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Macroprudential Regulation and Misallocation*

Enoch Hill† David Perez-Reyna‡

Abstract

In this paper, we study the macroeconomic effects of banking capital requirements. We provide a theoretical explanation for why decreasing capital requirements may lead to lower average leverage ratio among banks. This counterintuitive result is an outcome of the general equilibrium effects on interest rates, which affects capital allocation across different types of banks. Additionally, we find that the optimal policy for capital requirements depends on the available equity in the banking sector. Countries with a relatively undeveloped financial sector should have a higher capital requirement. For countries in the middle the optimal policy is a relaxed capital requirement. Finally, countries with a large amount of domestic capital are unaffected by capital requirements.

JEL Classification: E44, G21, G28,
Keywords: Banking capital requirements, misallocation

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Regulación macroprudencial y asignación inadecuada de recursos

Enoch Hill       David Perez-Reyna

Resumen

En este artículo estudiamos los efectos macroeconómicos de tener requerimientos de capital para los bancos. Explicamos teóricamente por qué disminuir los requerimientos de capital puede resultar en bancos menos apalancados. Este resultado contraintuitivo es consecuencia de efectos de equilibrio general sobre las tasas de interés, que afecta la asignación de capital de diferentes tipos de banco. Adicionalmente, encontramos que la política óptima de requerimientos de capital depende del capital bancario disponible. Economías con un sector financiero relativamente subdesarrollado deberían tener mayores requerimientos de capital. Para economías con un nivel de desarrollo medio la política óptima es tener requerimientos de capital bajos. Finalmente, las economías con una alta disponibilidad de capital bancario no se ven afectadas por los requerimientos de capital.

JEL Classification: E44, G21, G28

Palabras clave: Requerimientos de capital bancario, asignación inadecuada de recursos
1 Introduction

The systemic failure of the financial sector during the Great Recession highlighted the shortcomings of existing Basel II regulation and heightened the push for general reform of macroprudential policy leading to the endorsement of Basel III in November of 2010. Throughout the proceedings, the chief motive for macroprudential regulation was and continues to be to reduce aggregate risk. Consequently, the vast majority of the literature focusing on macroprudential regulation analyzes its effects on aggregate risk.\(^1\) However, the primary function of the financial sector is to allocate capital to its most productive uses. Macroprudential regulation not only affects aggregate risk but also interacts with the efficient allocation of capital. Several recent papers have highlighted the importance of capital allocation and its contribution to aggregate TFP.\(^2\) This paper links these strands by providing a theoretical framework from which to better understand the interplay between minimum capital requirements and the allocation of capital to and across firms.\(^3\)

We use a simple model with asymmetric information that allows us to fully characterize the effects of macroprudential regulation on general equilibrium outcomes. In our model, macroprudential regulation takes the form of minimum capital requirements which limit the maximum permissible leverage ratio of banks. In order to secure capital, firms borrow from banks. We model banks as intermediaries with the ability to accurately assess the risk of firms. They are subject to capital requirements, and have limited liability. Additionally, banks are \textit{ex-ante} homogeneous, so their expected return must be the same. Finally, while banks can differentiate the risk of firms, depositors cannot or choose not to differentiate the risk of banks resulting in a single deposit rate.

This simple framework leads to a surprising result: both the quantity of capital allocated to firms and expected aggregate output are not monotonically decreasing in the minimum capital requirement. In fact, for certain economies, decreasing the minimum capital requirement leads to a decrease in the total capital allocated to firms, an increase in misallocation across firms and a decrease in expected output. This result is consistent with the mixed

\(^1\)Despite this interest, there is no consensus in data or in theory about the effects of augmenting capital requirements: Tanda (2015) reviews the main empirical contributions of capital regulation in the behavior of banks and finds that, in general, decisions of capital and risk seem to be influenced by regulation, but the results are not consistent across time, country or capital analyzed. VanHoose (2007) reviews theoretical studies of bank capital regulation. His review finds mixed predictions on asset risk and of soundness of the banking system as a whole. Among papers which analyze the interaction of macroprudential regulation and risk are Kim and Santomero (1988).


\(^3\)There are papers which consider the interaction of prudential regulation with the aggregate capitalization of firms but we are not aware of work that considers the interaction with misallocation across firms. See Van den Heuvel (2008) and Klimenko et al. (2016) for examples of the former.
empirical findings following revisions to macroprudential regulation observed in the literature. In economies where the financial system is sufficiently developed, capital provided to firms, expected output and welfare are maximized if the banking sector has lax regulation (i.e. low minimum capital requirements). On the other hand, if the banking sector is not sufficiently developed, a smaller minimum capital requirement not only affects systemic risk, but can also lead to a smaller level of firm capitalization as well as lower expected output and welfare.

The intuition for this result is as follows. Limited liability incents banks to diverge across risk profiles. Some banks become riskier than others. A result of depositors not choosing to differentiate among banks is that there is a single deposit rate. This single deposit rate requires less risky banks to subsidize riskier banks in order to access additional capital. As capital requirements are lowered, several things happen. First, a smaller amount of banking equity is required to satiate the demand of risky firms (as additional loans can be financed through more leveraged deposits). This results in a shift of banking equity towards less risky banks which reduces the quantity of deposits needed to satisfy the demand of less risky firms. The combination of more leveraged risky banks and a reduction in the leverage of less risky banks results in an increased risk profile and an increased equilibrium rate on deposits. This larger deposit rate is passed through to firms through the rate on loans which lowers the demand for loans by firms, reduces the capital acquired by firms, and reduces both expected output and welfare. Further, the increased deposit rate exacerbates misallocation across firms as less risky firms face an even larger cost of capital relative to their riskier counterparts.

For economies with a more developed banking sector, sufficiently relaxed capital requirements result in a separating equilibrium and an efficient allocation of resources is obtained. Less risky firms are fully capitalized directly through banks and the rate on deposits is appropriately priced (i.e. no cross bank subsidization is occurring).

Recent work including Restuccia and Rogerson (2008) and Hsieh and Klenow (2014) has demonstrated the ability of misallocation of resources across firms to explain large portions of the differences in TFP observed across countries. Looking at Korean manufacturing plant level data, Midrigan and Xu (2014) find that differences in the average productivity of capital (as opposed to labor or intermediate goods) form the largest share of these misallocations. While there have been many different plausible explanations for these observed differences in the average productivity across plants, there is little work which ties misallocation across

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5Within the framework of our model, a more developed banking sector is a banking sector with a larger amount of banking capital.
6Including Restuccia and Rogerson (2008), Hsieh and Klenow (2014), and Midrigan and Xu (2014) as
firms to macroprudential regulation. This paper combines these two strands of literature by providing a framework from which to begin thinking about the interaction of macroprudential regulation with the misallocation of capital to and across firms in light of information asymmetries between depositors and banks.

There is also a sizeable literature which links the development of the financial sector with misallocation. That information asymmetries can result in quantitatively large misallocations across firms is demonstrated in Greenwood et al. (2010) and Greenwood et al. (2013). Our mechanism also generates misallocation through information asymmetries although the asymmetric information exists between depositors and banks rather than between banks and firms. This information friction faced by depositors causes the expected marginal productivity of capital to vary across firms.

