected by peers. In the same way, systems encouraging group work can be interpreted as cooperative ones, in which all members of a particular group have the same grade. Analogously, tournament grading systems, in which individuals show concern for their ranking, constitute competitive systems due to inequalities in students' grades. Thus, teachers can modify individual preferences through changes in the grading system.

School financing, scholarship programs, and college and university admissions processes should consider results derived in this paper for a specific reason: increasing competition levels between students beyond a certain level can diminish human capital accumulation. This is related to incentives generated by the educational system. Specifically we should wonder how human capital accumulated and test scores results are modified if we change students preferences. This paper shows that at some points increasing concerns for rankings would be negative for educational outcomes, especially for those in the lower percentiles of the initial ability distribution, as they do not benefit from the positive externalities of cooperation from their peers.

An additional important point is that this paper only considers a mechanism in which cooperative attitudes from students affect learning: technological positive externalities in educational production functions. However, it is important to point out that cooperative attitudes from students can positively alter other factors in the production of learning. In general, cooperative or pro-social actions in a classroom can be related to better relationships among students and higher levels of confidence and self-esteem. Taking these elements into account and the effect they can have over learning, it would be important to determine what is the optimal level of competition between individuals that maximizes human capital acquired by a particular group.

Finally, it is important to test the effects of competition over academic performance under other contexts. Data included in this analysis is based on fifteen year-old students from countries evaluated by PISA. It is important to state if the findings of this paper are similar in other groups. For example, to test whether the same effects operate for younger kids. Increasing cooperation levels for them can have effects besides improvements in learning that can be positive for their later life. Similarly, testing whether cooperative or competitive environments have different effects over learning in developing countries, can be used to identify lower costs mechanisms to increase human capital in these societies.

6 Conclusions

Most economic analyses of peer effects have focused on empirical strategies that allow for an unbiased estimation of these effects. Nevertheless, empirical evidence on the mechanisms through which they operate is still scarce. Some have pointed out the importance of different agents, their decisions and their effects on educational outcomes. This article considers a direct mechanism in which peers can affect a student's educational outcomes: cooperation and competition between them. A theoretical analysis shows that individual effort levels depend positively on individual concern for ranking and cooperation levels depend negatively on the same dimension. The existence of positive technological externalities in educational production functions and the importance of individual effort decisions in educational outcomes causes the existence of an optimal level of competition for maximizing total human capital accumulated by the group. Using data from PISA (2000) in which students are tested in mathematics, reading and sciences, and in which they report cooperation and competition preferences, I find empirical evidence that supports the theoretical conclusion.

The main aim of this paper is to show a non-linear effect of competition on educational outcomes. I find robust evidence of an inversed U-relationship between competition and academic performance. Specifically, I find an interior optimal value of competition that maximizes academic results. However, this paper does not focus on the determinants of this optimal value. In particular, it is important to analyze how this optimal value depends on the parameters of the initial ability distribution of the group. Future research will focus on this topic[21].

In conclusion, this paper tries to determine one possible mechanism through which peer effects operate: cooperation and competition between classmates. It is possible to show the existence of positive technological externalities between group members, due to the fact that increases in competitive attitudes or concern for ranking can have a detrimental effect on learning.

[21] I expect to find a negative relationship between optimal competition level and the mean of the initial ability distribution. Positive externalities on educational production functions should be higher for groups with a greater initial ability mean. Then, it is necessary to encourage more cooperative attitudes among the members of these groups. I also want to find a relationship with initial ability dispersion. I expect an inversed U-relationship between optimal level of competition and this parameter. The reasons for this relationship is that, Mejía & St-Pierre (2007) hold that, under tournament systems individuals exert higher levels of effort if the group they are competing with is more homogeneous. Then, if the degree of competition is constant increases in ability dispersion would have a detrimental effect on individuals' effort levels. To compensate this negative effect, the optimal competition level should be higher. However, if this dispersion is too high, individuals perceive that changes in effort levels would not change their relative ranking, and then, increases in competition would only diminish positive externalities. Due to these reasons, it is necessary to decrease optimal competition level to obtain increases in the positive externalities of educational production functions.

This is particularly important, not only for the allocation of students but also for the mechanism designs which can alter their preferences. It can be concluded that it is important to establish a correct design of grading systems and incentives within the educational system. Finally, it is essential to analyze the benefits and costs in terms of educational outcomes resulting from increases in competitive preferences. This should be a concern for parents, teachers, schools and policy makers.

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Table 1: Summary Statistics for Test Scores, Competition Variables, and Individual Covariates.

Variable	Obs	Mean	Std. Dev.	Min	Max
Test scores					
Mathematics test score	78,586	483.715	107.377	151.67	815.9
Reading test score	78,616	482.570	105.930	22.24	890.18
Science test score	31,570	485.440	98.885	168.6	801.85
Competition variables					
$better_1$ (%)	78,031	0.623	0.485	0	1
$help_1$ (%)	77,389	0.759	0.427	0	1
$better_2$ (%)	78,031	0.238	0.426	0	1
$help_2$ (%)	77,389	0.282	0.450	0	1
$Competition_1$ (Author calculation)	78,499	0.445	0.115	0	1
$Competition_2$ (Author calculation)	75,176	0.448	0.254	0	1
Individual covariates					
Younger siblings (%)	78,157	0.639	0.480	0	1
Older siblings (%)	78,157	0.642	0.479	0	1
Same age siblings (%)	78,835	0.029	0.168	0	1
Age (months)	78,619	187.974	4.076	171	195
Familiy structure single (%)	78,206	0.144	0.351	0	1
Familiy structure nuclear (%)	78,206	0.753	0.431	0	1
Familiy structure mixed (%)	78,206	0.065	0.246	0	1
Familiy structure other (%)	78,206	0.038	0.191	0	1
Number of siblings	78,157	1.850	1.388	0	12
Native (%)	75,542	0.934	0.248	0	1
Mom native (%)	75,360	0.879	0.327	0	1
Dad present (%)	78,835	0.889	0.314	0	1
Native language at home (%)	78,835	0.821	0.384	0	1
First child (%)	78,157	0.361	0.480	0	1
Second child (%)	78,157	0.220	0.415	0	1
Third or more child (%)	78,157	0.333	0.471	0	1
Socio economic index	75,423	43.043	16.653	16	90
Highest socio economic index between mom and dad	75,423	47.907	16.695	16	90
Mom level of education	75,364	4.248	1.440	1	6
Instrumental motivation index	78,692	0.077	0.984	-2.44	1.48
Interest in reading index	78,670	0.092	0.975	-1.8	1.76
Interest in math index	78,835	0.234	3.474	-1.93	97
Control strategies index	78,748	0.058	0.985	-3.56	2.61
Memorizing index	78,681	0.078	0.976	-2.82	2.61
Elaboration strategies index	78,684	0.099	0.999	-2.8	2.49
Self concept (verbal) index	78,664	0.028	0.994	-2.62	1.81
Maths self concept index	78,132	0.024	0.989	-1.62	1.74
Academic self concept index	78,788	0.017	0.996	-2.51	1.85
Self efficacy index	78,748	0.025	0.989	-2.9	2.28
Control expectations index	78,728	0.054	0.988	-3.38	2.24

Source: PISA (2000). Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Index corresponds to warm-estimates calculated by OECD researchers. For additional information consult PISA (2000) Technical Report.

Table 2: Summary Statistics for Household, Academic Environment and School Covariates

Variable	Observations	Mean	Std. Dev.	Min	Max
Household characteristics					
Parental academic interest index	77,763	0.044	1.027	-2.2	2.72
Cultural possession index	78,195	0.029	0.985	-1.65	1.16
Wealth index index	78,692	-0.401	1.183	-5.05	3.38
Home educational resources index	78,634	-0.260	1.164	-5.93	0.76
Academic Environment					
Time spent on homework index	77,946	0.089	1.016	-2.35	2.18
Teacher support index	78,255	0.033	0.961	-3.03	1.95
Achievement press index	78,298	0.062	1.015	-4.35	2.75
School disciplinary climate index	78,324	-0.058	0.989	-2.92	2.96
Teacher-student relationship index	78,259	0.099	1.026	-2.9	2.84
Sense of belonging index	78,253	-0.009	0.979	-3.4	2.33
School Characteristics					
Total number of schooling hours per year	71,069	929.076	192.401	120	1,680
Number of students in the school	74,207	800.942	623.384	2	9815
Percentage of girls in the school (%)	74,316	0.514	0.203	0	1
Public school (%)	74,485	0.855	0.352	0	1
School autonomy index	74,788	-0.053	0.976	-3.22	1.72
Teacher participation to decision making index	74,788	-0.028	0.982	-1.57	4.45
Teacher behaviors index	76,706	-0.025	1.025	-2.41	3.64
Teacher morale index	76,765	-0.089	0.976	-3.4	1.78
Instructional resources index	76,769	0.166	1.093	-1.9	3.22
Material resources index	76,624	-0.026	0.990	-1.12	3.38
Shortage of teachers index	76,428	0.030	1.022	-0.95	3.47

Source: PISA (2000). Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Index corresponds to warm-estimates calculated by OECD researchers. For additional information consult PISA (2000) Technical Report.