Finally, our paper is linked to the large and growing literature which seeks to understand the effects of macroprudential capital regulation on banks. Most of this literature focuses on the trade-offs between welfare gains achieved through a reduction in the inefficient risk-taking incentives of limited liability and the welfare losses of reduced lending, output and liquidity. In addition to consideration of aggregate output and lending, our work also considers the effects on capitalization across firms through the introduction of heterogeneous firms. In this sense, our work is related to Hill and Perez-Reyna (2016) and Erosa (2001) which consider the effects of banking regulation on occupational choice with the inclusion of firm heterogeneity. Our work builds on this previous work through the introduction of risk to firms.

2 Model

We begin by presenting our benchmark model. Consider a two-period economy with a representative household, banks, and firms. In this economy there are two types of firms that produce a final consumption good: risky and riskless. The production of risky firms depends on the state of the world, while the production of riskless firms does not. Both well as Guner et al. (2008), Banerjee and Moll (2010), Banerjee and Duflo (2005), and Lagos (2006).

7 See for instance Buera et al. (2011), Greenwood et al. (2010), Greenwood et al. (2013)
8 This friction raises an interesting observation relative to standard papers on misallocation (i.e. Restuccia and Rogerson (2008), Hsieh and Klenow (2014), Banerjee and Moll (2010)). Namely, that observed differences in marginal products across firms is not sufficient to demonstrate misallocation. Our model introduces heterogeneous risk across firms. Variation in ex-ante expected marginal productivity is a valid demonstration of misallocation. However, variation in the ex-post observed marginal productivities across firms is not sufficient to demonstrate misallocation. For example, a firm observed to have a higher marginal productivity ex-post may have received a positive idiosyncratic shock relative to a firm with a smaller observed marginal productivity even under an efficient allocation of capital.
types of firms use capital to produce. However, they do not own it, so they must rely on loans from banks. Consequently, total capital available to firms is equal to the aggregate supply of loans from banks. In the first period, no production occurs, the household makes deposits with banks or it invests in the outside option and banks lend to firms. In the second period, firms produce and pay back loans if able, banks pay back on deposits if able, and the household purchases and consumes. We assume that the liability of banks is limited in the standard sense: they will only pay back depositors if their revenues from loans are greater than or equal to the amount they owe on deposits. Each bank is ex-ante homogenous, starts with a fixed amount of equity, and is owned by the representative household. Banks can leverage their capital through deposits in order to lend in excess of their equity, subject to capital requirements.

Specifically, assume that there is a unit measure of both risky and riskless firms. Both types use capital $K$ to produce the final good according to a production function $f(K)$, which satisfies regular assumptions: strictly increasing, strictly concave and satisfies Inada conditions. The only difference between risky and riskless firms is that the production of the former depends on the state of the world. To clarify, assume that risky firms produce $f(K)$ with probability $p$ and produce 0 with probability $1 - p$. We assume that all agents are able to fully diversify across any idiosyncratic risk. Idiosyncratic risk is not compensated in excess of actuarially fair amounts and we abstract from it entirely. Consequently, in this model all risk is aggregate. For expository purposes we will call the state where risky firms produce, “the good state,” and the state where risky firms do not produce, “the bad state.”

We assume that neither type of firm owns capital. Therefore they must borrow it from banks in the first period in order to produce in the second period.\(^\text{10}\) Risky firms will only be able to pay back their loans in the good state, while riskless firms are always able to pay back. Banks can differentiate between types of firms and this implies that the interest rate charged on loans will depend on the firm. We will denote by $R$ the interest rate charged on loans to risky firms and will refer to it as the “risky interest rate.” Similarly, the riskless interest rate, $r$, will denote the rate charged on loans to riskless firms. The problem faced by firms is to maximize profits, given by production minus loan repayment. We will denote the demand of capital for riskless and risky firms by $K_r$ and $K_R$, respectively and defined as:

$$K_r(r) \equiv \arg\max_K f(K) - rK,$$

and

$$K_R(R) \equiv \arg\max_K E[\max\{1_p f(K) - RK, 0\}] = \arg\max_K f(K) - RK.$$  

\(^{10}\)The household is unable to lend directly to firms.
Firms are owned by the representative household. In the good state, total profit from firms is given by \( \pi_f^g \equiv f(K_r) - rK_r + f(K_R) - RK_R \) and in the bad state by \( \pi_b^f \equiv f(K_r) - rK_r \).

Now consider banks. We assume that there is a measure one continuum of ex-ante identical banks. Each bank intermediates capital by taking deposits from the household and lending to firms through a constant returns to scale technology. We assume that all banks are subject to a capital requirement: for every dollar lent, the bank must have at least \( \kappa \) dollars as equity. \( \kappa \) is an exogenous policy decision in the model and later we will analyze what happens as \( \kappa \) changes. We also assume that banks have limited liability; that is, they will only pay back the household as long as the revenue from loans is greater than or equal to amounts owed on deposits. They are not required to pay back on deposits in excess of revenues from loans.

We assume that the household cannot monitor which firms banks are lending to. Therefore the household is offered the same return from all banks. We denote this return by \( r^D \). The household has access to an outside option with exogenous return \( r^* \). These assumptions imply that the supply of deposits is perfectly elastic and that the expected utility from domestic deposits must be equivalent to a risk free return of \( r^* \).\(^{11}\)

Each bank \( i \) is endowed with \( K^B_i \) units of investment capital where \( K^B \) is exogenously given.\(^{12}\) It chooses how much capital to receive via deposits. Then it allocates its total available capital across firms in order to maximize expected investor utility. Formally, bank

\(^{11}\)The representative household is perfectly diversified across banks and the risk it bears on deposits is aggregate risk.

\(^{12}\)We assume that the cost for banks of raising additional equity is high enough that they keep their equity level at \( K^B \).
i’s problem is given by

$$\max_{K^B, K^D} pU(C_g) + (1 - p)U(C_b)$$

s.t. $C_g = \pi^f_g + \int \pi^B_g(j) dj$

$$+ \max \{(1 + r)K^L_r + (1 + R)K^L_R - (1 + r^D)(K^D_r + K^D_R), 0\}$$

$$+ (1 + r^D)K^D \psi_g + (1 + r^*) (K^* - K^D)$$

$C_b = \pi^f_b + \int \pi^B_b(j) dj$

$$+ \max \{(1 + r)K^L_r - (1 + r^D)(K^D_r + K^D_R), 0\}$$

$$+ (1 + r^D)K^D \psi_b + (1 + r^*) (K^* - K^D)$$

$$K^B_r + K^D_r \geq K^L_r, \quad K^B_R + K^D_R \geq K^L_R$$

$$K^B_r + K^D_r \leq K^B, \quad K^B \geq \kappa (K^L_r + K^L_R)$$

$$K^L_r, K^D_r, K^L_R, K^D_R \geq 0,$$

where $C_g$ represents the consumption of the representative consumer in the good state and $C_b$ in the bad state involving the integration across banks $j$. $K^*$ is the available capital the household has to deposit or invest in the outside option. Finally, $\psi_s$ is the fraction of banks which repay deposits in state $s$. Consumption in each state is given by the total wealth of the household including profits of firms and banks. Denote by $K^B_r(i), K^D_r(i), K^B_R(i), K^D_R(i)$ the choice variables which solve the above problem. $K^B$ denotes bank equity and $K^D$ denotes deposits. Subscripts $R$ and $r$ stand for resources related to loans to risky and riskless firms.