Table 3: OLS Estimations of Competition on Academic Performance

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Mathematic Test Scores						
Competition	0.010***	0.006***	0.009***	0.008***	0.007***	0.007***
	[0.002]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Competition^2	-0.014***	-0.010***	-0.011***	-0.010***	-0.009***	-0.008***
	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Observations	73,235	68,956	68,956	68,073	67,144	54,870
Optimal value ($\widehat{\gamma}$)	0.372	0.318	0.386	0.394	0.383	0.401
Standard error	0.010	0.010	0.010	0.010	0.010	0.020
p-value $\gamma = 0$	0.000	0.000	0.000	0.000	0.000	0.000
p-value $\gamma = 1$	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Reading Test Scores						
Competition	0.007***	0.004**	0.008***	0.008***	0.006***	0.007***
	[0.002]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Competition^2	-0.014***	-0.010***	-0.013***	-0.012***	-0.010***	-0.009***
	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Observations	73,272	69,014	69,014	68,154	67,232	54,994
Optimal value ($\widehat{\gamma}$)	0.242	0.168	0.321	0.327	0.307	0.364
Standard error	0.010	0.010	0.010	0.010	0.010	0.020
p-value $\gamma = 0$	0.000	0.000	0.000	0.000	0.000	0.000
p-value $\gamma = 1$	0.000	0.000	0.000	0.000	0.000	0.000
Child characteristics		Yes	Yes	Yes	Yes	Yes
Region fixed effects			Yes	Yes	Yes	Yes
Household characteristics				Yes	Yes	Yes
Academic environment					Yes	Yes
School characteristics						Yes

Note: PISA (2000). Beta standardized coefficients presented for an increase of competition in one percentage point. Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Clustered standard errors at group level presented in brackets. Standard error for optimal values estimated with delta method. *** p<0.01, ** p<0.05, * p<0.1.

Table 4: First Stage for Competition Levels

	(1)	(2)	(3)	(4)	(5)	(6)
			Panel A: Competition			
No siblings	0.027***	0.019***	0.027***	0.026***	0.027***	0.025***
	[0.005]	[0.005]	[0.005]	[0.005]	[0.005]	[0.005]
Older Siblings	-0.013***	-0.016***	0.001	0.001	0.000	-0.006**
	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]
Observations	74,801	70,417	70,417	69,499	68,537	56,001
F-statistic	51.06	37.52	17.55	17.06	18.07	21.02
			Panel B: Competition^2			
No siblings	0.029***	0.026***	0.024***	0.024***	0.023***	0.022***
	[0.004]	[0.005]	[0.004]	[0.004]	[0.004]	[0.005]
Older Siblings	-0.004	-0.008***	0.006**	0.006**	0.005*	0.001
	[0.002]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]
Observations	74,801	70,417	70,417	69,499	68,537	56,001
F-statistic	39.00	33.62	14.09	13.91	13.08	11.61
Child characteristics		Yes	Yes	Yes	Yes	Yes
Region fixed effects			Yes	Yes	Yes	Yes
Household characteristics				Yes	Yes	Yes
Academic environment					Yes	Yes
School characteristics						Yes

Note: PISA (2000). Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Robust standard errors presented in brackets. *** p<0.01, ** p<0.05, * p<0.1.

Table 5: IV Estimations of Competition on Academic Performance

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Mathematic Test Scores						
Competition	0.737***	0.468***	1.004***	0.912***	0.866***	0.709**
	[0.206]	[0.180]	[0.365]	[0.342]	[0.328]	[0.278]
$Competition^2$	-0.627***	-0.328*	-1.031**	-0.929**	-0.911**	-0.772**
	[0.240]	[0.192]	[0.406]	[0.378]	[0.388]	[0.383]
Observations	73,229	68,956	68,956	68,073	67,144	54,870
Optimal value ($\widehat{\gamma}$)	0.588	0.714	0.487	0.491	0.475	0.459
Standard error	0.15	0.47	0.13	0.14	0.14	0.16
p-value $\gamma = 0$	0.000	0.128	0.000	0.001	0.001	0.004
p-value $\gamma = 1$	0.005	0.541	0.000	0.000	0.000	0.001
Hansen Test	3.882	2.883	0.000	0.010	0.000	0.051
p-value	0.049	0.09	0.988	0.922	0.997	0.822
Endogeneity test	181.4	92.48	260	219.3	206.6	162.5
p-value	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Reading Test Scores						
Competition	0.675***	0.380**	1.172***	1.056***	0.982***	0.746***
	[0.202]	[0.160]	[0.437]	[0.404]	[0.372]	[0.275]
$Competition^2$	-0.549**	-0.252	-1.162**	-1.033**	-0.994**	-0.768**
	[0.228]	[0.167]	[0.478]	[0.436]	[0.426]	[0.363]
Observations	73,267	69,014	69,014	68,154	67,232	54,994
Optimal value ($\widehat{\gamma}$)	0.615	0.754	0.505	0.511	0.494	0.486
Standard error	0.19	0.65	0.15	0.16	0.15	0.16
p-value $\gamma = 0$	0.001	0.247	0.001	0.002	0.001	0.003
p-value $\gamma = 1$	0.04	0.705	0.001	0.003	0.001	0.001
Hansen Test	4.232	3.813	0.01	0.04	0.004	0.002
p-value	0.040	0.051	0.919	0.841	0.950	0.964
Endogeneity test	166.5	73.1	300.7	261	245.3	189.4
p-value	0.000	0.000	0.000	0.000	0.000	0.000
Child characteristics		Yes	Yes	Yes	Yes	Yes
Region fixed effects			Yes	Yes	Yes	Yes
Household characteristics				Yes	Yes	Yes
Academic environment					Yes	Yes
School characteristics						Yes