The profit of bank $i$ in the good state is

$$\pi^B_g(i) \equiv K^L_r(i)(1 + r) + K^L_R(i)(1 + R) - (K^D_r(i) + K^D_R(i))(1 + r^D).$$

Similarly the profit of bank $i$ in the bad state is

$$\pi^B_b(i) \equiv \max \{K^L_r(i)(1 + r) - (K^D_r(i) + K^D_R(i))(1 + r^D), 0\},$$

since risky firms don’t pay back their loans in this state. Finally, note that all banks are ex-ante identical and consequently each bank selects portfolio which offers the same expected utility to investors.

The representative household owns the banks and firms and is exogenously endowed with additional capital in the amount $K^*$ with which to either deposit with banks or to invest in
the outside option. In the second period it consumes all wealth.

We now define an equilibrium in our economy:

**Definition 1.** Given outside option return $r^*$, household capital endowment $K^*$, banking capital requirement $\kappa$ and banking equity $K^B$, an equilibrium in this economy is interest rates $\{r, R, r^D\}$, loans to riskless and risky firms, $K_r$ and $K_R$, as well as capital, loan and deposit allocations for each bank $i$, $\{K^B_r(i), K^L_r(i), K^D_r(i), K^B_R(i), K^L_R(i), K^D_R(i)\}$, such that

1. firms maximize profits;

2. banks maximize investor utility;

3. the household is indifferent between depositing in banks or investing in the outside option;

4. loan markets clear

$$\int K^L_r(i)di = K_r$$
$$\int K^L_R(i)di = K_R;$$

Notice that banks will find it profitable to lend positive amounts of capital to both types of firms in every equilibrium since the production function of the firms satisfies Inada conditions. Additionally, every bank offers the same expected utility to the household. Since risky firms only produce in one state, then in equilibrium it must be the case that $R > r$.

We will now argue that in equilibrium there are only going to be two types of banks: banks that exclusively lend to riskless firms, and banks that exclusively lend to risky firms. We will refer to these as riskless and risky banks.

To understand this result, consider a bank that lends to risky firms. Risky firms do not produce in the bad state, so this bank cannot use revenues from risky loans to pay back on deposits in this state. Any dollar lent to a riskless firm will imply one less dollar that can be lent to a risky firm, and it will not earn any extra income in the bad state: the revenue from riskless loans would have to be used to pay back on deposits in the bad state leaving the amount remaining for the bank unchanged. Further, since $R > r$ it is strictly preferred to lend to risky firms in the good state. We formally prove this in the following proposition.

**Proposition 1.** As long as there are banks that take deposits, in equilibrium there will only be two types of banks: banks that only lend to risky firms and banks that only lend to riskless firms. If no bank takes deposits, then banks are indifferent between any loan portfolio.
Proposition 1 allows us to characterize the equilibrium. In equilibrium banks maximize investor utility. Since the production function of firms satisfies Inada conditions, in equilibrium both types firms will produce, so both risky banks and riskless banks will exist. Since banks are ex-ante homogenous, the expected utility on the margin of one more risky bank must equal the expected utility of one more riskless bank. That is,

\[(pU'(C_g) + (1 - p)U'(C_b)) (1 + r^B_r) = pU'(C_g)(1 + r^B_R). \tag{2}\]

In words, an additional unit of capital invested with a riskless bank has a gross return equal to 1 + \(r^B_R\) in every state. On the other hand, an extra unit of capital invested in a risky bank has a gross return equal to 1 + \(r^B_R\), but only in the good state.

If \(R > r^D\), a risky bank will demand as many deposits as possible, so the capital requirement will bind. If \(R = r^D\), the bank will be indifferent between demanding any amount of deposits up to the capital requirement. If \(R < r^D\) the bank will not take any deposits. Similar relations hold for riskless banks. Therefore we can express \(r^B_r\) and \(r^B_R\) as

\[1 + r^B_r = \begin{cases} 1 + r^D + \frac{r-r^D}{\kappa} & \text{if } r > r^D \\ 1 + r & \text{if } r \leq r^D \end{cases}\]

\[1 + r^B_R = \begin{cases} 1 + r^D + \frac{R-r^D}{\kappa} & \text{if } R > r^D \\ 1 + R & \text{if } R \leq r^D. \end{cases}\]

Additionally, in equilibrium households are indifferent between depositing in banks or in their outside option. Since deposits in risky banks are only paid in the good state, we can express this condition as

\[(pU'(C_g) + (1 - p)U'(C_b)) (1 + r^*) = pU'(C_g)(1 + r^D) + (1 - p)U'(C_b)(1 + r^D) \frac{K^D}{K^B}. \tag{3}\]

Note that \(\frac{K^D}{K^B}\) is the ratio of deposits with riskless banks. These are the only banks that will repay in the bad state.

To simplify the exposition of our setup we assume that households are risk neutral. However our qualitative results still hold under the assumption of risk aversion (see Section 4.1). In this case, analyzing production is equivalent to analyzing consumption with the only difference being a constant. This allows us to use consumption/welfare and production interchangeably in our analysis of results. To demonstrate this, recall that in this economy
the household consumes all wealth at the end of the second period. The household’s wealth includes their endowed capital, profit from banks and firms as well as any return on deposits and investments taking into account that in the bad state only riskless banks have positive profits and pay back depositors. Specifically, the household consumes

\[ C_g = \pi^f_g + (1 + r^B_R)K^B_R + (1 + r^B_r)K^B_r + (1 + r^D)(K^D_r + K^D_R) + (1 + r^*) (K^* - K^D_r - K^D_R) \]

in the good state and

\[ C_b = \pi^f_b + (1 + r^B_r)K^B_r + (1 + r^D)K^D_r + (1 + r^*) (K^* - K^D_r - K^D_R) \]

in the bad state.

We can express welfare as

\[ C \equiv pC_b + (1 - p)C_g \]

\[ = p\pi^f_b + \pi^f_g + p(1 + r^B_R)K^B_R + (1 + r^B_r)K^B_r + (1 + r^D)(pK^D_R + K^D_r) + (1 + r^*) (K^* - K^D_r - K^D_R) \]

On the other hand, total production in the economy is given by production from the firms minus capital that is “used up” in the process and adding any returns on investment in the outside option. By “used up” we account for the fact that in the bad state, not only is nothing produced by bad firms but the initial capital investment disappears.

\[ Y = pf(K_R) - (1 - p)K_R + f(K_r) + r^* (K^* - K^D) \]

Lemma 1 summarizes this result.

**Lemma 1.** \( Y = C - (K^B + K^*) \).

**Proof.** The proof follows from the definition of \( \pi^f_g \) and \( \pi^f_b \) and from

\[ (1 + r^B_r)K^B_r = (1 + r)K_r - (1 + r^D)K^D_r \]

\[ (1 + r^B_R)K^B_R = (1 + R)K_R - (1 + r^D)K^D_R \]

\[ \square \]
3 Results

3.1 An overview of results

The aim of our model is to understand the interplay between capital requirements and the efficient allocation of capital to firms. In our benchmark model, we use risk neutral agents in order to isolate the effects of capital requirements on the allocation of capital to and across firms. In extensions, we revise preferences to incorporate risk aversion and to incorporate risk weighted capital requirements.

The optimal capitalization of firms occurs when their expected marginal product of capital is equated across firms and is equal to the return on the outside option for capital. The most surprising result of our paper is that even in the risk neutral setting, the level of misallocation is not monotonically increasing in the tightening of capital requirements. In other words, it is possible that tightening capital requirements not only increases equity ratios for risky banks reducing systemic risk, but it can also decrease misallocation. Our model isolates the effects of macroprudential regulation on misallocation and implies that the optimal minimum capital requirement is a function of the capital in the banking sector $K^B$. We characterize this finding by splitting economies into groups in increasing order of $K^B$.