Note: PISA (2000). Beta standardized coefficients presented for an increase of competition in one percentage point. Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Clustered standard errors at group level presented in brackets. Standard error for optimal values estimated with delta method. *** p<0.01, ** p<0.05, * p<0.1.

Table 6: Optimal Competition Values for OLS

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Mathematics below the median value						
Optimal value ($\widehat{\gamma}$)	0.405	0.406	0.374	0.387	0.294	0.326
Standard error	0.010	0.020	0.030	0.030	0.090	0.130
p-value $\gamma = 0$	0.000	0.000	0.000	0.000	0.001	0.015
p-value $\gamma = 1$	0.000	0.000	0.000	0.000	0.000	0.000
Observations	35,925	33,350	33,350	32,651	32,031	26,664
Panel B: Mathematics above the median value						
Optimal value ($\widehat{\gamma}$)	0.338	0.483	0.400	0.425	0.432	0.432
Standard error	0.060	0.010	0.380	0.680	2.520	3.000
p-value $\gamma = 0$	0.000	0.000	0.296	0.534	0.864	0.886
p-value $\gamma = 1$	0.000	0.000	0.117	0.401	0.822	0.850
Observations	36,320	34,780	34,780	34,611	34,314	27,656
Panel C: Reading below the median value						
Optimal value ($\widehat{\gamma}$)	0.302	0.273	0.291	0.300	0.177	0.214
Standard error	0.010	0.020	0.010	0.020	0.030	0.060
p-value $\gamma = 0$	0.000	0.000	0.000	0.000	0.000	0.000
p-value $\gamma = 1$	0.000	0.000	0.000	0.000	0.000	0.000
Observations	35,508	32,846	32,846	32,151	31,474	26,284
Panel D: Reading above the median value						
Optimal value ($\widehat{\gamma}$)	0.029	-0.541	0.337	0.331	0.309	0.351
Standard error	0.060	0.040	0.060	0.080	0.140	0.160
p-value $\gamma = 0$	0.624	2.000	0.000	0.000	0.026	0.033
p-value $\gamma = 1$	0.000	0.000	0.000	0.000	0.000	0.000
Observations	36,755	35,327	35,327	35,179	34,945	28,149
Child characteristics		Yes	Yes	Yes	Yes	Yes
Region fixed effects			Yes	Yes	Yes	Yes
Household characteristics				Yes	Yes	Yes
Academic environment					Yes	Yes
School characteristics						Yes

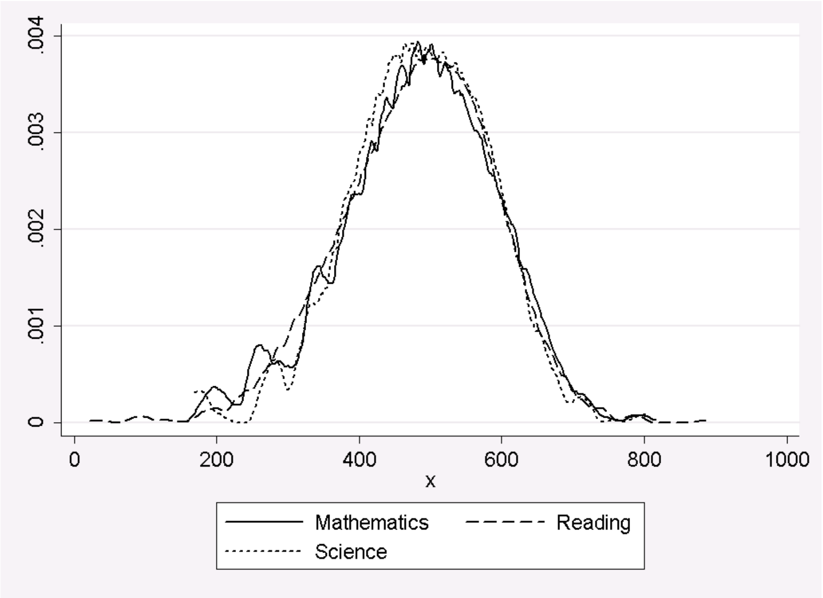
Note: PISA (2000). Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Clustered standard errors at group level. Standard error for optimal values estimated with delta method.

Table 7: Optimal Competition Values for IV

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Mathematics below the median value						
Optimal value ($\widehat{\gamma}$)	1.38	15.91	0.50	0.52	0.46	0.41
Standard error	20.53	501,488	0.14	0.15	0.17	0.33
p-value $\gamma = 0$	0.95	1.00	0.00	0.00	0.01	0.21
p-value $\gamma = 1$	1.02	1.00	0.00	0.00	0.00	0.07
Observations	35,921	33,350	33,350	32,651	32,031	26,664
Panel B: Mathematics above the median value						
Optimal value ($\widehat{\gamma}$)	0.60	0.64	0.51	0.52	0.52	0.52
Standard error	1.96	1.80	0.80	0.82	0.51	0.47
p-value $\gamma = 0$	0.76	0.72	0.52	0.53	0.31	0.27
p-value $\gamma = 1$	0.84	0.84	0.54	0.55	0.35	0.31
Observations	36,319	34,780	34,780	34,611	34,314	27,656
Panel C: Reading below the median value						
Optimal value ($\widehat{\gamma}$)	0.89	1.88	0.45	0.47	0.43	0.39
Standard error	2.92	78.75	0.48	0.50	0.70	0.88
p-value $\gamma = 0$	0.76	0.98	0.36	0.35	0.54	0.66
p-value $\gamma = 1$	0.97	1.01	0.25	0.29	0.41	0.49
Observations	35,504	32,846	32,846	32,151	31,474	26,284
Panel D: Reading above the median value						
Optimal value ($\widehat{\gamma}$)	0.62	0.63	0.56	0.55	0.54	0.56
Standard error	0.55	0.41	0.25	0.22	0.18	0.21
p-value $\gamma = 0$	0.26	0.12	0.02	0.01	0.00	0.01
p-value $\gamma = 1$	0.48	0.36	0.07	0.04	0.01	0.03
Observations	36,755	35,327	35,327	35,179	34,945	28,149
Child characteristics		Yes	Yes	Yes	Yes	Yes
Region fixed effects			Yes	Yes	Yes	Yes
Household characteristics				Yes	Yes	Yes
Academic environment					Yes	Yes
School characteristics						Yes

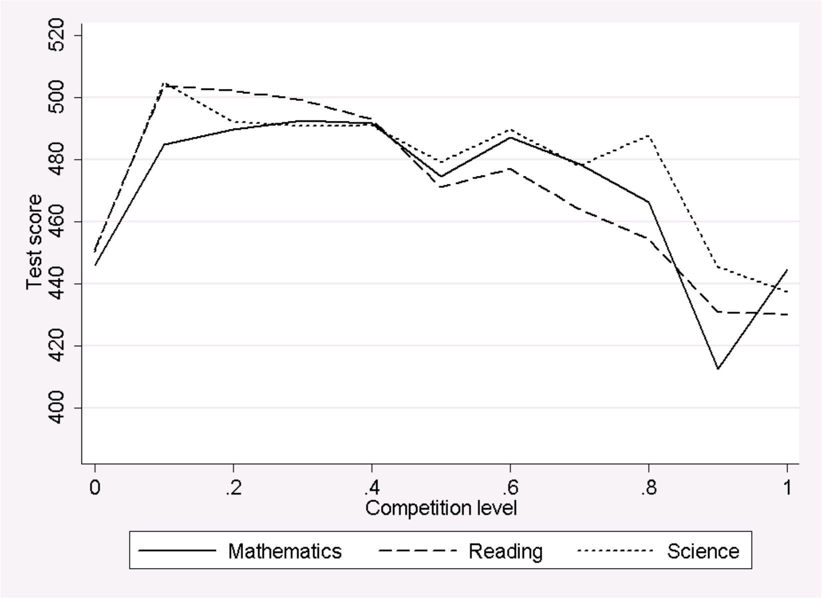
Note: PISA (2000). Unit of observations are students who presented PISA tests, respond cross-curricular competencies questionnaire and are in schools selected for the school questionnaire. Clustered standard errors at group level. Standard error for optimal values estimated with delta method.