Our result implies that there is not a single policy in $\kappa$ that minimizes misallocation across all $K^B$. Economies with small quantities of capital in the banking sector $K^B$ minimize misallocation and achieve the greatest expected output by setting $\kappa$ to be neither too small nor too large. Expected output is non-monotonic in $\kappa$ and the maximum achievable output occurs at an intermediate value. The reason for not setting $\kappa$ too small is straightforward in that it results in firms being undercapitalized. The reason for not setting $\kappa$ too large is less intuitive. It arises from the general equilibrium effect on the deposit rate. The deposit rate is a function of two things, the rate on the outside option and the risk of domestic deposits. In our model, the rate on the outside option is exogenous and fixed. As the capital requirement is relaxed there comes a point where risky banks continue to increase their leverage ratios but riskless banks are satiated. This results in an increased fraction of deposits being supplied to risky firms. Consequently, the risk of domestic deposits is increased which causes the rate on deposits to rise. This increased rate on deposits is passed through in the rate on loans to firms and also results in firms being undercapitalized relative to the first best allocation.

For economies with an intermediary range of capital in the banking sector $K^B$, the greatest expected output is achieved by setting $\kappa$ to any value sufficiently low. In addition to the composition effect outlined in the preceding paragraph, a second effect is occurring as $\kappa$ is relaxed (decreased). Namely, a smaller fraction of banking equity is required to satisfy
the demand for loans to risky firms and more banking capital is diverted to supplying riskless firms. If there is sufficient banking capital $K^B$, then for low enough $\kappa$, enough capital is directly supplied to riskless firms to drive the rate on loans $r$ below the rate on deposits and riskless banks fully stop accepting deposits. Instead riskless firms are directly funded through banking equity. At this point, the composition effect is no longer occurring and the deposit rate is fixed. Further relaxing of $\kappa$ results in additional banking capital being supplied to riskless firms which pushes the economy towards the efficient capitalization rate.

The final group of economies has sufficient domestic capital in the banking sector to optimally capitalize their firms without any additional deposits. In these economies, there are no frictions or informational issues since banks can differentiate the risk of firms and do not need to accept any deposits to optimally finance firms. This group achieves the efficient outcome and is unaffected by the choice of $\kappa$ since it is not a binding constraint. Further explanation is included below.

3.1.1 Sources of Misallocation

Misallocation in our model can be decomposed into two categories; namely, internal and external misallocation. Internal misallocation arises when the capital invested in domestic firms is split sub-optimally between riskless and risky firms. In the optimal allocation, capital is split so that the expected marginal return on investment between each type of firm is equated. In any equilibrium, the marginal product of each type of firm can be determined directly from the interest rates on loans to risky and riskless firms, $R$ and $r$. Specifically, the expected marginal return of a unit investment in a risky firm at interest rate $R$ is $(1 + R)p$ and the marginal return of a unit investment in a riskless firm at interest rate $r$ is $(1 + r)$. If these two expected marginal returns are not equal, then output could be increased by shifting resources from the firm with a lower marginal output to the firm with higher marginal output and internal misallocation is occurring. Our measure of internal misallocation, $\mu_I$ is defined as

$$\mu_I = |(1 + r) - (1 + R)p|. \quad (4)$$

There are two sources of internal misallocation. The first one is that banks might allocate too much capital to one type of firm and too little to the other type. This arises from the informational asymmetry in our model. Banks are able to differentiate the risk of firms but depositors cannot differentiate the risk of banks. While banks can differentiate risk, in some cases they do not have sufficient capital to optimally supply firms apart from deposits. Since depositors cannot differentiate the risk of the banks they deposit with, the rate on deposits
is uniform across banks. As long as both types of banks are accepting deposits, the risky banks receive a benefit through limited liability, since riskless banks implicitly share part of the burden of compensating investors for the bad realizations of risky banks.

Since risky and riskless banks earn the same expected return per unit of capital invested, the benefit arising from limited liability to risky banks is passed through to risky firms in the form of a lower interest rate on loans. Conversely, since riskless banks share the cost of default of risky banks through a higher interest rate on deposits, they pass through this additional cost to riskless firms in the form of a higher interest rate on loans. As a result, in any equilibrium where both types of banks are accepting deposits, too much capital is allocated to risky firms and too little to riskless firms compared with the optimal split of capital between risky and riskless firms.

A second cause for internal misallocation comes as a result of the differences in the leverage ratios across types of banks. In certain equilibria, risky bankers enjoy a margin of intermediation between the rate they lend at and their deposit rate whereas riskless bankers do not. Since bankers are free to choose whether to lend to riskless or risky firms, the expected return per unit of capital lent to riskless firms must be larger than the expected return per unit of capital lent to risky firms as risky bankers are also enjoying benefits from leverage.

The second source of misallocation is external misallocation, $\mu_E$. External misallocation occurs when the marginal return of capital in the outside option is not equal to the expected marginal product of capital used by domestic firms. External misallocation is defined as the difference between the expected return from domestic firms and the outside return on capital. Similar to internal misallocation, if these expected returns are not equal, output could be increased by moving capital from the source providing the lower expected return to the source providing the higher expected return. The expected return from domestic firms is calculated using a weighted average of the marginal productivities in domestic firms. The formula for external misallocation is given in equation (5). In any equilibrium where banks are earning returns on banking capital in excess of the rate on the outside option, external misallocation is occurring.

$$\mu_E = \frac{p(1 + R)K_R + (1 + r)K_r}{K_R + K_r} - (1 + r^*)$$

(5)

3.2 Five Cases for the Equilibrium

In order to solve for the equilibrium, we begin by analyzing the supply and demand for deposits. We observe that there are five possible cases for the way deposits are allocated
among banks. Each case is identified by the relationship between three endogenous interest rates; namely $r^D$ the interest on deposits that make a depositor indifferent between depositing in a domestic bank and depositing in the outside option, $R$ the interest paid on loans to risky firms, and $r$ the interest paid on loans to riskless firms. The supply of deposits is perfectly elastic at $r^D$. Demand for deposits is a five section downward step function. An example demand curve for deposits is provided in Figure 1. Each line of the step function corresponds to one of the cases below. As a final introductory note to this section, it is useful to recall that the interest rate paid on loans by risky firms $R$ is always greater than the interest rate paid on loans by riskless firms $r$.

In case 1, the rate for deposits is larger than the loan rate to risky firms. In this case there is no demand for deposits since even risky banks would lose on any deposits intermediated. Banks can determine the risk of firms and price loans accordingly and so there are no frictions and the equilibrium is efficient. Banks optimally supply loans to firms and invest any excess capital in the outside option. In this case banks have a supply of capital in excess of demand for loans, limited liability is not a concern, and banks are unable to earn returns on banking capital above the rate on the outside option.

In case 2, $r^D = R$. This implies that risky banks are indifferent towards accepting deposits. Deposits to risky banks can be anywhere between zero and $\frac{1-\kappa}{\kappa}K^R$, which is the

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13 We can solve for $r^D$ by using the equation $1 + r^* = (1 + r^D) \frac{bK^R + K^D}{K^R}$.
14 In this section we are making some partial equilibrium assumptions in order to improve the clarity of exposition. In our general equilibrium solution, all variables $R, r, K^B_R, K^B_r, K^D_R, K^D_r$ and $r^D$ are interrelated.
maximum amount of deposits the risky banks are permitted to accept. The quantity of
loans is pinned down by demand from firms which set marginal productivity equal to $R$. Riskless banks do not accept deposits since $r < R = r^D$. In this sense, case 2 is a separating
equilibrium as only risky banks accept deposits. Since $r^D = R$ there are no profits from
intermediating capital for banks, loans are efficiently priced and again any equilibrium in
case 2 is optimal in the sense that there is no internal or external misallocation occurring.