Figure 1: Distribution of Test Scores



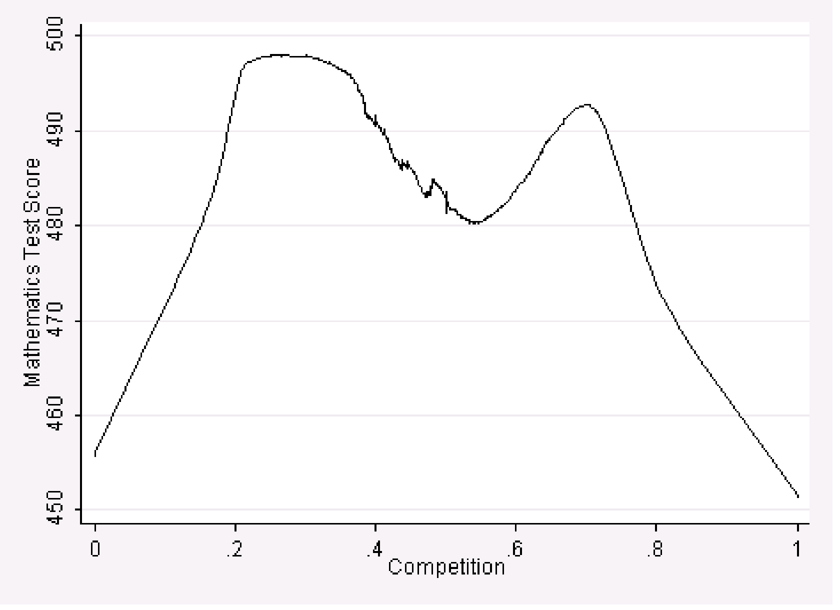
Note: Test scores distribution for each cognitive are using the whole sample.

Figure 2: Relationship Between Test Scores and Competition Levels



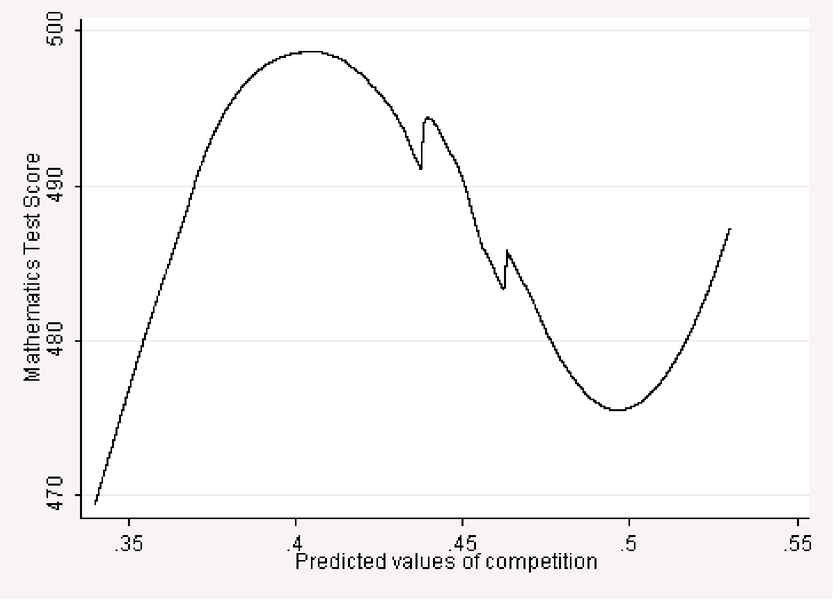
Note: Competition measure is number 1 without using instrumental variables. For each competition value the mean of each test score is graphed.

Figure 3: No-parametric Estimation of Competition on Mathematics Test Scores



Note: Test scores distribution for each cognitive are using the whole sample.

Figure 4: No-parametric Estimation of Competition Predicted Values on Mathematics Test Scores



Note: Competition measure is predicted from column 6 of table 4 using instrumental variables. The graph is shown for the 90% of the whole sample, eliminating data below the percentile 5^{th} or above the percentile 95^{th} of the predicted values of competition.