In case 3 the rate on deposits are between the rate on loans to risky and riskless firms $r < r^D < R$. In this case, risky banks will fully leverage and riskless banks will not accept deposits
since they still lose per unit of intermediation. This results in a perfectly inelastic demand
curve between these interest rates at quantity $\frac{1-\kappa}{\kappa} K_R^B$. Even though this is a separating
equilibrium in the sense that only risky banks accept deposits and must fully compensate
depositors for their risk, the supply of loans to firms is constrained by $\kappa$ and the quantity
of capital in the banking sector $K_R^B$. This causes the equilibrium risk adjusted rate on loans
to risky firms to be driven above the rate on the outside option and the quantity of loans to
risky firms to be inefficiently small. Due to the indifference condition across banks, riskless
banks must also be earning a return in excess of the outside option which implies loans to
riskless firms are also below the efficient level. Therefore, in case 3, external misallocation
is occurring. Further, since the rate on deposits is greater than the rate on loans to riskless
firms, riskless banks are not able to leverage their capital. Consequently, in order to incent
banks to lend to riskless firms, they must receive larger excess returns on loans to compensate
for inability to leverage. This results in a difference in the expected marginal productivity
of capital across firms, which is internal misallocation by definition.

In cases 4 and 5 $r^* < r^D \leq r < R$ which implies that both types of banks accept
deposits. As discussed in section 3.1.1, internal misallocation exists in any scenario where
this occurs and the rate on loans to riskless firms is greater than $r^*$. The excessive rate on
loans to riskless firms implies that capital provided directly by riskless banks, as opposed to
capital supplied by depositors and intermediated by banks, provides returns in excess of the
outside option. At minimum, riskless banks can earn a return on their own banking capital
without accepting deposits. As banks are free to choose their loan portfolio, the indifference
condition for banks requires that risky banks also earn a return in excess of the outside
option. As discussed in 3.1.1, this immediately implies that firms are undercapitalized and
external misallocation is also occurring.
3.3 Sample Equilibria Across $\kappa$

An economy with a large amount of capital in the banking sector can optimally capitalize firms without the use of deposits (case 1). By optimally capitalizing firms, we mean that the expected marginal product of capital on firms is equivalent to the outside return on capital $r^*$. Formally, an economy of this type has domestic banking capital $K^B$ which satisfies the following:

$$K_r(r^*) + K_R\left(\frac{1 + r^*}{p} - 1\right) \leq K^B.$$ 

In this economy, $\kappa$ is irrelevant as no bank would be willing to pay above $1 + r^*$ for deposits and all depositors would utilize the outside option. Note that a loan rate of $\frac{1 + r^*}{p} - 1$ sets the expected return for risky banks equal to the return on the outside option. Henceforth we define

$$R^* \equiv \frac{1 + r^*}{p} - 1$$

where $R^*$ is the efficient interest rate on loans to risky firms.

An economy with an intermediary amount of capital in the banking sector can span four cases as $\kappa$ progresses from 0 to 1. This is the most cases any economy can span by only changing the minimum capital requirement $\kappa$. We use this example throughout the remainder of this section since it provides insight across the largest number of cases. Formally, any economy with capital in the banking sector which satisfies

$$K_r(r^*) \leq K^B < K_r(r^*) + K_R(R^*)$$

is an economy which can span the 4 cases.

In Figure 2 we graph welfare against $\kappa$. Throughout the remainder of this section we will assume a standard decreasing returns to scale production function of the form $Y = zK^\alpha$. For sufficiently relaxed constraints on minimum capital requirements $\kappa$, the economy is in case 2. Risky banks are indifferent between accepting deposits or not and so the minimum capital requirement is not binding. The interest rate on loans to riskless firms $r$ is less than the rate on deposits which implies that riskless banks do not accept deposits. As the constraint is not binding on either type of bank, the resulting equilibrium allocation is equivalent for any $\kappa$ small enough to reach case 2. As explained earlier, this allocation is efficient from a social planners perspective.

There is a cutoff $\kappa_{23}$ above which the minimum capital requirement is binding for risky banks, and above which case 3 begins. The second condition for case 3 is that $r^D > r$.

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15With risk neutral utility, this is equivalent to graphing output against $\kappa$. 
This condition implies that riskless banks will not accept deposits. As all deposits are with risky banks, \( r^D = \frac{1+r^*}{p} - 1 \), the rate which makes depositors indifferent between depositing and using the outside option. For \( \kappa > \kappa_{23} \), demand for loans is larger than the supply of loans at the risk adjusted deposit rate \( r^D = \frac{1+r^*}{p} - 1 \) and so \( r^D < R \) in equilibrium and all risky banks maximize profits by fully leveraging. In order to incent banks to lend to riskless firms and satisfy the indifference condition on banks it must also be the case that \( r > r^* \). In this case, the expected marginal productivity of capital rises above the outside option as can be seen in Figure 3a and so external misallocation exists. Further, risky banks are able to leverage while riskless banks are not. The ability of risky banks to leverage lowers their required return on loans relative to riskless banks and internal misallocation also occurs as can be seen in Figure 3b.

There is a second cutoff \( \kappa_{34} \) above which the limited access to loans for riskless firms drives \( r \) to or above the deposit rate \( r^D \). At this point, riskless banks begin to accept deposits and as long as \( r = r^D \) the economy is in case 4. As riskless banks accept deposits, the probability of default by banks is reduced and there is a general equilibrium effect on the deposit rate. Within case 4, increases in the capital requirement \( \kappa \) increase the fraction of deposits with riskless banks relative to risky banks and lowers \( r^D \). As the minimum capital constraint for riskless banks in not binding in case 4, the supply of loans to riskless firms is perfectly elastic.
at $r = r^D$ with the counterintuitive result that increased regulation in $\kappa$ increases access to loans for riskless firms and consequently lowers misallocation, as can be seen in Figure 3.

If $\kappa$ increases sufficiently there comes a point when the constraint on riskless banks is also binding and consequently the interest rate to riskless firms $r$ is driven above the deposit rate. In case 5 both loan rates $r$ and $R$ are greater than the rate on deposits $r^D$, which implies that both types of banks will choose to hold the minimum capital requirements (in other words, to leverage fully.) This can be observed in Figure 4 where deposited capital is decreasing as the minimum capital requirement $\kappa$ is tightened and is equal to $\frac{1-\kappa}{\kappa} K^B$. In other words, within case 5 the total capital supplied to firms is directly tied to $\kappa$. Increases in $\kappa$ directly reduce capital available to firms and lead to increases in external misallocation.

We now analyze internal and external misallocation. Figure 5 shows $\mu_I$ and $\mu_E$ that arise from the equilibria displayed in Figure 2. Most notable is the fact that both internal and external misallocation are rising when the minimum capital requirement $\kappa$ is tightened in case 4.

In cases 1 and 2, risky banks fully internalize their risk since no riskless banks subsidize the deposit rate. Further, risky banks do not receive the benefit of a positive margin of intermediation so the second source of internal misallocation is also non-existent in these cases. Therefore, cases 1 and 2 do not experience any internal misallocation. In cases 3 and 4, risky banks enjoy a positive margin of intermediation while riskless banks do not. This implies risky banks are able to accept a lower expected return through leveraging and internal misallocation is occurring. In cases 4 and 5, both types of banks are accepting
Figure 4

(a) Risky Capital

(b) Riskless Capital

Figure 5: Misallocation across $\kappa$
deposits which implies that subsidization of risky banks is occurring, an additional cause of internal misallocation.

To summarize our results on internal misallocation $\mu_I$:

1. Cases 1 and 2: $\mu_I = 0$ for these cases.

2. Case 3: $\mu_I \geq 0$ and $\frac{\partial \mu_I}{\partial \kappa} \geq 0$ within this case.

3. Cases 4 and 5: $\mu_I > 0$ and $\frac{\partial \mu_I}{\partial \kappa} \leq 0$ within these cases.

External misallocation is not occurring in cases 1 and 2. In case 1, banks do not intermediate funds, they directly supply funds to firms with perfect information and there is no friction. They lend to each type of firm in the quantity that equates the expected marginal return on capital with the firm to the outside option and place any additional capital in the outside option. In case 2, only risky banks accept deposits. This separating equilibrium gives full information to depositors who demand full compensation for the risk involved with depositing with risky banks who in turn demand full compensation from risky firms. In case 2, there is sufficient banking capital to optimally capitalize riskless firms directly and so there is no external misallocation. In cases 3 through 5 the leverage constraint is binding for at least some of the banks and there is underinvestment in the domestic firms.

To summarize our results on external misallocation $\mu_E$:

1. Case 1: $\mu_E < 0$ and $\frac{\partial \mu_E}{\partial \kappa} = 0$ within this case.

2. Case 2: $\mu_E = 0$ and $\frac{\partial \mu_E}{\partial \kappa} = 0$ within this case.

3. Cases 3 and 5: $\mu_E \geq 0$ and $\frac{\partial \mu_E}{\partial \kappa} > 0$ within these cases.

4. Case 4: $\mu_E \geq 0$ and $\frac{\partial \mu_E}{\partial \kappa} < 0$ within this case.

Additional intuition for misallocation can be obtained by focusing on the behavior of equilibrium interest rates across $\kappa$. Figure 6 displays equilibrium interest rates across $\kappa$ for this example. In this figure, instead of graphing $R$ we graph $(1 + R)p - 1$ which is equal to the expected marginal product of capital at risky firms. Internal misallocation is given by the difference in the expected marginal product of capital across firms. This can be observed in the figure as the difference between the solid black line, $r$, and the dotted purple line, $(1 + R)p - 1$. As long as the weighted average marginal product of capital used domestically is greater than $1 + r^*$, the economy is experiencing external misallocation.

A final measure of misallocation is the difference between the optimal capitalization of firms and the actual capital supplied to firms. This can be observed in Figure 4 which
Figure 6: Interest Rates across $\kappa$

$displays the equilibrium capital allocations across $\kappa$ and the optimal capital amount. The capital allocated to risky and riskless firms can be decomposed into capital from deposits $K^D$ and capital supplied directly from the bank $K^B$.

### 3.4 Optimal Policy in $\kappa$

As observed in section 3.3, welfare is not monotonic in $\kappa$. Consequently the optimal policy in $\kappa$ is dependent on the amount of equity in the banking sector $K^B$ relative to the amount of capital required to optimally capitalize firms. For economies with large amounts of banking capital, setting $\kappa$ to be small is optimal, for economies with smaller amounts of banking capital, a more restrictive capital requirement can actually increase aggregate output compared with a more relaxed policy.

As outlined in prior sections, for economies with sufficient banking capital to fully capitalize firms without deposits, $\kappa$ has no effect on outcomes and the selection of $\kappa$ is irrelevant. For economies with insufficient banking capital to fully capitalize firms, there are two policies in minimum capital requirements $\kappa$ which locally maximize welfare and are candidates for the globally optimal policy. Specifically, the candidates for optimal policy are setting $\kappa$ to zero or to select $\kappa$ such that the economy falls on the border of cases 4 and 5.
Figure 7 displays the welfare generated by selecting these two $\kappa$ which result in local maxima across a range of $K^B$. The labeled ranges refer to the resulting case when $\kappa = 0$.

Throughout the remainder of this section it is useful to provide additional notation in $K^B$ in order to increase the clarity of exposition. Economies can be classified based on their banking capital $K^B$ by the resulting case when capital requirements $\kappa$ are removed (i.e. $\kappa = 0$.) We denote $K^B_{ab}$ as the banking capital which places the economy on the border of cases $a$ and $b$ in the absence of capital requirements.\footnote{Formally, $K^B_{12} = K_r(r^*) + K_R(R^*)$, $K^B_{23} = K^B \leq K_r(r^*)$, $K^B_{34} = K^B = K_r \left( \frac{1+r^*}{p} - 1 \right)$, and $K^B_{45} \rightarrow 0$.}

For economies in the class of the sample economy of section 3.3, case 2 is obtainable for any sufficiently small $\kappa$. Since there is no misallocation in this case, this is the optimal policy in terms of capital allocation. Specifically, this scenario occurs for $K^B$ which satisfy $K^B_{23} \leq K^B \leq K^B_{12}$ and any policy in $\kappa$ where $\kappa \leq \frac{K^B - K_r(r^*)}{K_R(R^*)}$ achieves the optimal welfare. These economies constitute the area labeled, “range 2” in Figure 7.

If $K^B < K^B_{34}$. Then, setting $\kappa$ to zero will result in an equilibrium in case 4. Cases 2 and 3 are both unattainable. In this scenario the optimal policy is to set $\kappa$ on the border of case 4 and 5 which we will call $\kappa_{45}$.\footnote{Formally, $\kappa_{45}$, $R$, and $r$ simultaneously solve $K^B = (K_r(r) + K_R(R)) \kappa$, $(1 + r^*) = (1 + r)\kappa \frac{K_r(r) + K_R(R)}{K^B}$, and $p \left( 1 + r + \frac{R - r}{\kappa} \right) = 1 + r$ where the first equation is the minimum capital constraint, the second is the indifference condition for depositors and the third is the indifference condition for banks.} These economies constitute the area labeled, “range 4” in Figure 7. As can be observed in the figure, the welfare resulting from the more restrictive capital requirement $\kappa_{45}$ is larger than when the capital requirement is removed ($\kappa = 0$).

For economies with insufficient capital in the banking sector to directly capitalize riskless firms but with sufficient capital to reach case 3 we have no analytical solution for the optimal policy. These economies have banking capital $K^B$ which falls in the range $K^B_{34} < K^B \leq K^B_{23}$. Numerical results in this range suggest that there is a single cutoff for an economy in $K^B$, above which setting $\kappa = 0$ is optimal and below which setting $\kappa = \kappa_{45}$ is optimal from an aggregate welfare perspective. This idea is demonstrated graphically in range 3 of Figure 7.

### 4 Extensions

In this section we consider a number of extensions to our model. We first analyze our model when the household is risk averse and demonstrate that our results are not dependant on the
assumption of risk neutrality. Next we introduce deposit insurance. In our model, if both types of banks accept deposits in equilibrium, then riskless banks are subsidizing risky banks through the deposit rate. By including deposit insurance we highlight that our result does not rely on this subsidy. Finally, we consider risk weighted capital requirements. One of the main outcomes from the Basel Accords is that capital requirements depend on the riskiness of assets. With this extension we analyze the effects on output when capital requirements don’t reflect correctly the riskiness of loans.