Appendix

1. **Proof of proposition P1**

If $\gamma_1 < \gamma_2 \Rightarrow k_{1,i} < k_{2,i}$

Suppose by contradiction that $k_{1,i} \geq k_{2,i}$. Then, the two following inequalities hold:

$$\begin{aligned} U(\gamma_1, k_{1,i}, \cdot) \geq U(\gamma_1, k_{2,i}, \cdot) \\ U(\gamma_2, k_{1,i}, \cdot) \leq U(\gamma_2, k_{2,i}, \cdot) \end{aligned} \tag{1}$$

From first inequality of equation 1 we have that:

$$\begin{aligned} \overline{w}h(k_{1,i}, \mu_i, \cdot) + \gamma_1 R_i(k_{1,i}, \mu_i, \cdot) + f(1 - k_{1,i} - \mu_i) \geq \\ \overline{w}h(k_{2,i}, \mu_i, \cdot) + \gamma_1 R_i(k_{2,i}, \mu_i, \cdot) + f(1 - k_{2,i} - \mu_i) \end{aligned} \tag{2}$$

By reorganizing the terms of equation 2, we have that:

$$\begin{aligned} \gamma_1(R_i(k_{1,i}, \mu_i, \cdot) - R_i(k_{2,i}, \mu_i, \cdot)) \geq \\ \overline{w}(h(k_{2,i}, \mu_i, \cdot) - h(k_{1,i}, \mu_i, \cdot)) + f(1 - k_{2,i} - \mu_i) - f(1 - k_{1,i} - \mu_i) \end{aligned} \tag{3}$$

From second inequality of equation 1 we have that:

$$\begin{aligned} \overline{w}h(k_{1,i}, \mu_i, \cdot) + \gamma_2 R_i(k_{1,i}, \mu_i, \cdot) + f(1 - k_{1,i} - \mu_i) \leq \\ \overline{w}h(k_{2,i}, \mu_i, \cdot) + \gamma_2 R_i(k_{2,i}, \mu_i, \cdot) + f(1 - k_{2,i} - \mu_i) \end{aligned} \tag{4}$$

By reorganizing the terms of equation 4, we have that:

$$\begin{aligned} \gamma_2(R_i(k_{1,i}, \mu_i, \cdot) - R_i(k_{2,i}, \mu_i, \cdot)) \leq \\ \overline{w}(h(k_{2,i}, \mu_i, \cdot) - h(k_{1,i}, \mu_i, \cdot)) + f(1 - k_{2,i} - \mu_i) - f(1 - k_{1,i} - \mu_i) \end{aligned} \tag{5}$$

From inequalities 3 and 5 we have that:

$$\gamma_2(R_i(k_{1,i}, \mu_i, \cdot) - R_i(k_{2,i}, \mu_i, \cdot)) \leq \gamma_1(R_i(k_{1,i}, \mu_i, \cdot) - R_i(k_{2,i}, \mu_i, \cdot)) \tag{6}$$

Due to the fact that R_i is an increasing function in k_i and that $k_{1,i} \geq k_{2,i}$, then $R_i(k_{1,i}, \mu_i, \cdot) \geq R_i(k_{2,i}, \mu_i, \cdot)$ which implies that $\gamma_2 \leq \gamma_1$. $\wp$ This is a contradiction. Then, $k_{1,i} < k_{2,1}$.

2. **Proof of proposition P2**

If $\gamma_1 < \gamma_2 \Rightarrow \mu_{1,i} \geq \mu_{2,i}$

Suppose by contradiction that $\mu_{1,i} < \mu_{2,i}$. Then, the two following inequalities hold:

$$\begin{aligned} U(\gamma_1, \mu_{1,i}, \cdot) \geq U(\gamma_1, \mu_{2,i}, \cdot) \\ U(\gamma_2, \mu_{1,i}, \cdot) \leq U(\gamma_2, \mu_{2,i}, \cdot) \end{aligned} \tag{7}$$

From first inequality of equation 7 we have that:

$$\begin{aligned} \overline{w}h(k_i, \mu_{1,i}, \cdot) + \gamma_1 R_i(k_i, \mu_{1,i}, \cdot) + f(1 - k_i - \mu_{1,i}) \geq \\ \overline{w}h(k_i, \mu_{2,i}, \cdot) + \gamma_1 R_i(k_i, \mu_{2,i}, \cdot) + f(1 - k_i - \mu_{2,i}) \end{aligned} \tag{8}$$

By reorganizing the terms of equation 8, we have that:

$$\begin{aligned} \gamma_1(R_i(k_i, \mu_{1,i}, \cdot) - R_i(k_i, \mu_{2,i}, \cdot)) \geq \\ \overline{w}(h(k_i, \mu_{2,i}, \cdot) - h(k_i, \mu_{2,i}, \cdot)) + f(1 - k_i - \mu_{2,i}) - f(1 - k_i - \mu_{1,i}) \end{aligned} \tag{9}$$