4.1 Risk aversion

In this extension we assume that households are risk averse. In particular, we assume that $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and analyze what happens as $\sigma$ changes. Our baseline scenario occurs when households are risk neutral, (i.e. $\sigma = 0$.)

Figure 8 shows welfare across $\kappa$ for different values of $\sigma$. The continuous black line is generated from our benchmark model. We show welfare for two levels of risk aversion: $\sigma = 2$ (blue-dashed line) and $\sigma = 4$ (red-dotted line). To compare welfare for different levels of risk aversion, in this figure we show the percentage of equivalent consumption that is attained for each value of $\kappa$, relative to the maximum equivalent consumption attained. That is, for
each level of $\kappa$ we calculate $\tilde{C}(\kappa; \sigma)$ such that

$$U\left(\tilde{C}(\kappa; \sigma)\right) = E\left[U\left(C(\kappa; \sigma)\right)\right]$$

and we graph

$$\frac{\tilde{C}(\kappa; \sigma)}{\max_{\kappa'} \tilde{C}(\kappa'; \sigma)} \times 100.$$ 

We observe from Figure 8 that our main result is not dependant on the assumption of risk neutrality: Welfare is non-monotonic over $\kappa$ across different levels of risk aversion. In the figure we parameterize the economy such that $K^B$ is large enough so that setting $\kappa \to 0$ maximizes welfare. As explained in Section 3, if $K^B$ wasn’t as large, welfare would be maximized by setting $\kappa = \kappa_{45}$.

Despite the similarities, we do observe some changes when analyzing our model across different levels of risk aversion. An deeper analysis of these differences is provided in the Appendix.

### 4.2 Deposit insurance

We now consider a scenario with deposit insurance. In this revised economy, our primary results remain unchanged compared to the risk neutral case: welfare is non-monotonic over $\kappa$. Deposit insurance is set so that the household gets its deposit plus interest back in both states. As a result, deposits are riskless, so in equilibrium $r^D = r^*$. We assume that there is a banking authority that manages the insurance. To be able
to pay for the insurance, the authority taxes banks at a fixed proportion $\tau$ of deposits accepted. If the authority was able to accurately observe the risk of a bank, it would only tax risky banks removing the information asymmetry. Therefore, we assume that the banking authority taxes all banks equally.

In this economy, the riskless bank’s problem is

$$\max_{K_r^L, K_r^D, K_r^B} (1 + r)K_r^L - (1 + r^D + \tau)K_r^D$$

s. t. $K_r^L = K_r^D + K_r^B$

$$K_r^B \geq \kappa K_r^L$$

and the risky bank’s problem is

$$\max_{K_R^L, K_R^D, K_R^B} p((1 + R)K_R^L - (1 + r^D + \tau)K_R^D)$$

s. t. $K_R^L = K_R^D + K_R^B$

$$K_R^B \geq \kappa K_R^L.$$

The return from investing in a bank is

$$1 + r^B = \begin{cases} 
1 + r^D + \tau + \frac{r - r^D - \tau}{\kappa} & \text{if } r > r^D + \tau \\
1 + r & \text{if } r \leq r^D + \tau
\end{cases}$$

$$1 + r^R = \begin{cases} 
1 + r^D + \tau + \frac{R - r^D - \tau}{\kappa} & \text{if } R > r^D + \tau \\
1 + R & \text{if } R \leq r^D + \tau
\end{cases}$$

In equilibrium, $\tau$ has to be equal to the deposits that risky banks are unable to pay back in the bad state:

$$\tau = \frac{1}{W}(1 - p)(1 + r^D)\frac{K_R^D}{K^D},$$

where

$$W \equiv \frac{K_r^D + pK_R^D}{K^D}$$

accounts for the fact that risky banks are only able to pay $\tau$ in the good state. Figure 9 shows $\tau$ for different values of $\kappa$. $\tau$ is largest for cases 2 and 3, since in these cases only risky banks are accepting deposits. Because risky banks are only able to pay depositors back in the good state, $\tau$ needs to be higher to provide a return of $r^*$ on deposits. As $\kappa$ increases, riskless banks also begin to accept deposits, and $\tau$ decreases.
4.3 Risk-weighted capital

In the 2008 recession it became clear that risk-weights did not reflect actual risk. In fact, the low risk-weighting assigned to mortgage backed securities certainly contributed to the accumulation of certain types of risk before the mortgage crisis in the US. If risk-weights do reflect actual risk it must be the case that the government and other institutions are able to identify risky firms and banks. As long as there is a way for this information to be communicated to depositors, then in the context of our model the information asymmetry would disappear, interest rates would accurately reflect risk and the optimal allocation would be achieved.

We explore another possibility in this section. Specifically, suppose risk-weights do reflect actual risk but that depositors are still unable to differentiate between the risk of banks. We include this in our model as having different capital requirements for risky and riskless loans. Our result that banks fully specialize does not change, so this is equivalent to having different capital requirements for risky and riskless banks, which we denote by $\kappa_R$ and $\kappa_r$. As a result, we can re-write problem of riskless banks as

$$\max_{K_r^L, K_r^D, K_r^B} (1 + r)K_r^L - (1 + r^D)K_r^D$$

s. t. $K_r^L = K_r^D + K_r^B$

$K_r^B \geq \kappa_r K_r^L$
and the problem of risky banks as

\[
\max_{K^L_R, K^D_R, K^B_R} p \left( (1 + R)K^L_R - (1 + r^D)K^D_R \right)
\]

s. t. \( K^L_R = K^D_R + K^B_R \)

\( K^B_R \geq \kappa_R K^L_R \).

In the scenario of risk-weighted capital requirements, the return from investing in a bank becomes

\[
1 + r^B_R = \begin{cases} 
1 + r^D + \tau + \frac{r - r^D}{\kappa_r} & \text{if } r > r^D \\
1 + r & \text{if } r \leq r^D
\end{cases}
\]

\[
1 + r^B_R = \begin{cases} 
1 + r^D + \tau + \frac{R - r^D}{\kappa_R} & \text{if } R > r^D \\
1 + R & \text{if } R \leq r^D.
\end{cases}
\]

Using this updated environment we are also able to analyze the impact of incorrectly setting capital requirements. To do this we assume that \( \kappa_r = \mu \kappa_R \) and we vary \( \mu \). Values of \( \mu \) less than 1 imply that banks need more banking capital to lend to risky firms, while \( \mu > 1 \) means that capital requirements are set incorrectly, in the sense that banks require less capital to lend to risky firms than to riskless firms.

Notice that \( \kappa_r \) and \( \kappa_R \) will only play a role as long as there are banks whose capital requirements bind. In other words, if \( K^B \) is high enough so that cases 1 or 2 can be reached, then capital requirements play no role. Regardless of \( \mu \), the production level will be the same. Therefore, in order to analyze the impact of \( \mu \) we consider \( K^B \) low enough so that there is an optimal \( \kappa_R \). As seen in Section 3, this corresponds to analyzing output for \( \kappa_{R,45} \) for each value of \( \mu \). We denote this output by \( Y_{45} \).