From second inequality of equation 7 we have that:

$$\begin{aligned} \overline{w}h(k_i, \mu_{1,i}, \cdot) + \gamma_2 R_i(k_i, \mu_{1,i}, \cdot) + f(1 - k_i - \mu_{1,i}) \leq \\ \overline{w}h(k_i, \mu_{2,i}, \cdot) + \gamma_2 R_i(k_i, \mu_{2,i}, \cdot) + f(1 - k_i - \mu_{2,i}) \end{aligned} \tag{10}$$

By reorganizing the terms of equation 10, we have that:

$$\begin{aligned} \gamma_2(R_i(k_i, \mu_{1,i}, \cdot) - R_i(k_i, \mu_{2,i}, \cdot)) \leq \\ \overline{w}(h(k_i, \mu_{2,i}, \cdot) - h(k_i, \mu_{1,i}, \cdot)) + f(1 - k_i - \mu_{2,i}) - f(1 - k_i - \mu_{1,i}) \end{aligned} \tag{11}$$

From inequalities 9 and 11 we have that:

$$\gamma_2(R_i(k_i, \mu_{1,i}, \cdot) - R_i(k_i, \mu_{2,i}, \cdot)) \leq \gamma_1(R_i(k_i, \mu_{1,i}, \cdot) - R_i(k_i, \mu_{2,i}, \cdot)) \tag{12}$$

Due to the fact that R_i is a decreasing function in μ_i and that $\mu_{1,i} < \mu_{2,i}$, then $R_i(k_i, \mu_{1,i}, \cdot) \geq R_i(k_i, \mu_{2,i}, \cdot)$ which implies that $\gamma_2 \leq \gamma_1$. $\wp$ This is a contradiction. Then, $\mu_{1,i} \geq \mu_{2,1}$.

3. **Proof of proposition P3**

$$\frac{\partial \sum_{i=1}^{N} h_i}{\partial \gamma} = \sum_{i=1}^{N} \frac{\partial h_i}{\partial k_i}\frac{\partial k_i}{\partial \gamma} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial h_i}{\partial \mu_j}\frac{\partial \mu_j}{\partial \gamma} \tag{13}$$

The first term of equation 13 is negative while the second one is positive. Then, in order to prove the existence of an interior competition value $\tilde{\gamma}$, it is necessary to prove concavity of $\sum_{i=1}^{N} h_i$ in γ.

$$\frac{\partial^2 \sum_{i=1}^{N} h_i}{\partial \gamma^2} = \sum_{i=1}^{N} \frac{\partial^2 h_i}{\partial k_i^2}\left(\frac{\partial k_i}{\partial \gamma}\right)^2 + \frac{\partial^2 k_i}{\partial \gamma^2}\frac{\partial h_i}{\partial k_i} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial^2 h_i}{\partial \mu_j^2}\left(\frac{\partial \mu_j}{\partial \gamma}\right)^2 + \frac{\partial^2 \mu_j}{\partial \gamma^2}\frac{\partial h_i}{\partial \mu_j} \tag{14}$$

By proving that equation 14 is negative we show that $\sum_{i=1}^{N} h_i$ is concave in γ. $\frac{\partial^2 h_i}{\partial k_i^2}$ and $\frac{\partial^2 h_i}{\partial \mu_j^2}$ are negative as it was shown in assumption A2. They are multiplied by squared terms so the product is also negative. Then, it is necessary to prove that $\frac{\partial^2 k_i}{\partial \gamma^2}\frac{\partial h_i}{\partial k_i}$ and $\frac{\partial^2 \mu_j}{\partial \gamma^2}\frac{\partial h_i}{\partial \mu_j}$ are also negative. $\frac{\partial h_i}{\partial k_i}$ and $\frac{\partial h_i}{\partial \mu_j}$ are assumed positive as it was shown in assumption A1. So, the problem is reduced to prove that:

$$\begin{aligned} \tfrac{\partial^2 k_i}{\partial \gamma^2} &< 0 \\ \tfrac{\partial^2 \mu_j}{\partial \gamma^2} &< 0 \end{aligned} \tag{15}$$

In order to prove the first inequality of equation 15, it is necessary to show that the second derivative of utility function presented in equation 5 depends negatively on γ. This second derivative[22] is given by:

$$\frac{\partial^2 U_i}{\partial k_i^2} = \overline{w}\frac{\partial^2 h_i}{\partial k_i^2} + \gamma\frac{\partial^2 R_i}{\partial k_i^2} + \frac{\partial^2 f}{\partial l_i^2} \tag{16}$$

Given the fact that $\frac{\partial^2 R_i}{\partial k_i^2} < 0$ as was assumed in A6, equation 16 depends negatively on γ. So the first inequality of equation 15 holds. For the second inequality the proof is analogous.

[22]Derivative of equation 7.