Figure 10 shows \( Y_{45} \) for different values of \( \mu \). We can see that output is higher for low values of \( \mu \). That is, if capital requirements for risky loans, \( \kappa_R \), are more strict than for riskless loans (low \( \mu \)), then the optimal output that is reached is higher. However, if capital requirements for risky loans are more relaxed than for riskless loans, then the economy will achieve a lower level of production.

This result highlights the importance of setting up capital requirements in a way that reflects the actual risk of loans. If these requirements are not set correctly, output will be reduced.
5 Conclusion

In this paper we study the macroeconomic effects of banking capital requirements, specifically reviewing the interaction of capital requirements with misallocation. Our model demonstrates that it is theoretically possible to tighten capital requirements and have a resulting equilibrium with a higher average leverage ratio among banks. This counterintuitive result is an outcome of the general equilibrium effects on interest rates. In particular, a higher capital requirement causes the share of deposits in riskless banks to grow, which results in a decrease in the riskiness of deposits and a corresponding decrease in the interest rate on loans to firms. Since the quantity of loans demanded by firms is determined by setting the marginal productivity equal to the rate on loans, this also raises the total loans demanded by firms. Our primary takeaway is that general equilibrium effects are of first order importance and affect qualitative outcomes.

Additionally, using our simple model, we find that the optimal policy for capital requirements is dependent on the available equity in the banking sector. For countries with a relatively undeveloped financial sector, our model indicates that a stricter (larger) capital requirement will result in the largest amount of capital inflow, the greatest productivity, and the highest welfare. For countries in the middle, the optimal policy is a relaxed (low) capital requirement. Finally, for countries with a large amount of domestic capital, allocations are unaffected by the capital requirement. Instead, the optimal allocation arises when banks are freely allowed to lend their capital internationally.

Our main result does not rely on risk neutrality nor on the absence of deposit insurance.
Furthermore, we can use our model to highlight the importance of adequately setting risk-weights.
References


Appendix A Selected proofs

Appendix A.1 Proof of Proposition 1

Consider (1). First, notice that

\[(1 + r)K^L_r + (1 + R)K^L_R > (1 + r^D)(K^D_r + K^D_R)\]

since otherwise the bank could always choose \(K^D_r = K^D_R = 0\) to achieve a higher value of the objective function. Second,

\[K^B_r + K^D_r = K^L_r\]
\[K^B_R + K^D_R = K^L_R\]
\[K^B_r + K^B_r = K^B(i)\]

since the objective function is strictly increasing in \(K^L_r\) and \(K^L_R\).

Third, if \((1 + r)K^L_r \leq (1 + r^D)(K^D_r + K^D_R)\) then the bank is unable to repay depositors in the bad state and its return in this state is 0. If this is the case, there is no reason to lend to riskless firms, since \(R > r\) in equilibrium and it will only receive a positive return in the good state. Therefore, if a bank lends to riskless firms it must hold that \((1 + r)K^L_r > (1 + r^D)(K^D_r + K^D_R)\) or otherwise the bank will choose to lend exclusively to risky firms.

To prove that banks will fully specialize we are only pending to prove that if \((1 + r)K^L_r > (1 + r^D)(K^D_r + K^D_R)\) then the bank will not lend to risky firms. That is, \(K^B_R = 0\). So consider a bank that is lending to riskless firms in an amount sufficient that \(K^L_r > (1 + r^D)(K^D_r + K^D_R)\). Now, the marginal benefit of lending to each type of firm is constant. If this bank could offer the investor a greater expected utility by lending an extra dollar to a risky firm, then the bank would only want to lend to risky firms. But then it cannot be the case that \((1 + r)K^L_r > (1 + r^D)(K^D_r + K^D_R)\). Therefore, if this inequality holds it must be because the bank can offer the investor a greater expected utility by lending to riskless firms. But then there is no reason to lend to risky firms, so \(K^L_R = 0\) and \(K^B_R = 0\).

Appendix A.2 Understanding the Effects of Risk Aversion on Outcomes

The most evident difference as observed in Figure 8 is that for larger levels of risk aversion (larger \(\sigma\)), \(\kappa_{34}\) is higher. In other words, if households are more risk averse, riskless banks require higher levels of \(\kappa\) before they begin to accept deposits in equilibrium.

To understand the role that risk aversion plays in determining \(\kappa_{34}\), we analyze interest rates in equilibrium across \(\kappa\) for different levels of risk aversion, \(\sigma\). Figure 11 shows \(r^D\), \(r\) and \(R\) across \(\kappa\) for different values of \(\sigma\). As we mentioned, \(r\) and \(R\) are increasing for low values of \(\kappa\). At the point when riskless banks begin to accept deposits these interest rates begin to decrease in \(\kappa\).

Beginning with the left panel of Figure 11, notice that \(r^D\) is strictly increasing in \(\sigma\). Recall from Equation (3) that \(r^D\) is determined by an indifference condition: households must be
Figure 11: Interest rates for different $\sigma$’s

-indifferent between depositing with banks and investing in the outside option. Deposit
in banks has some risk, since risky banks are only able to pay back depositors in the good
state. For more risk averse households, $r^D$ must be higher to compensate for the risk of
deposits not being paid back. The greater the risk of deposits, the larger the differences
across $\sigma$. The differences in $r^D$ are greatest when $\kappa$ is small enough to result in only risky
banks accepting deposits (cases 1 to 3).

Shifting attention to the right panel of Figure 11 we analyze the effects of different levels
of risk aversion on the rate of loans to risky firms, $R$. In cases 1 and 2 $R = r^D$, so it makes
sense that $R$ is also increasing in $\sigma$, at least for low levels of $\kappa$. In case 3, $R > r^D$, but we
observe that $R$ continues to increase in $\sigma$. To understand why, recall that in case 3, riskless
banks don’t accept deposits and risky banks have binding capital requirements. Therefore
equation (2) can be written for case 3 as

$$R - r^D = \kappa \left[ \left( p + (1 - p) \left( \frac{C_g}{C_b} \right)^\sigma \right) \frac{1 + r}{p} - (1 + r^D) \right].$$

To understand how sigma affects this equation we plot consumption in the good state
relative to the bad state in Figure 12. Observe that $\left( \frac{C_g}{C_b} \right)^\sigma$ is increasing in $\sigma$. This is because
a higher level of risk aversion drives households to more strongly prefer smooth consumption
across states. As a result, the return from lending to risky firms must be greater for larger
$\sigma$ in order to compensate the lower consumption in the bad state.

Finally, focusing on the center panel of Figure 11, it is worth mentioning that $r$ is not
very sensitive to $\sigma$. As $\sigma$ changes, it is primarily the other interest rates, $R$ and $r^D$, that
adjust in equilibrium. The only major difference we see in $r$ for different values of $\sigma$ arises
from the fact that as $\sigma$ increases, case 4 is reached at a larger $\kappa$. To understand why, we
analyze the border between cases 3 and 4.

At the border of cases 3 and 4 $r = r^D$. We can use Equation (3) to solve for the value of
$\kappa$ that hits this border:

$$\kappa_{3,4}(\sigma) \equiv p \frac{R - r}{1 + r} \left( \frac{C_g}{C_b} \right)^\sigma.$$
As mentioned, $r$ does not explain much of the changes we observe in $\kappa_{3,4}(\sigma)$. The difference is primarily explained by $\left(\frac{C_y}{C_b}\right)^\sigma$ and $R$, both of which are increasing in $\sigma$. 

Figure 12: $\frac{C_y}{C_b}$ for different $\sigma$’s